Emergence of Space-time Structures

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Talk addressed primarily to non-experts, i.e. the audience outside quantum gravity.

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Main Issues

• The term *emergence* has many facets and is often used rather loosely. In this talk: Physics of interest is emergent if it is formulated in terms of notions which can be derived from a fundamental theory based on entirely different degrees of freedom. In *this* sense: space-time structures of general relativity are almost certainly emergent.

• Standard example: QCD. Fundamental DOF: Quarks and gluons while directly observable physics: Hadrons and leptons.

• But, contrary to a wide-spread belief inspired by QCD, exactly solvable models show that general relativity can still serve as a powerful guide to arrive at the fundamental DOF.

Organization: Discussion through illustrative examples of emergence of space-time structures in fundamental theories.

1. CGHS Model: Emergence of BH Formation and Evaporation
2. AdS/CFT: Emergence of Space-Time from its (almost) Holographic Image.
3. LQG: Emergence of Geometry from Discrete Fundamental Structures.
4. Summary and Discussion.

1. Quantum BHs Emergent Phenomena

• Spherical collapse of a scalar field f in 4-d GR:

Writing ${}^4g_{ab} = g_{ab} + r^2\, s_{ab} \equiv g_{ab} + \frac{e^{-2\phi}}{\kappa^2}\, s_{ab}$, the action reduces to

$$S(g,\phi,f) = \frac{1}{2G}\int d^2x\,\sqrt{|g|}\,\left[e^{-2\phi}\,(R + 2\nabla^a\phi\nabla_a\phi + 2e^{-2\phi}\kappa^2) + Ge^{-\phi}\nabla^a f\nabla_a f\right]$$

• The 2-d Callen-Giddings-Harvey-Strominger (CGHS) Black hole:

$$S(g,\phi,f) := \frac{1}{2G}\int d^2x\,\sqrt{|g|}\,\left[e^{-2\phi}\,(R + 4\nabla^a\phi\nabla_a\phi + 4\kappa^2) + G\,\nabla^a f\nabla_a f\right]$$

f: scalar field; Setting $g^{ab} = \Omega\,\eta^{ab}$, gravitational sector: (ϕ, Ω).

• 4-d and 2-d rather similar but CGHS is technically much simpler: for, the matter field f now satisfies $\Box_\eta f = 0$, and, given any solution f we can write down the solution for ϕ, Ω in a closed form algebraically !

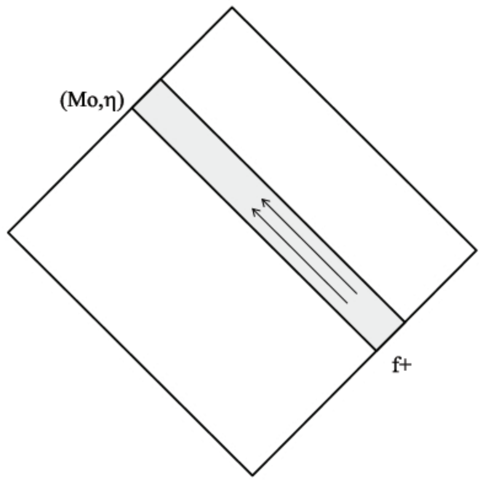
Setting $\eta_{ab} = -\partial_{(a}z^+\,\partial_{b)}z^-$, $\kappa\, x^\pm = \pm e^{\pm\kappa z^\pm}$, $\Phi = e^{-2\phi}$, $\Omega = \Theta^{-1}\Phi$

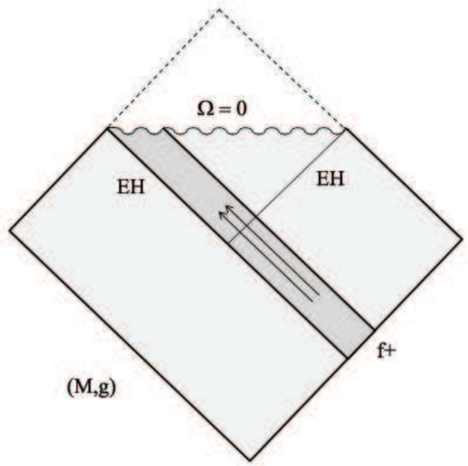
The solution is: $f = f_+(z^+) + f_-(z^-)$, $\Theta = -\kappa^2 x^+\, x^-$ and

$$\Phi = \Theta - \frac{G}{2}\int_0^{x^+} d\bar{x}^+ \int_0^{\bar{x}^+} d\bar{\bar{x}}^+\,(\partial f_+/\partial\bar{\bar{x}}^+)^2 - \frac{G}{2}\int_0^{x^-} d\bar{x}^- \int_0^{\bar{x}^-} d\bar{\bar{x}}^-\,(\partial f_-/\partial\bar{\bar{x}}^-)^2$$

$\Rightarrow$ True DOF is in the scalar field f.

