Applications of an exact counting formula in the Bousso-Polchinski Landscape

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Overview

The Bousso-Polchinski model is a proposal for solving the Cosmological Constant Problem. The simple BP method of counting low Λ states is reviewed and then we give an exact formula with two simple asymptotic regimes (BP and non-BP). We finally give some applications of the extended formula.

Contents

A Huge Discrepancy

Standard Model of Particle Physics: $\rho_{\text{vac}} = \Lambda \approx 1$

$+$

Standard Model of Cosmology: $\Lambda \approx 10^{-120}$

$=$

Cosmological Constant Problem!

We use units in which $8\pi G = \hbar = c = 1$.

The Bousso-Polchinski Model

The BP Landscape [hep-th/0004134] is a finite (but *enormous* $\approx 10^{500}$) subset of

$$\mathcal{L} = \left\{(n_1 q_1, \cdots, n_J q_J) \in \mathbb{R}^J : n_1, \cdots, n_J \in \mathbb{Z}\right\}$$

(a lattice in flux space) where

- The number of fluxes $J = N_3 + 1$ is determined by the number of three-cycles in the compactification manifold.
- The charges $q_1, \cdots, q_J$ are determined by the sizes of the three-cycles.
- A vacuum state λ is characterized by the quantum numbers $(n_1, \cdots, n_J)$.
- The cosmological constant of vacuum λ is ($\Lambda_0 \approx -1$)

$$\Lambda(\lambda) = \Lambda_0 + \frac{1}{2}\sum_{j=1}^{J} n_j^2 q_j^2 = \Lambda_0 + \frac{1}{2}\|\lambda\|^2$$

The BP count

- Each vacuum λ lies at the center of its Voronoi cell, a translate of a fundamental cell Q of volume $\operatorname{vol} Q = \prod_{j=1}^{J} q_j$.
- Each $\Lambda > \Lambda_0$ defines a ball $\mathcal{B}^J(\Lambda)$ of radius $\sqrt{2(\Lambda - \Lambda_0)}$.
- The number of states in the Weinberg Window is computed by dividing volumes:

$$\mathcal{N}_{\text{WW}} = \frac{\operatorname{vol} \mathcal{B}^J(\Lambda_{\text{WW}}) - \operatorname{vol} \mathcal{B}^J(0)}{\operatorname{vol} Q} \approx \operatorname{vol} S^{J-1} \frac{R^{J-2} \Lambda_{\text{WW}}}{\operatorname{vol} Q}$$

with $R = \sqrt{2|\Lambda_0|}$ and $\operatorname{vol} S^{J-1} = \frac{2\pi^{\frac{J}{2}}}{\Gamma\left(\frac{J}{2}\right)}$.

- For large J, the volume of a single cell surpasses the volume of the ball, no matter how small the charges might be:

$$\frac{R^J}{J} \operatorname{vol} S^{J-1} < \operatorname{vol} Q \quad \Rightarrow \quad \frac{Jq^2}{R^2} > 2\pi e \approx 17.079$$

In this regime, volume quotient cannot be used for counting.

Number of nodes inside a sphere

We start using an exact formula for the number of nodes of the lattice $\mathcal{L}$ characterized by the J charges q_i inside a sphere of radius r in flux space,

$$\Omega_J(r) = \sum_{\lambda \in \mathcal{L}} \chi_{[0,r]}(\|\lambda\|)$$

where $\lambda = (n_1 q_1, \cdots, n_J q_J)$ are the nodes of $\mathcal{L}$ and $\chi_I(t)$ is the indicator function for the interval I. Its derivative is the Landscape degeneracy *density*,

$$\omega_J(r) = \frac{\partial \Omega_J(r)}{\partial r} = 2r \sum_{\lambda \in \mathcal{L}} \delta(r^2 - \|\lambda\|^2) \, .$$

