Crossroads between

Relativistic Astrophysics and Numerical Relativity

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Bowdoin College

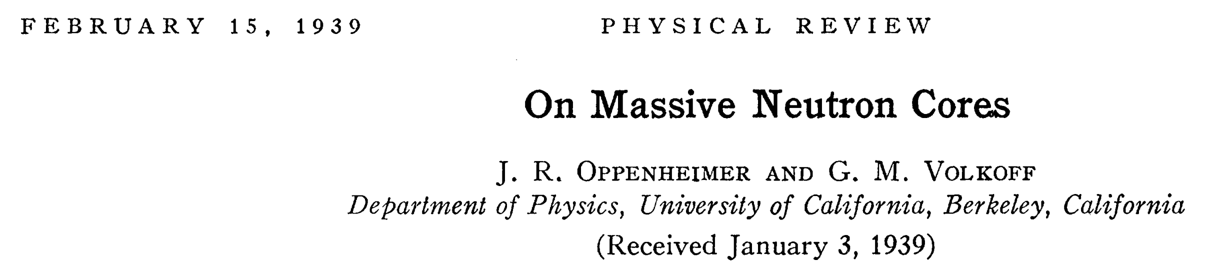
ERE, Granada, 2010

Outline

- A brief history of numerical relativity
  - spherical symmetry
  - axisymmetry
  - no symmetries...
- Obstacles in 3D numerical relativity
  - formulations of Einstein's equations
  - treating black hole singularities
- Binary black holes
  - black hole recoil
  - black hole spin
  - consequences for galaxy mergers
- Binary neutron stars / mixed binaries
  - the central engines of short gamma-ray bursts?
- New/future developments

Spherical Symmetry

- Hydrostatic equilibrium



FEBRUARY 15, 1939 PHYSICAL REVIEW VOLUME 55

On Massive Neutron Cores

J. R. Oppenheimer and G. M. Volkoff
Department of Physics, University of California, Berkeley, California
(Received January 3, 1939)

$\Longrightarrow$ maximum mass of neutron stars

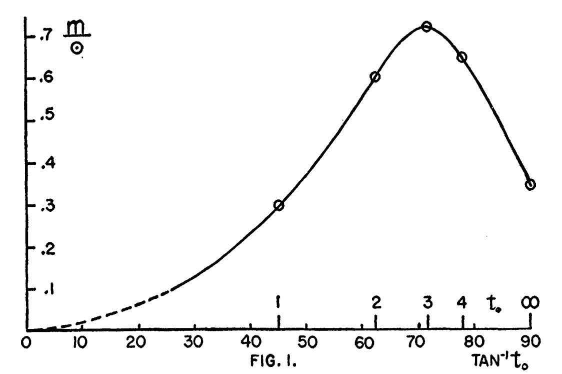
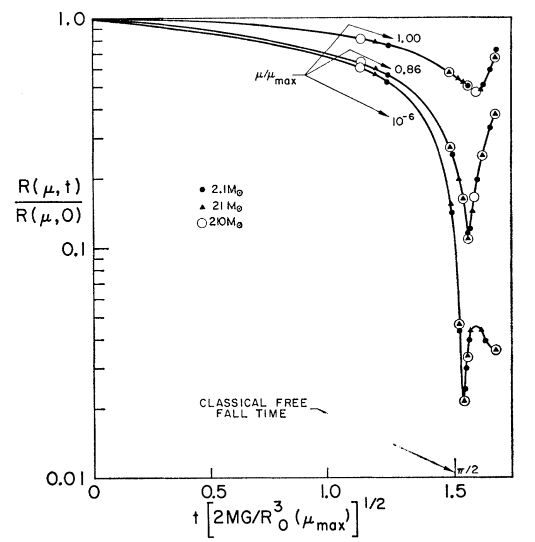


Fig. 1. Dependence of m on t_0 for neutrons.

- Hydrodynamics

$\Longrightarrow$ stellar collapse,
supernovae

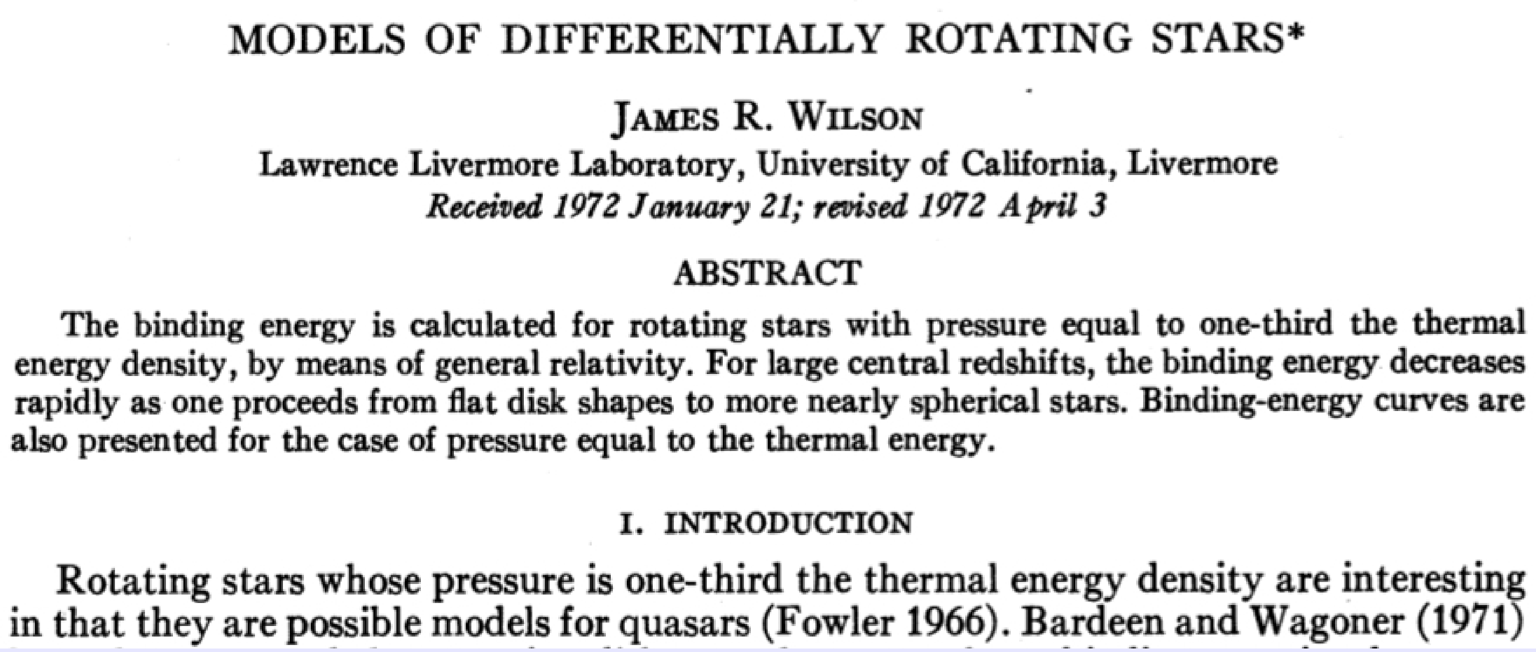


[May & White, 1966]

- Critical collapse, boson stars...