Reinterpretation of the collapse





• 'Fundamental' Theory: $\Box f = 0$ on Minkowski space (M_0, η)!
A general solution $f_+(z^+)$ determines *composite fields* Φ, Θ and $g^{ab} = \Phi\Theta^{-1}\eta^{ab}$.
These composite fields capture the physics of interest. Analogy with QCD.

• How can there be a black hole if the fundamental theory is so trivial?

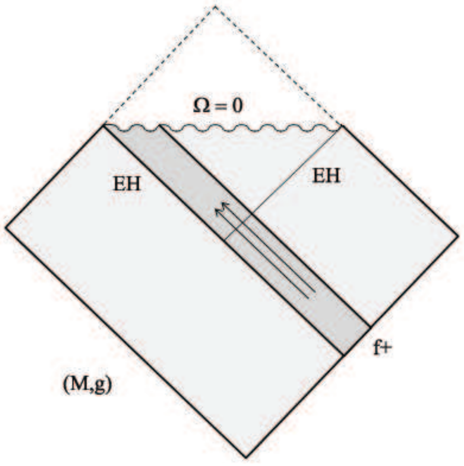
• Indeed, all fields regular on entire M_0. But Φ vanishes along a space-like line. Curvature of g blows up there. So the emergent space-time (M, g) is smaller.
But $\mathcal{I}_R^+$ is complete: The affine parameter y^- on $\mathcal{I}_R^+$ w.r.t. (M, g) runs from $-\infty$ to ∞. And past of $\mathcal{I}_R^+$ is not all of M. $\Rightarrow$ Black hole!
BH emerges when one interprets the composite field g as the emergent, physical space-time metric (AA, Taveras, Varadarajan (ATV)).

Emergence of Hawking radiation

• Black hole formed by the gravitational collapse of some left moving modes $f_+(z^+)$ which fall in from $\mathcal{I}_R^-$. Consider the right moving mode as a test quantum field $\hat{f}_-(z^-)$ on this dynamical BH space-time. Since $f_- = 0$ on $\mathcal{I}_L^-$, natural to assume that $\hat{f}_-(z^-)$ is in a vacuum state on $\mathcal{I}_L^-$. What is the outgoing state on $\mathcal{I}_R^+$ for this test field $\hat{f}_-$ on the dynamical BH background (M, g)?

• Fundamental dynamics of $\hat{f}_-$ is trivial on the entire (M_o, η): $\Box_{(\eta)} \hat{f}_- = 0$. On $\mathcal{I}_R^{o,+}$, state is just the vacuum state $|0_R^+\rangle$ in the $\pm$ freq. decomposition w.r.t. inertial observers $\partial/\partial z^-$ of η.

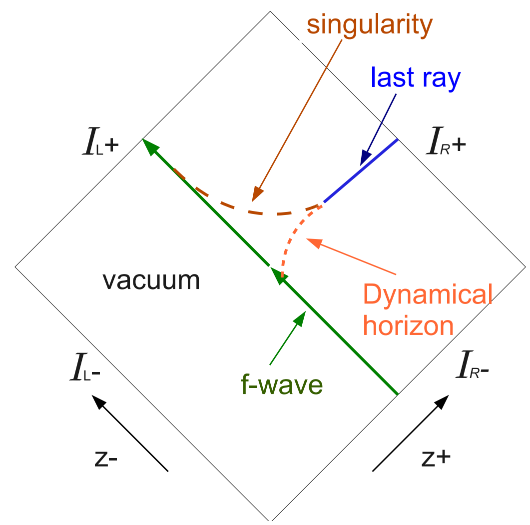
• Interpretation w.r.t. emergent (M, g)?
i) Need $\pm$frequency decomposition
w.r.t. inertial observers $\partial/\partial y^-$ of (M, g); and,
ii) trace over modes on $\mathcal{I}_R^{o,+} - \mathcal{I}_R^+$.
Result: On $\mathcal{I}_R^+$ of (M, g), we now have
a thermal state at temperature $\kappa\hbar$!
Information loss in the approximate
emergent theory! (Back reaction ignored.)