Number of nodes inside a sphere

Using the contour integral for the δ:

$$\delta(r^2 - \|\lambda\|^2) = \frac{1}{2\pi i} \int_\gamma e^{s(r^2 - \|\lambda\|^2)} \,\mathrm{d}s$$

for $\gamma = \{c + i\tau : \tau \in \mathbb{R}, c > 0\}$, we have

$$\begin{aligned} \omega_J(r) &= \frac{2r}{2\pi i} \int_\gamma e^{sr^2} \left[\sum_{\lambda \in \mathcal{L}} e^{-s\|\lambda\|^2} \right] \mathrm{d}s \\ &= \frac{2r}{2\pi i} \int_\gamma e^{sr^2} \left[\prod_{j=1}^{J} \vartheta(sq_j^2) \right] \mathrm{d}s \,, \end{aligned}$$

with $\vartheta(s) = \sum_{n \in \mathbb{Z}} e^{-n^2 s} \xrightarrow{s \to 0} \sqrt{\frac{\pi}{s}}$.

Asymptotic approximation

In the small s regime the integral can be done in closed form:

$$\omega_J(r) \approx \frac{2r}{2\pi i} \int_\gamma e^{sr^2} \left[\prod_{j=1}^{J} \sqrt{\frac{\pi}{q_j^2 s}} \right] \mathrm{d}s = \frac{r^{J-1} \operatorname{vol} S^{J-1}}{\operatorname{vol} Q} \,.$$

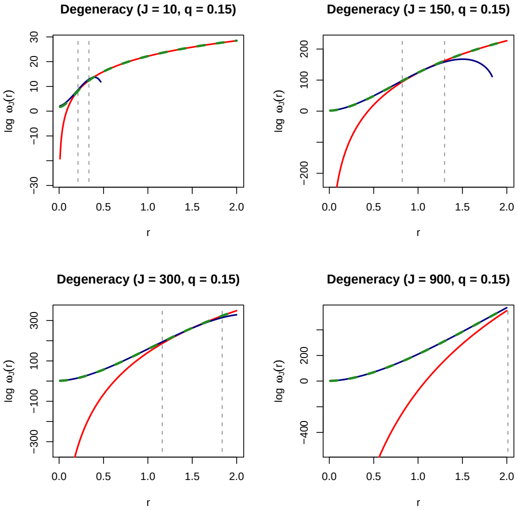
This is the Bousso-Polchinski count. It is valid when the saddle point is in the range of validity of the asymptote:

$$s^* q_j^2 = \frac{J q_j^2}{2r^2} < 1 \,.$$

In the large s regime, the integral shoud be approximated using the saddle point method. The saddle point $u = q^2 s^*$ depends only on the parameter

$$h = \frac{J q^2}{R^2}$$

Asymptotic approximation: large and small distances



Asymptotic regimes of $\omega_J(r)$ for equal charges $q = 0.15$ and different J as functions of r. For small J the large distance (BP count) regime dominates, but for large J the small distance regime spans the whole $[0, 2]$ interval. Vertical dashed lines delimit the crossing regime, which is above $r = 2$ in the $J = 900$ panel.

Number of states in the Weinberg Window

The number of states inside a thin shell $[0, \Lambda_\varepsilon]$ is

$$\Omega_J(R_\varepsilon) - \Omega_J(R) \approx \omega_J(R)\frac{\Lambda_\varepsilon}{R}\,,$$

valid if

$$R_\varepsilon - R = \sqrt{2(\Lambda_\varepsilon - \Lambda_0)} - \sqrt{2|\Lambda_0|} \approx \frac{\Lambda_\varepsilon}{R} \ll R\,.$$

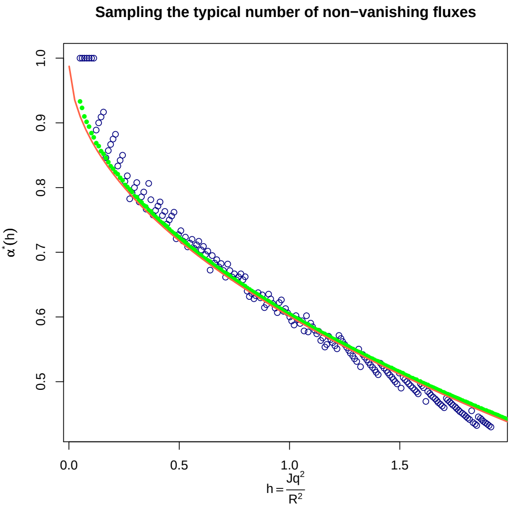
If Λ_ε is the width of the anthropic range (the Weinberg Window), then the number of states in it is