Axisymmetry

- Rotating stars: equilibrium models



MODELS OF DIFFERENTIALLY ROTATING STARS*

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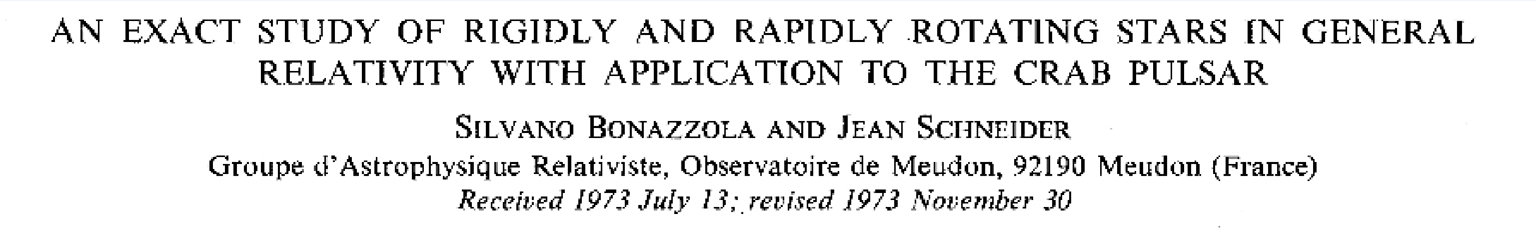
Received 1972 January 21; revised 1972 April 3

ABSTRACT

The binding energy is calculated for rotating stars with pressure equal to one-third the thermal energy density, by means of general relativity. For large central redshifts, the binding energy decreases rapidly as one proceeds from flat disk shapes to more nearly spherical stars. Binding-energy curves are also presented for the case of pressure equal to the thermal energy.

I. INTRODUCTION

Rotating stars whose pressure is one-third the thermal energy density are interesting in that they are possible models for quasars (Fowler 1966). Bardeen and Wagoner (1971)



AN EXACT STUDY OF RIGIDLY AND RAPIDLY ROTATING STARS IN GENERAL RELATIVITY WITH APPLICATION TO THE CRAB PULSAR

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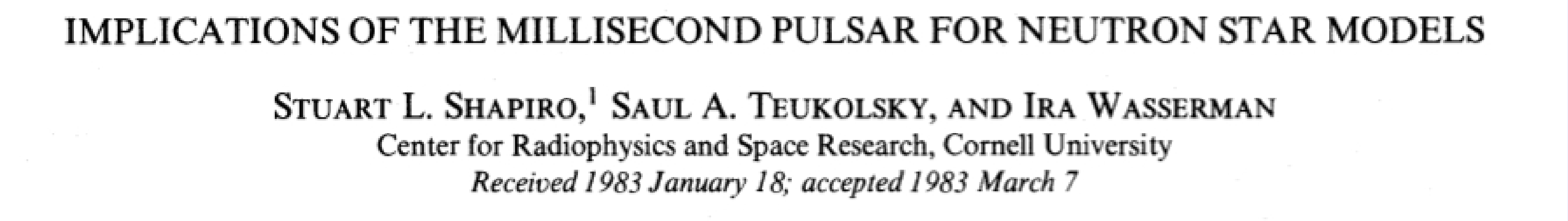
Received 1973 July 13; revised 1973 November 30

- Rotating stars cont...



IMPLICATIONS OF THE MILLISECOND PULSAR FOR NEUTRON STAR MODELS

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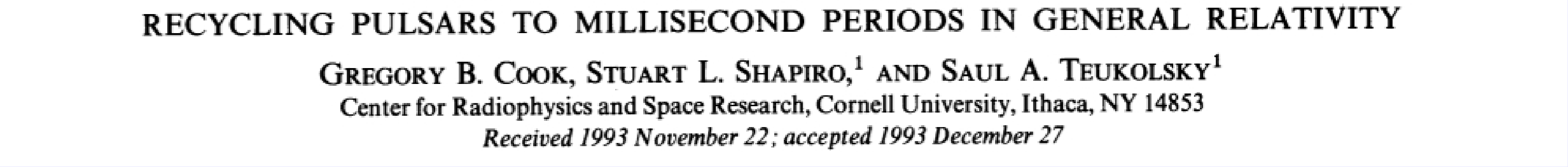
Received 1983 January 18; accepted 1983 March 7



RECYCLING PULSARS TO MILLISECOND PERIODS IN GENERAL RELATIVITY

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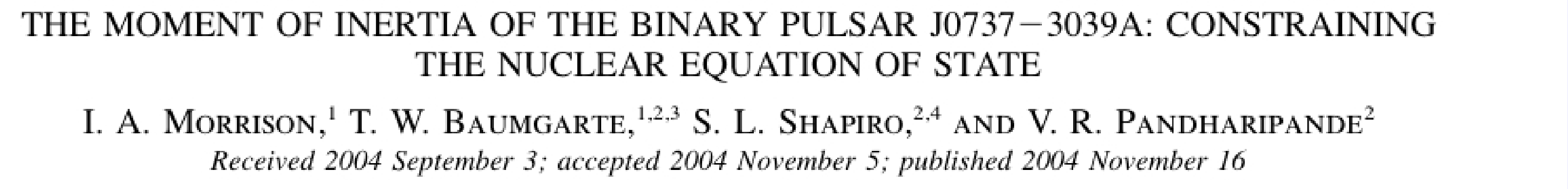


THE MOMENT OF INERTIA OF THE BINARY PULSAR J0737−3039A: CONSTRAINING THE NUCLEAR EQUATION OF STATE

I. A. MORRISON,[1] T. W. BAUMGARTE,[1,2,3] S. L. SHAPIRO,[2,4] AND V. R. PANDHARIPANDE[2]
Received 2004 September 3; accepted 2004 November 5; published 2004 November 16