Mean field approximation (semi-classical gravity)

• To include back reaction, one can use the mean field approximation: Ignore quantum fluctuations of geometry but not of matter (large number N of matter fields). Emergent geometry still classical but metric coefficients depend on $\hbar$. Theory given by hyperbolic evolution equations and constraints which are preserved in time. The singularity persists. But it is weak. g is C^o but not C^1 there.

• Detailed numerics: Emergent geometry now exhibits *formation and evaporation of a dynamical horizon* with unforeseen scaling and universality, of interest to mathematical GR community (AA, Pretorius, Ramazanoglu). No event horizon because $\mathcal{I}_R^+$ is incomplete.



Full quantum theory

• Fundamental *quantum* theory again very simple: $\Box \hat{f}_\pm = 0$ on (M_0, η).
Initial state: Coherent state $|\Psi_{f_+}\rangle$ on $\mathcal{I}_R^{o-}$ and $|0_-\rangle$ on $\mathcal{I}_L^{o-}$.
Composite operators $\hat{\Phi}, \hat{\Theta}$ on the Fock space of $\hat{f}_\pm$ determine a quantum metric $\hat{g}$ (AA, Taveras, Varadarajan (ATV)). These are non-trivial.

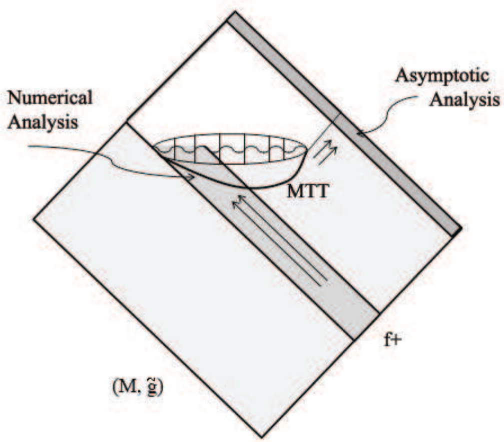
• In the fundamental theory, the state $|0_-\rangle$ on $\mathcal{I}_L^{o-}$ trivially evolves to $|0_-\rangle$ on $\mathcal{I}_R^{o+}$. S-matrix identity and no information loss in the fundamental theory. Key question: What is the situation in the emergent space-time?

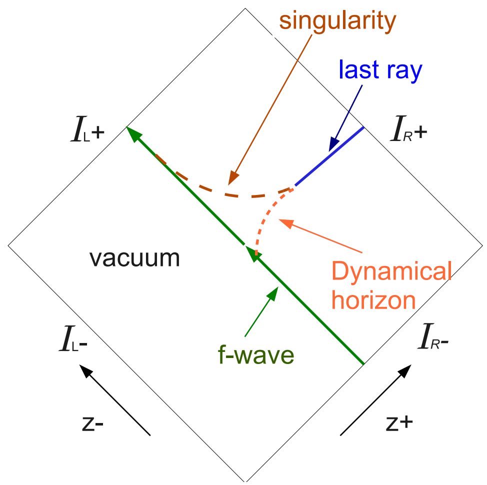
• Standard, well-motivated assumptions on $\bar{g} = \langle \hat{g} \rangle$ imply (ATV):
i) $\mathcal{I}_R^{+} = \mathcal{I}_R^{o,+}$; no tracing over modes; and,
ii) y^- related to z^- in such a way that the Bogoluibov transform is well defined; i.e., the vacuum state at $\mathcal{I}_R^{+}$ w.r.t. η does belong to the Fock space at $\mathcal{I}_R^{+}$ of the physical geometry $\bar{g}$.