$$\mathcal{N}_{\mathrm{WW}} = \frac{\omega_J(R)}{R}\Lambda_{\mathrm{WW}}\,.$$

Typical number of non-vanishing components

- If a state λ has f_0 vanishing fluxes, the fraction of non-vanishing components is $\alpha(\lambda) = 1 - f_0/J$.
- Numerically we find that $\alpha = 1$ is typical for $J \leq 7$.
- The typical value α^* for large J has maximum probability accounting only abundances of states.
- $P(\alpha)$ is Gaussian around α^* with variance $\approx \frac{1}{4J}$.
- The maximum α^* depends only on $h = \frac{Jq^2}{R^2}$.
- The $\alpha^*(h)$ curve is a robust property of the BP Landscape: generic subsets of the lattice result in analogous $\alpha^*(h)$ curves.
- $\omega_J(r)$ plays an essential role: replacing it by the BP regime results in a $P(\alpha)$ valid only for $h < \frac{8\pi}{27}$ with the same $\alpha^*(h)$.

Typical number of non-vanishing components



Samples of the typical number of non-vanishing fluxes. Two sampling methods have been used: The inside-shell, maximum-frequency method (blue hollow circles) and the inside-ball, average-frequency method (green bullets). The saddle point solution is also shown (red line).

Minimum Cosmological Constant

- Computation of the minimum Λ (Λ^*) for arbitrary J and charges is a difficult combinatorial optimization problem.
- For equal charges $q_i = q$, we have a lower bound:

$$\Lambda^* = \Lambda_0 + \frac{q^2}{2}\sum_{i=1}^{J} n_i^2 \geq \Lambda_\varepsilon = \Lambda_0 + \frac{q^2}{2}\left\lceil \frac{2|\Lambda_0|}{q^2} \right\rceil .$$

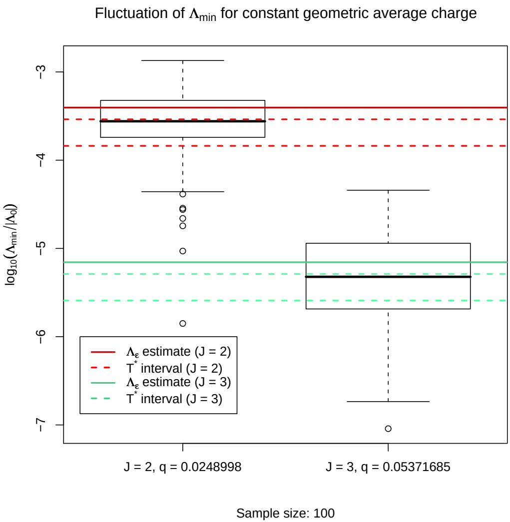
- The agreement with brute-force data is complete for $J > 4$.
- For incommensurate charges, the symmetry degeneracy is typically $2^{J\alpha^*}$, so that

$$\Lambda_\varepsilon \approx \frac{2^{J\alpha^*} R}{\omega_J(R)} .$$

- The "thermodynamic" estimator is more accurate and gives an interval:

$$T^* \in \Lambda_\varepsilon \times \left[e^{-1},\; 2\,e^{-1}\right] .$$

Minimum Cosmological Constant: Estimators



Each box plot represent a sample of 100 choices of J charges with constant geometric mean for which Λ^* has been computed by brute-force search. Thick lines correspond to Λ_ε, and cross inside the boxes in good agreement with numerical data. Dashed lines enclose the values provided by the thermodynamic estimator T^*, which are even better.

Conclusions

- We have expressed the BP Landscape degeneracy in closed form an given its two main asymptotic regimes.
- The Landscape degeneracy has been applied to low-Λ state counting in actual BP models, and in the determination of general properties of these states, as the distribution of non-vanishing fluxes and minimum Λ estimates.

Thank You!

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Table of Contents