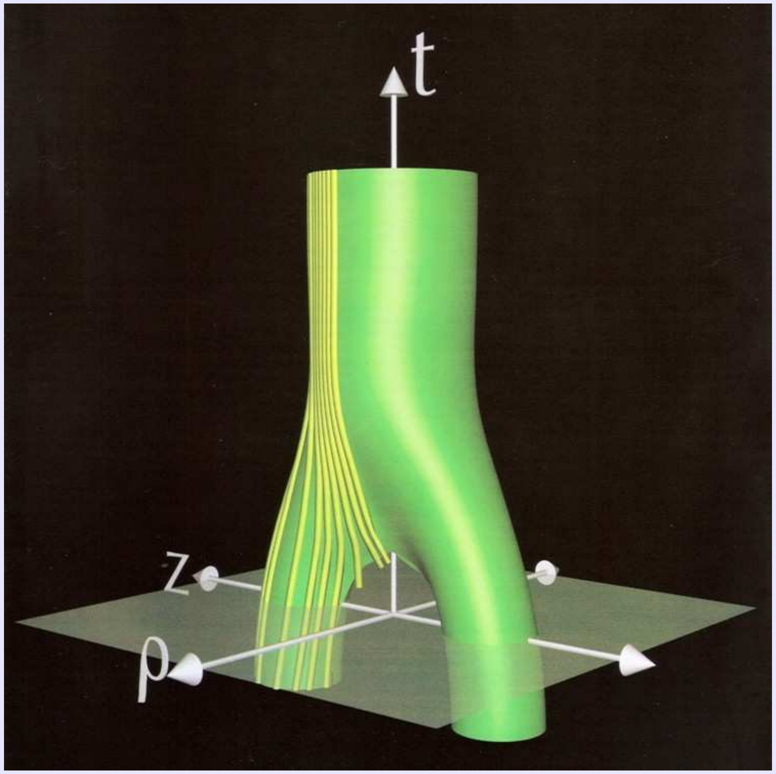
◦ Motivated by questions about quasars, pulsars, millisecond pulsars...
⟹ maximum masses, spin rates etc. for uniform and differential rotation

Other calculations in axisymmetry

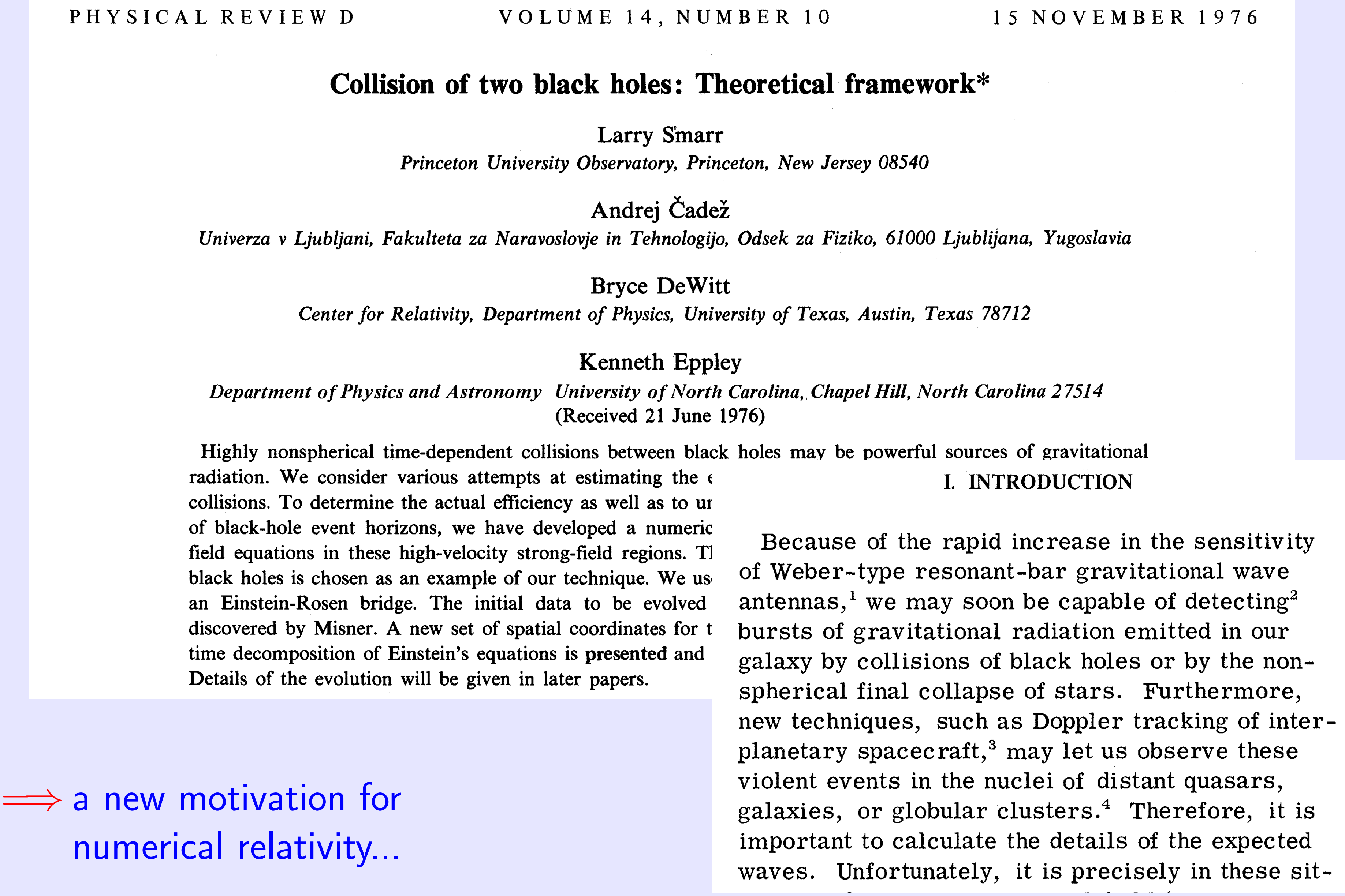
- Collapse of rotating stars:
  stability, collapse to black hole
  [Evans, 1986]

- Stellar clusters:
  equilibrium and stability
  [Shapiro & Teukolsky, 1991]

- Head-on collision of black holes
  [Smarr *et.al.*, 1976; Shapiro & Teukolsky, 1992; Anninos *et.al.*, 1993]



[Matzner *et.al.*, 1995]



PHYSICAL REVIEW D VOLUME 14, NUMBER 10 15 NOVEMBER 1976

Collision of two black holes: Theoretical framework*

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(Received 21 June 1976)