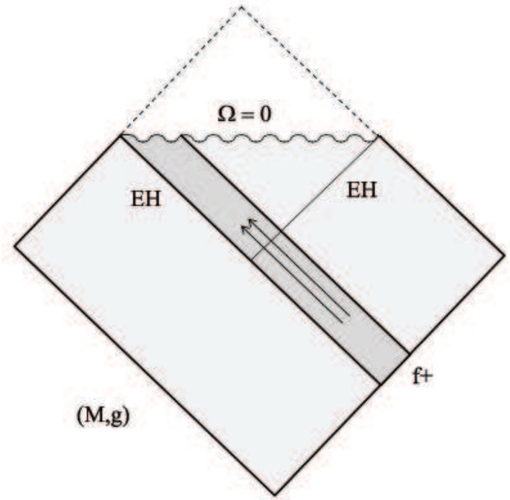
$\Rightarrow$ Dynamics w.r.t. $\bar{g}$ non-trivial but the S-matrix still unitary; No information loss in full quantum gravity also from the perspective of the emergent space-time!

Emergence: Successive approximations

• DOF of the fundamental quantum theory: $\hat{f}$ satisfying $\Box\hat{f} = 0$ on (M_0, η) with states of the type $|0_-\rangle \otimes |\Psi_{f_+}\rangle$. Very simple theory.
Directly observable & rich physics coded in composite operators, in particular $\hat{g}$ constructed from $\hat{f}$.







• Space-time description in full quantum theory: Singularity would be replaced by a 'quantum region' where the composite quantum field $\hat{g}$ is fine but has very large quantum fluctuations.
Supporting evidence: mini-superspace analysis (Ori); truncated theory (ATV); Resolution of 4-d BH singularity in LQG (AA, Bojowald; ...) .

2. Second Example: The AdS/CFT Conjecture

- Big subject. Will illustrate with the original, bold conjecture (Maldecena):

Consider a space-time $M^{(10)} = M^{(5)} \times S^5$ with

$M^{(5)}$: Asymptotically AdS space-time and S^5 a 5-sphere with radius $L = 1/\sqrt{-\Lambda}$.

Type IIB string theory on $M^{(10)}$ (with constants $\ell_s = \sqrt{\alpha'}$, g_s) :=

$SU(N)$ SYM theory (with $\mathcal{N}$ = 4) on $\mathcal{I} \equiv \partial M^{(5)} = \mathbb{R} \times S^4$

(with $N = L^4/4\pi g_s \ell_s^4$ and $\lambda_{\rm YM} = 4\pi g_s N$)

- String theory on a macroscopic space-time

$\Leftrightarrow$ L very large $\Leftrightarrow$ N large $\Leftrightarrow$ $\lambda_{\rm YM}$ large.

Weak coupling in string theory $\Leftrightarrow$ strong coupling in gauge theory.

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- Dictionary: relates quantities in the bulk (string/gravity side) with composite, local quantities on $\mathcal{I}$ of M^5 (gauge theory side)

Ex: Anti-deSitter space $\longleftrightarrow$ gauge theory vacuum

gravitons in the bulk $h_{\mu\nu}$ $\longleftrightarrow$ $T_{\mu\nu}$ on $\mathcal{I}$

supergravity dilaton ϕ in the bulk $\longleftrightarrow$ Tr F^2 on $\mathcal{I}$

(More in Álvarez's talk tomorrow)

AdS/CFT Conjecture: Strengths

• Idea: Space-time structures emerge through composite operators of a controlled (probably finite) quantum field theory, namely SYM theory in Minkowski space which knows nothing about gravity. Similar in spirit as the CGHS black holes.

• Major difference: Flat space QFT resides on the boundary $\mathcal{I}$ of $M^{(5)}$: physically interesting space-time constructed from its “holographic Image” (although not quite because the bdry of $M^{(10)}$ is $\mathcal{I} \times S^5$).

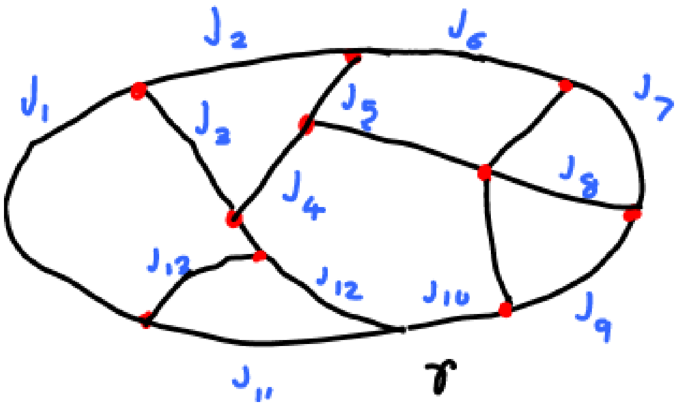
• This difference is powerful!

• Unexpected boon: The correspondence has tremendously widened the scope of gravitational physics. Black holes in particular provide astonishingly effective tools to explore areas of physics that seemed far removed from gravity! Namely, quark-gluon plasma, superconductors, fluids, ...

• This uncovering of deep unity of physics will have a conceptual impact even if it turns out that the systems that can be treated are idealized and not of direct interest to other areas of physics.

3. Last Example: Loop quantum gravity

• Canonical approach: Geometry built from spin networks. Fundamental excitations are 1-dimensional, to be thought of as quantum threads weaving the fabric of space.



$$\mathcal{H} = \bigoplus_{\gamma,\vec{j}} \underbrace{\mathcal{H}_{\gamma,\vec{j}}}_{\substack{\text{Finite-dim.} \\ \text{H.S. of a} \\ \text{spin-system}}}$$

• Each such spin network defines a quantum state of geometry. All geometric operators have discrete eigenvalues. Smooth, continuum geometries arises only as a coarse grained approximation.

• Path integral approach: Elementary histories are spin-foams — 'Evolution' of spin-networks. A decorated 2-complex that represents a quantum space-time geometry. Fundamental DOF very different from smooth g_{ab} often used as histories.

(More: Rovelli's talk tomorrow)

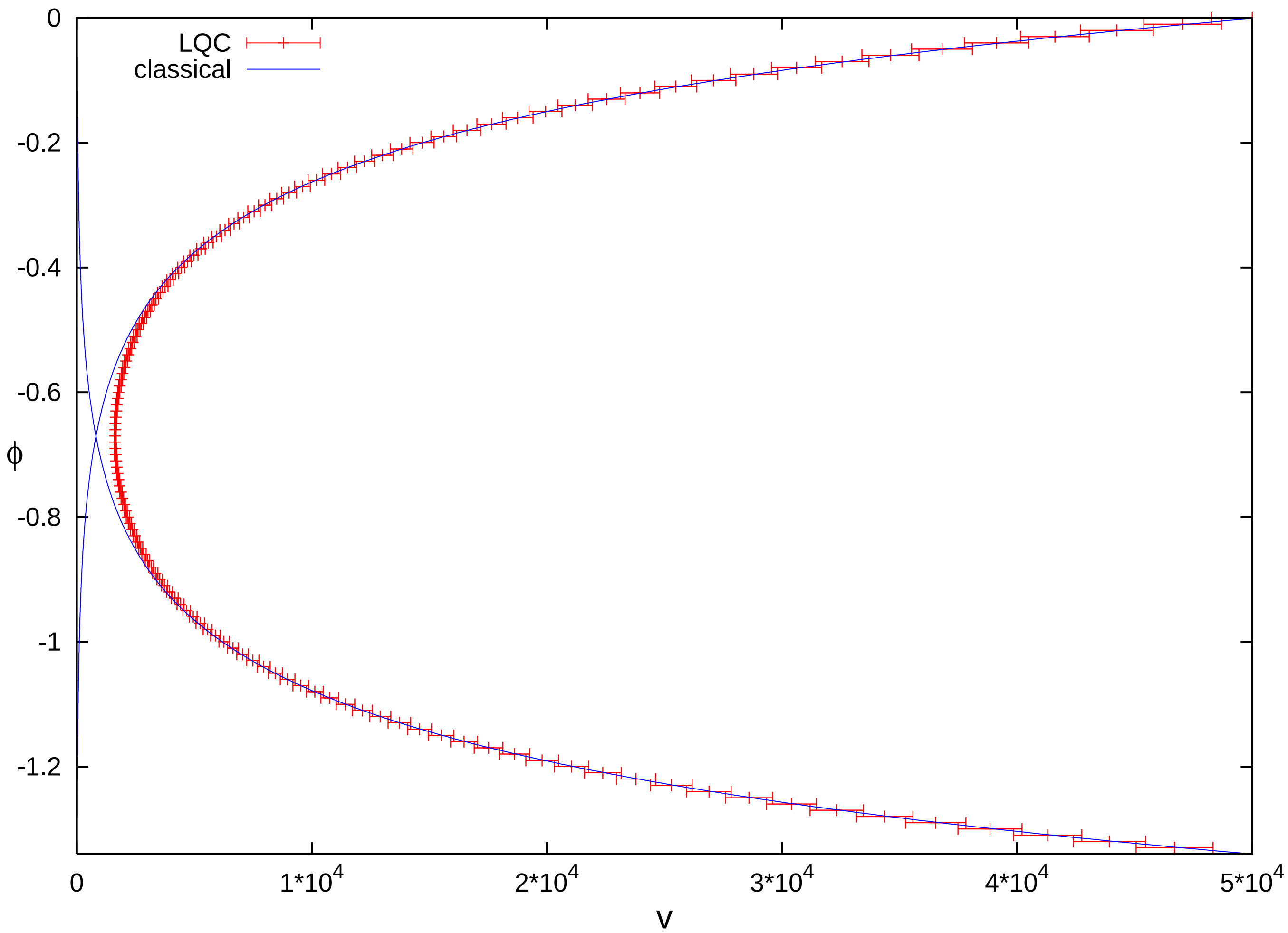
Importance of fundamental discreteness