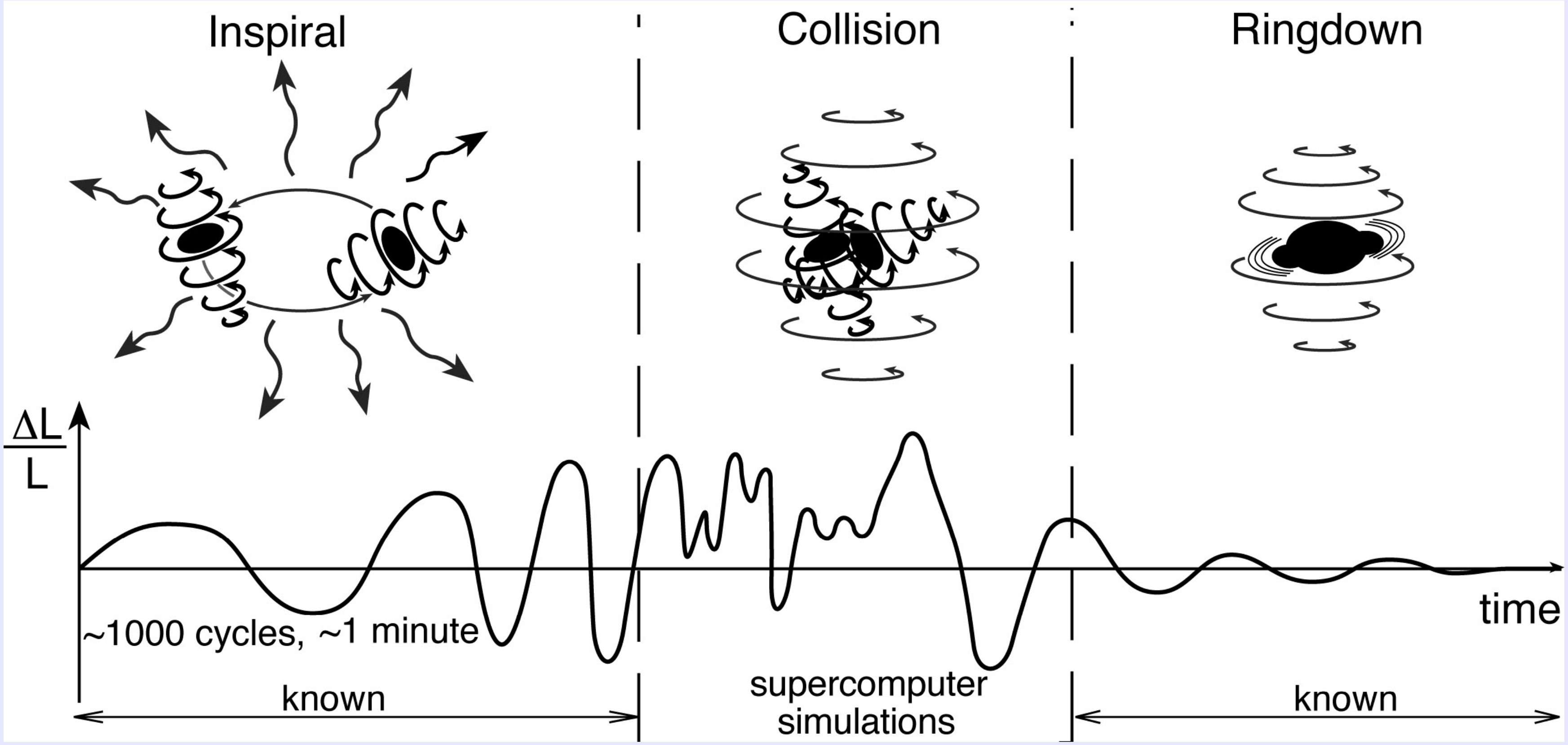
Highly nonspherical time-dependent collisions between black holes may be powerful sources of gravitational radiation. We consider various attempts at estimating the
collisions. To determine the actual efficiency as well as to u
of black-hole event horizons, we have developed a numeric
field equations in these high-velocity strong-field regions. T
black holes is chosen as an example of our technique. We us
an Einstein-Rosen bridge. The initial data to be evolved
discovered by Misner. A new set of spatial coordinates for t
time decomposition of Einstein's equations is presented and
Details of the evolution will be given in later papers.

I. INTRODUCTION

Because of the rapid increase in the sensitivity of Weber-type resonant-bar gravitational wave antennas,[1] we may soon be capable of detecting[2] bursts of gravitational radiation emitted in our galaxy by collisions of black holes or by the nonspherical final collapse of stars. Furthermore, new techniques, such as Doppler tracking of interplanetary spacecraft,[3] may let us observe these violent events in the nuclei of distant quasars, galaxies, or globular clusters.[4] Therefore, it is important to calculate the details of the expected waves. Unfortunately, it is precisely in these sit-

⟹ a new motivation for numerical relativity...

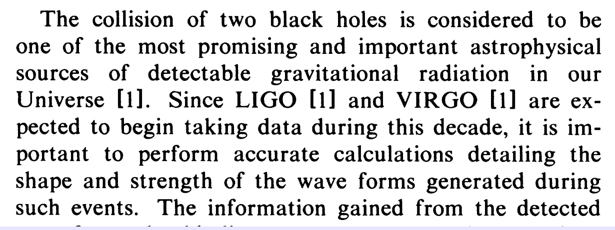
Numerical Relativity and Gravitational Radiation



[Thorne, 2002]

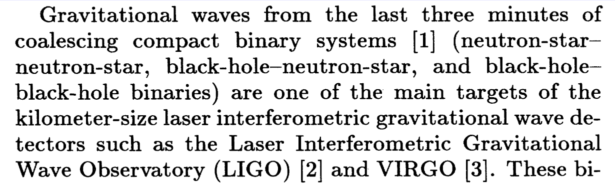
- Numerical relativity key tool for understanding transition from binary inspiral to ringdown

⟹ Prediction of gravitational waves from compact binary merger becomes prime motivation for development of 3D numerical relativity



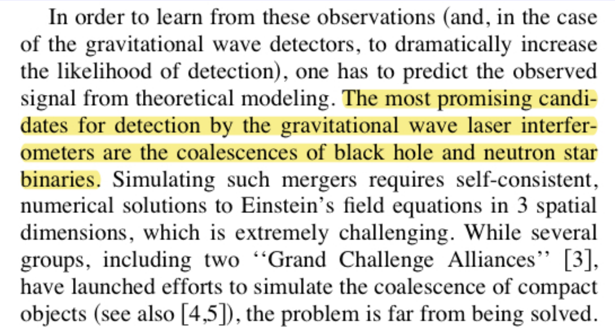
The collision of two black holes is considered to be one of the most promising and important astrophysical sources of detectable gravitational radiation in our Universe [1]. Since LIGO [1] and VIRGO [1] are expected to begin taking data during this decade, it is important to perform accurate calculations detailing the shape and strength of the wave forms generated during such events. The information gained from the detected

[Anninos *et. al.*, 1993]