- Loop Quantum Cosmology: Application of the LQG framework to homogeneous cosmology. When curvature is less than 1% of Planck scale, excellent agreement with GR. But when it is higher, a new repulsive force with origin in quantum geometry takes over and dominates classical attraction. Big bang, big crunch (as well as big rip, sudden death, ...) singularities resolved. Quantum space-times vastly larger than Einstein had us believe. Cosmological paradigm extended.

- Black hole entropy: A statistical mechanical accounting starting from isolated horizons. Bulk geometry polymer-like; intrinsic geometry of the IH described by a Chern-Simons theory of a punctured sphere. Micro-states: Quantum horizon geometry. Same calculation for all ordinary, astrophysical BHs and cosmological horizons. Discreteness of area eigenvalues essential for finiteness of entropy.

- Graviton Propagator: Non-perturbative calculation using spin foams. Non-triviality of the 2-point function & apparent tension with diffeomorphism invariance! Resolved by fixing a spin-network boundary state approximating flat space-time and summing over spin foams in the interior. To leading order, re-produces the perturbative answer.First signs that, although they first seem exotic, the fundamental DOF of LQG do capture familiar low energy physics.

Simplest Example: k=0 LQC



(AA, Pawlowski, Singh)

Expectations values and dispersions of volume $\sim a^3$ & classical trajectories.

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Quantum Horizon

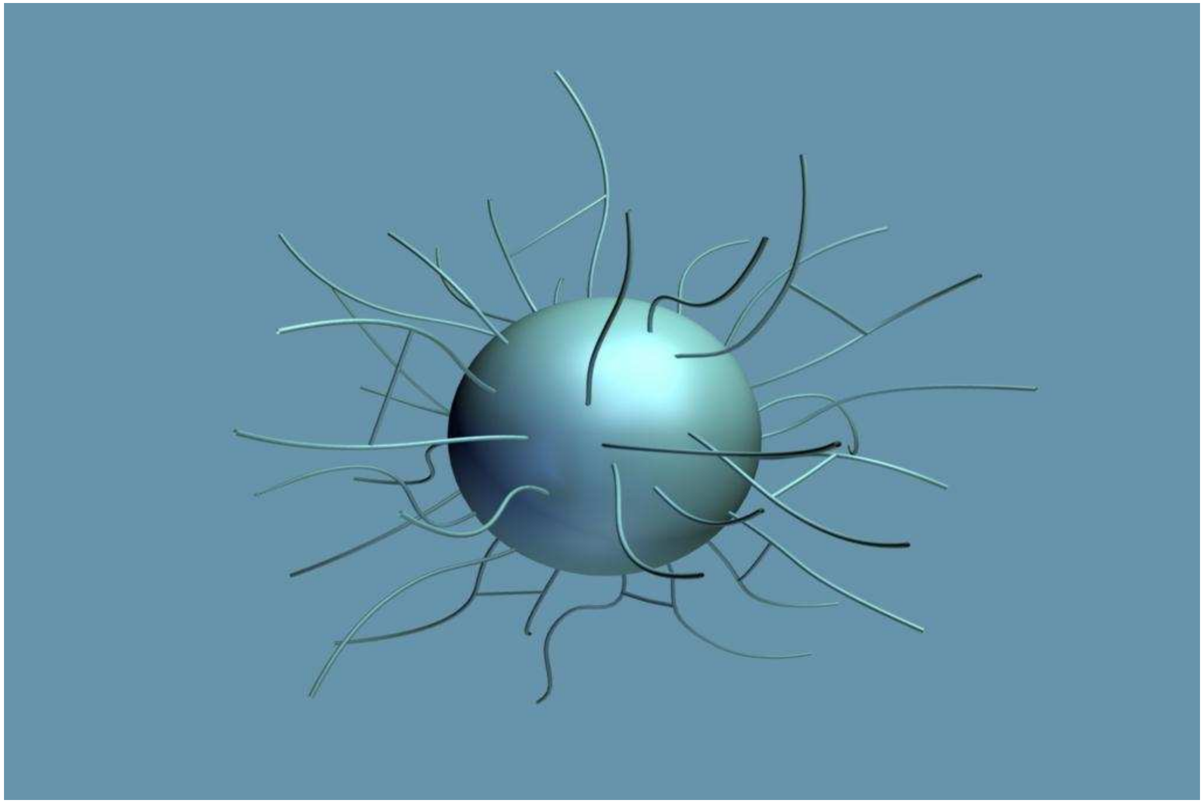

Image: Courtesy Alejandro Corichi.

Polymer excitations of geometry in the bulk puncture the horizon.
Quantum horizon geometry described by the $\mathrm{U}(1)$ Chern-Simons theory.

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4. Summary and Discussion

• I have discussed three examples. In all cases, fundamental DOF very different from space-time structures of classical GR. Surprising that familiar geometric concepts can emerge from them.

• In the first, one starts with a gravity theory but isolates the true degree of freedom and its dynamics. Space-time geometry is secondary, built from composite fields. Although the dynamics of the fundamental DOF is trivial, dynamics of composite fields is very rich!

• Another similar example: Einstein-Rosen waves! Again, fundamental DOF: scalar field satisfying wave equation in a flat space! Space-time metric: built from Composite fields. Very beautiful work by the Madrid group. (Barbero, Garay, Mena, Villaseñor)

• Although fundamental theory appears far removed from gravity, GR serves as a powerful guide to arrive at the fundamental DOF!

Second Example: Ads/CFT Conjecture

• Power: Because SYM is rather well-controlled, systematic calculations possible. Unforeseen bridges between gravity and other areas of physics.

• But the radical approach also leads to difficulties. (Dictionary not derived from first principles; a mixture of art and science. Moreover, so far, relation of this proposed fundamental description and perturbative string theory on the AdS background not known.)

• Other limitations from a gravitational perspective:

⋆ $\Lambda < 0$ not of direct physical interest.

⋆ In the compactifications envisaged, the extra dimensions are large; they are of cosmological size ($R_{S^5} = R_\Lambda$) and not tiny/Planck scale.

⋆ Mechanisms to obtain a positive Λ compatible with observations appear artificial at least from outside. "Concept of proof rather than a definitive solution" (Radu Roiban).

⋆ Except in some analytical space-times, difficult to see how physics behind horizons is encoded in the fundamental description on the boundary.

⋆ "tiny corner of quantum gravity" (Steve Shenker).

Last Example: LQG

• Now the fundamental theory is *not* a QFT in flat space! More radical than first two example. Recurring theme: space-time built from 2-d structures (also in Reuter et al's Asymptotic Safety).

• Power: Fundamental discreteness provides a natural uv cure. Can help address long standing issues related to physically most interesting singularities, BHs etc. Contrast to WDW Theory.

• Limitations: Distance from GR and absence of a familiar QFT in flat space-time is attractive and has the right 'radical flavor'. But this also makes it very difficult to:

⋆ Narrow down the freedom in dynamics of the fundamental DOF.

⋆ Extract low energy physics, e.g. scattering cross sections.

⋆ Systematically introduce effective theories, e.g. to discuss black hole evaporation or cosmological perturbations in a well-controlled fashion.

⋆ Use the rich QFT techniques such as RG group flow.

• Thus, approaches to Emergence have interesting trade-offs. More radical the choice of fundamental DOF, greater the potential but also greater the difficulties.