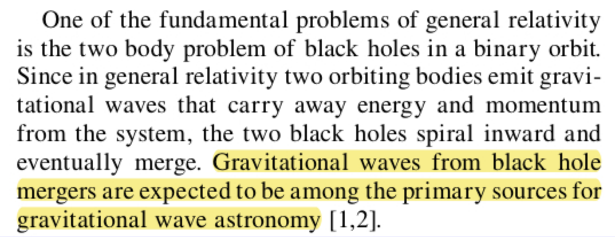
Gravitational waves from the last three minutes of coalescing compact binary systems [1] (neutron-star–neutron-star, black-hole–neutron-star, and black-hole–black-hole binaries) are one of the main targets of the kilometer-size laser interferometric gravitational wave detectors such as the Laser Interferometric Gravitational Wave Observatory (LIGO) [2] and VIRGO [3]. These bi-

[Shibata & Nakamura, 1995]



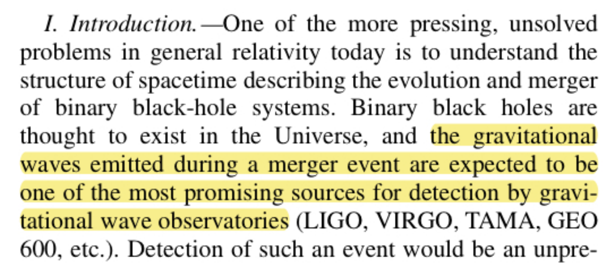
In order to learn from these observations (and, in the case of the gravitational wave detectors, to dramatically increase the likelihood of detection), one has to predict the observed signal from theoretical modeling. The most promising candidates for detection by the gravitational wave laser interferometers are the coalescences of black hole and neutron star binaries. Simulating such mergers requires self-consistent, numerical solutions to Einstein's field equations in 3 spatial dimensions, which is extremely challenging. While several groups, including two ''Grand Challenge Alliances'' [3], have launched efforts to simulate the coalescence of compact objects (see also [4,5]), the problem is far from being solved.

[Baumgarte & Shapiro, 1998]



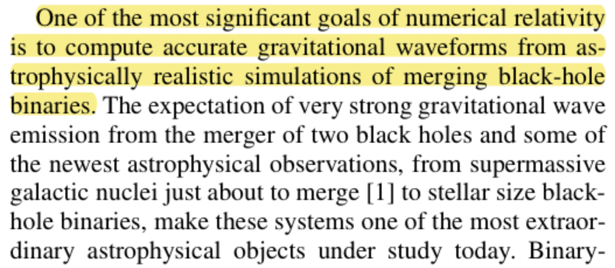
One of the fundamental problems of general relativity is the two body problem of black holes in a binary orbit. Since in general relativity two orbiting bodies emit gravitational waves that carry away energy and momentum from the system, the two black holes spiral inward and eventually merge. Gravitational waves from black hole mergers are expected to be among the primary sources for gravitational wave astronomy [1,2].

[Brügmann *et. al.*, 2004]



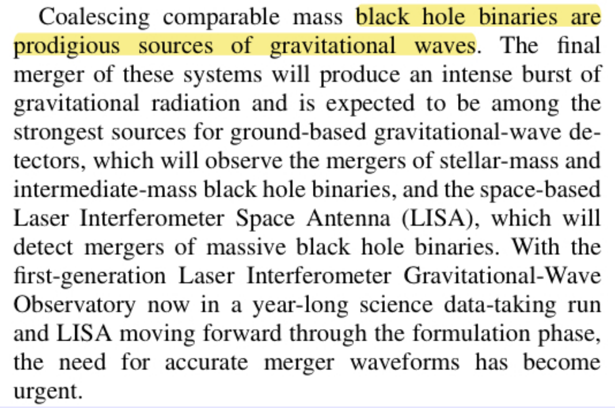
I. Introduction.—One of the more pressing, unsolved problems in general relativity today is to understand the structure of spacetime describing the evolution and merger of binary black-hole systems. Binary black holes are thought to exist in the Universe, and the gravitational waves emitted during a merger event are expected to be one of the most promising sources for detection by gravitational wave observatories (LIGO, VIRGO, TAMA, GEO 600, etc.). Detection of such an event would be an unpre-

[Pretorius, 2005]



One of the most significant goals of numerical relativity is to compute accurate gravitational waveforms from astrophysically realistic simulations of merging black-hole binaries. The expectation of very strong gravitational wave emission from the merger of two black holes and some of the newest astrophysical observations, from supermassive galactic nuclei just about to merge [1] to stellar size black-hole binaries, make these systems one of the most extraordinary astrophysical objects under study today. Binary-

[Campanelli *et. al.*, 2006]



Coalescing comparable mass black hole binaries are prodigious sources of gravitational waves. The final merger of these systems will produce an intense burst of gravitational radiation and is expected to be among the strongest sources for ground-based gravitational-wave detectors, which will observe the mergers of stellar-mass and intermediate-mass black hole binaries, and the space-based Laser Interferometer Space Antenna (LISA), which will detect mergers of massive black hole binaries. With the first-generation Laser Interferometer Gravitational-Wave Observatory now in a year-long science data-taking run and LISA moving forward through the formulation phase, the need for accurate merger waveforms has become urgent.

[Baker *et. al.*, 2006]

Obstacles in 3D Numerical Relativity

- Formulations of Einstein's equations
- Treatment of black hole singularities

Formulations of Einstein's equations

Einstein's equations

$$G_{\alpha\beta} = 8\pi T_{\alpha\beta}$$

for metric

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

can be split into ("3+1" decomposition)

- constraint equations (like "div" equations in E&M)

$$R + K^2 - K_{ij}K^{ij} = -16\pi\rho \qquad \text{(Hamiltonian constraint)}$$

$$D_i(K^{ij} - \gamma^{ij}K) = 8\pi j^j \qquad \text{(Momentum constraint)}$$

- evolution equations (like "curl" equations in E&M)

$$\frac{d}{dt}\gamma_{ij} \equiv \left(\frac{\partial}{\partial t} - \mathcal{L}_\beta\right)\gamma_{ij} = -2\alpha K_{ij}$$

$$\frac{d}{dt}K_{ij} = -D_iD_j\alpha + \alpha(R_{ij} - 2K_{ik}K^k{}_j + KK_{ij} - 8\pi M_{ij})$$

$\Longrightarrow$ "ADM" formulation [Arnowitt, Deser & Misner, 1962; York 1979]

$\Longrightarrow$ numerical implementations become unstable and crash

Reformulating Einstein's equations

Can reformulate Einstein's equations by

- introducing new auxiliary variables
  (e.g. $\bar{\Gamma}^i \equiv \bar{\gamma}^{jk}\bar{\Gamma}^i_{jk}$ in BSSN formulation)
- adding constraints to evolution equations

How can this affect numerical behavior???

- mathematical properties:
  - constraints contain as high derivatives as evolution equations
  - adding constraints to evolution equations affects principle operators

  $\Longrightarrow$ some reformulations are strongly or symmetric hyperbolic, while ADM is not

  $\Longrightarrow$ these reformulations are well-posed, while ADM is not

  [Alcubierre, 2008]
- consider constraint violations:
  - exact solutions must satisfy all reformulations of Einstein's equations,
  - but constraint violations may behave very differently

Evolution of constraint violations in electromagnetism

Can write Maxwell's equations as

$$\begin{aligned} \partial_t A_i &= -E_i - D_i \Phi \\ \partial_t E_i &= -D^j D_j A_i + D_i D^j A_j - 4\pi j_i \end{aligned}$$

(with $B_i = \epsilon_{ijk} D^j A^k$) with constraint equation

$$D_i E^i = 4\pi\rho.$$

Define constraint violation

$$\mathcal{C} \equiv D_i E^i - 4\pi\rho$$

Exercise 1

Show that the constraint violations $\mathcal{C}$ satisfy

$$\partial_t \mathcal{C} = 0.$$

"BSSN" formulation of electromagnetism

Introduce auxiliary variable $\Gamma \equiv D_i A^i$. Then

$$\partial_t E_i = -D_j D^j A_i + D_i \Gamma - 4\pi j_i$$

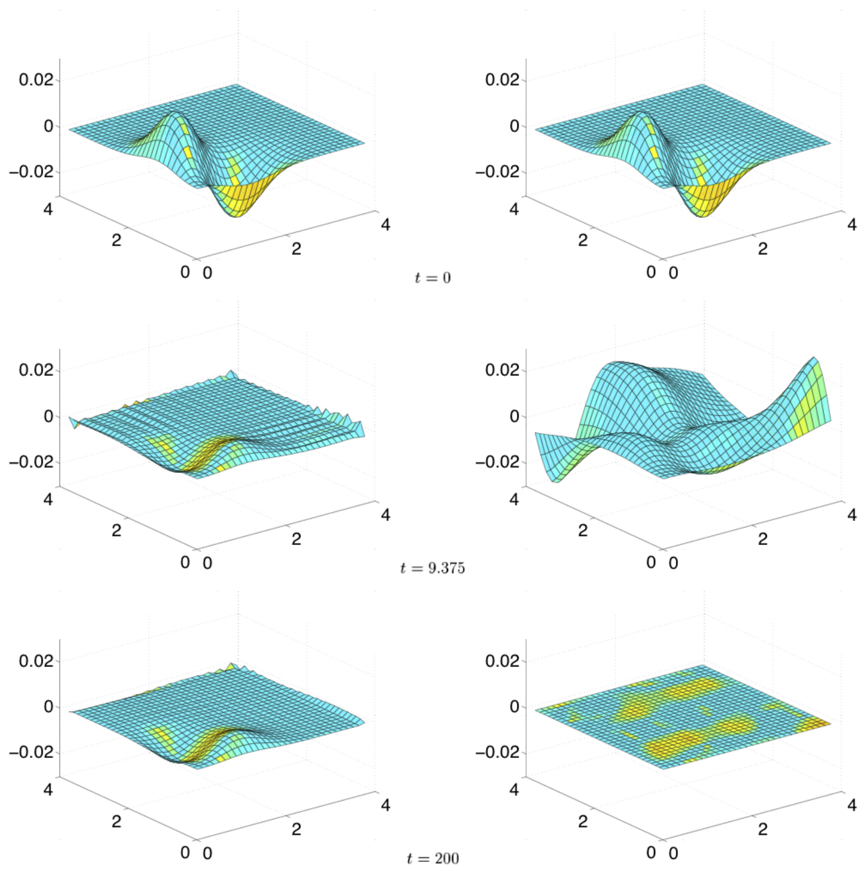
and

$$\begin{aligned} \partial_t \Gamma &= \partial_t D_i A^i = -D_i E^i - D_i D^i \Phi \\ &= -D_i D^i \Phi - 4\pi\rho \end{aligned}$$

Exercise 2

Show that the constraint violations $\mathcal{C}$ now satisfy the wave equation

$$(-\partial_t^2 + D_i D^i)\mathcal{C} = 0.$$



[Knapp *et.al.*, 2002]

Similar analysis in general relativity explains numerical stability properties of formulations in terms of propagation of constraints

[Alcubierre *et.al.*, 2000]

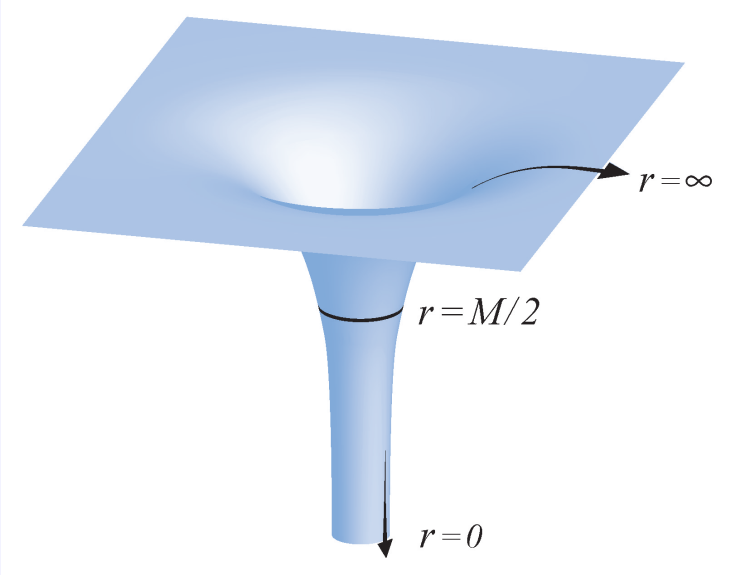
Reformulations of Einstein's equations

Some successful formulations of Einstein's equations:

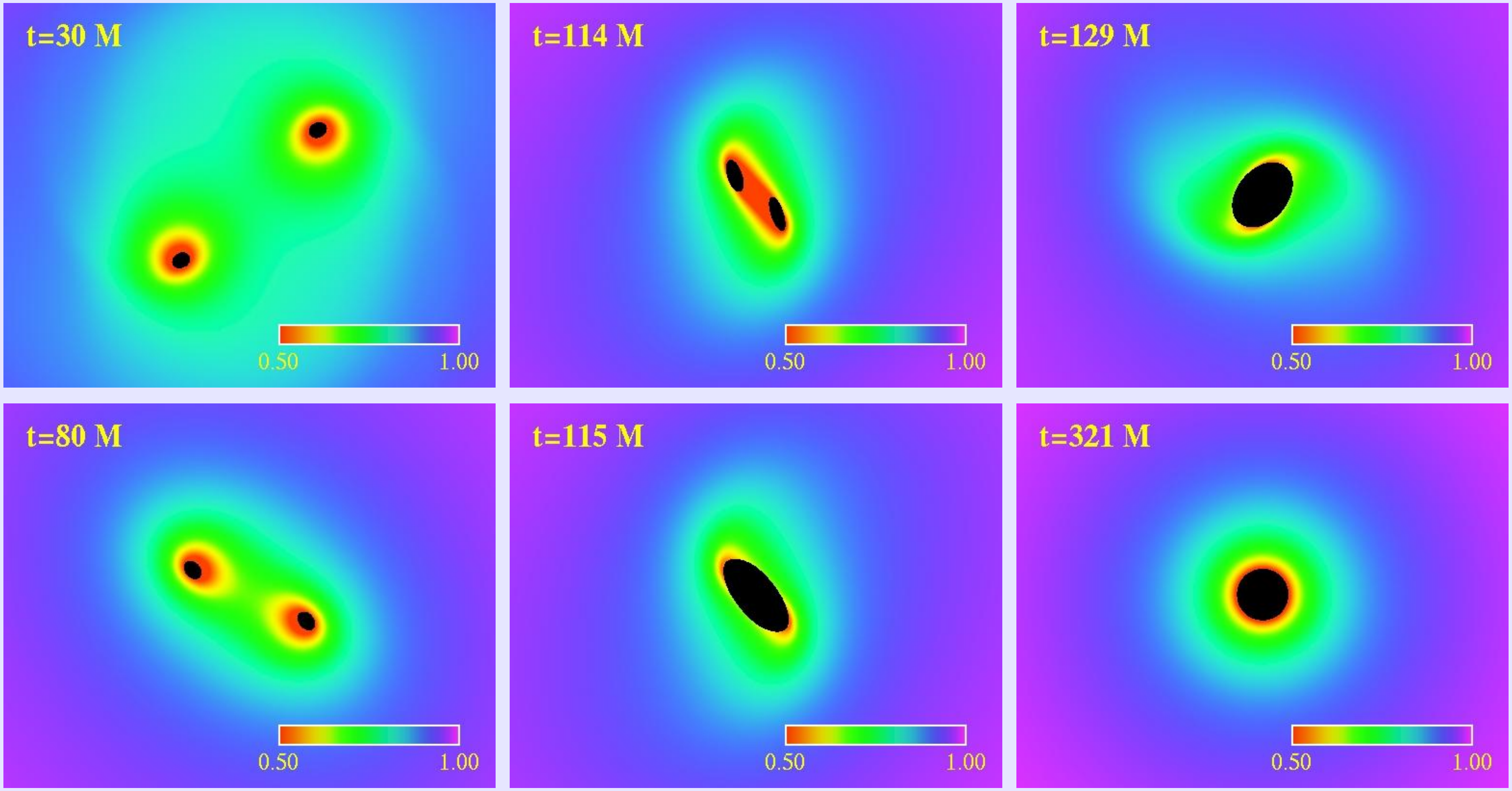
- BSSN formulation
  [Nakamura $et.al.$, 1987; Shibata & Nakamura, 1995; Baumgarte & Shapiro; 1998]
- Generalized harmonic formulation
  [Friedrich, 1985; Garfinkle, 2002; Pretorius, 2005; Lindblom $et.al.$, 2006]
- Z4 formulation (see Carles Bona's talk)
  [Bona $et.al.$, 2003]

Treating the black hole singularity

- Black hole excision:
  excise black hole interior from numerical grid
  [Unruh, 1984; Seidel & Suen, 1992; Alcubierre & Brügmann, 2001; Pretorius, 2005]
- Moving-puncture method:
  use 1+log slicing [Bona *et.al.*, 1995] and $\bar{\Gamma}^i$-freezing [Alcubierre *et.al.*, 2003] [Campanelli *et.al.*, 2006; Baker *et.al.*, 2006]
  $\Longrightarrow$ evolution of Schwarzschild settles down into "trumpet" geometry
  $\Longrightarrow$ slices do not reach singularity at $R = 0$
  [Hannam *et.al.*., 2007, 2008]

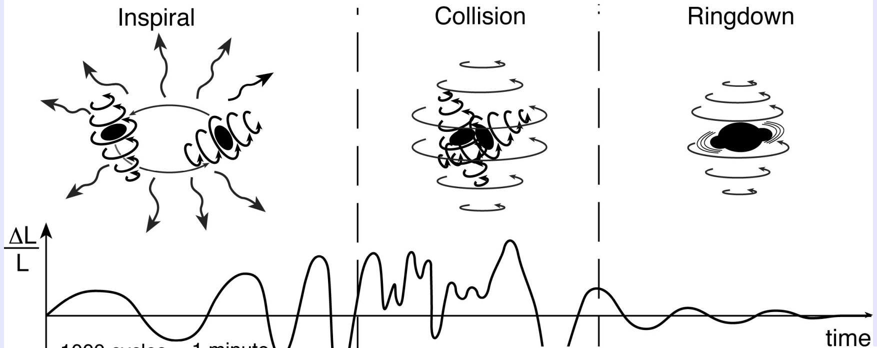


Simulations of binary black holes

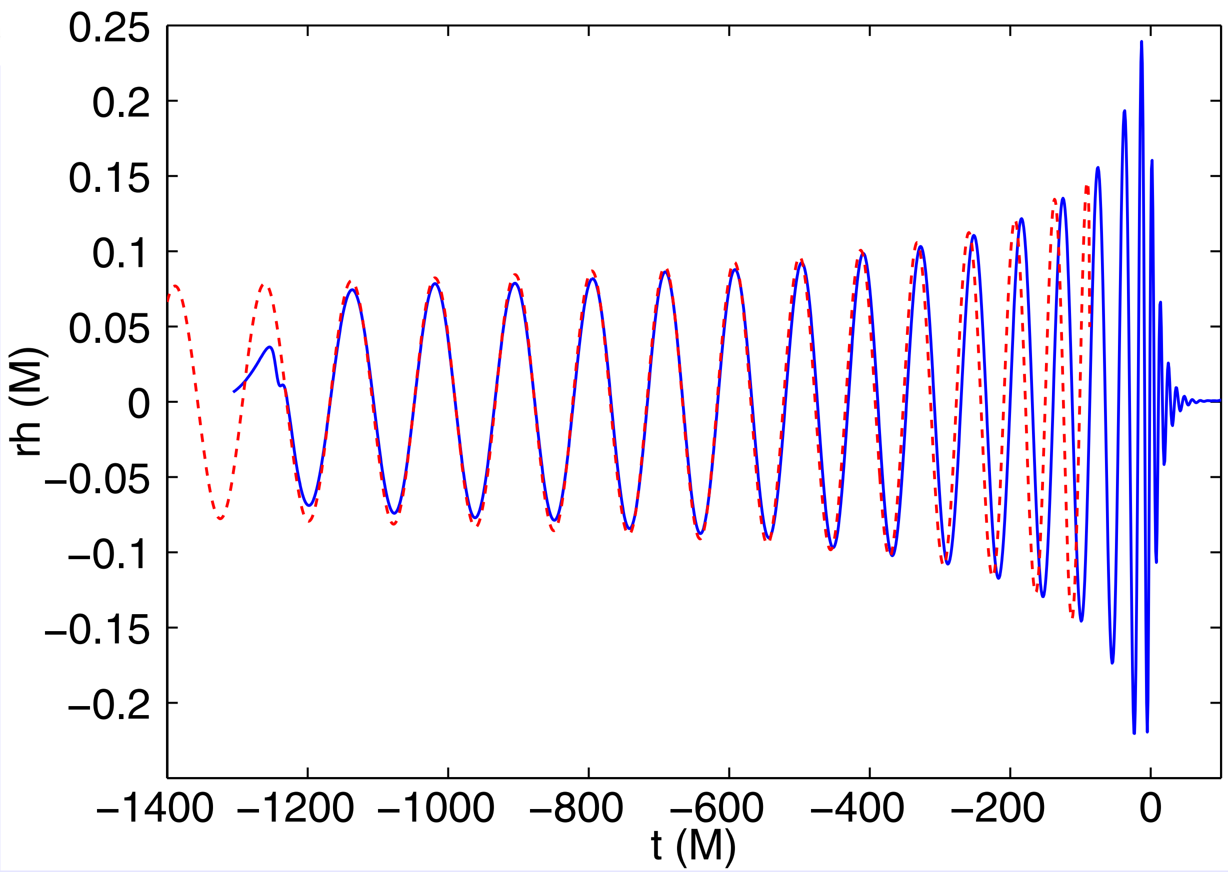


[Pretorius, 2005]

Gravitational waves from binary black holes



[Thorne, 2002]

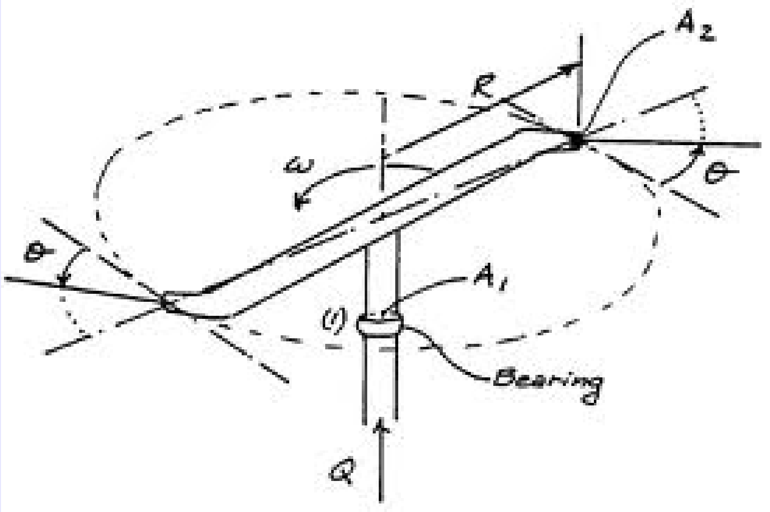


equal-mass,
nonspinning
black holes
[Baker *et.al.*, 2007]

Lawn sprinklers

equal flow from both jets
$\Longrightarrow$ center of mass (COM) static
unequal but steady flow
$\Longrightarrow$ COM describes circle
unequal and increasing flow
$\Longrightarrow$ COM describes outward spiral

Turn off water
$\Longrightarrow$ COM continues to coast in random direction
$\Longrightarrow$ sprinkler recoil...



Black hole recoil

- In asymmetric black hole binaries, one black hole radiates more linear momentum than the other
- Rate of linear momentum loss increases as binary separation decreases
  $\Longrightarrow$ center of mass describes outward spiral
- Process terminates with merger
  $\Longrightarrow$ remnant continues to coast in random direction
  $\Longrightarrow$ "black hole recoil"

Asymmetries can be introduced by unequal masses and/or black hole spin

Unequal masses

Analytical analysis based on lowest-order multipole moments for Newtonian point-mass binary

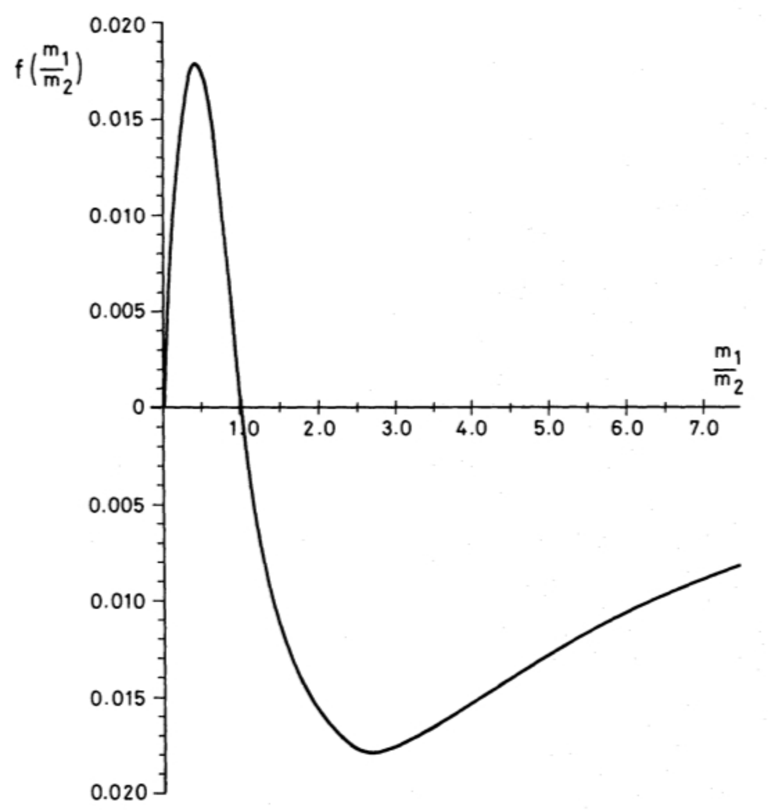
$$V_{\rm kick} \approx 1480 \text{ km/s} \, \frac{f(q)}{f_{\max}} \left(\frac{2M}{r_{\rm term}} \right)^4$$

with

$$f(q) = q^2 \frac{1-q}{(1+q)^5}$$

and $q \equiv m_1/m_2$. [Fitchett, 1983]

$\Longrightarrow$ maximum of possibly several 100 km/s for $q \approx 0.38$

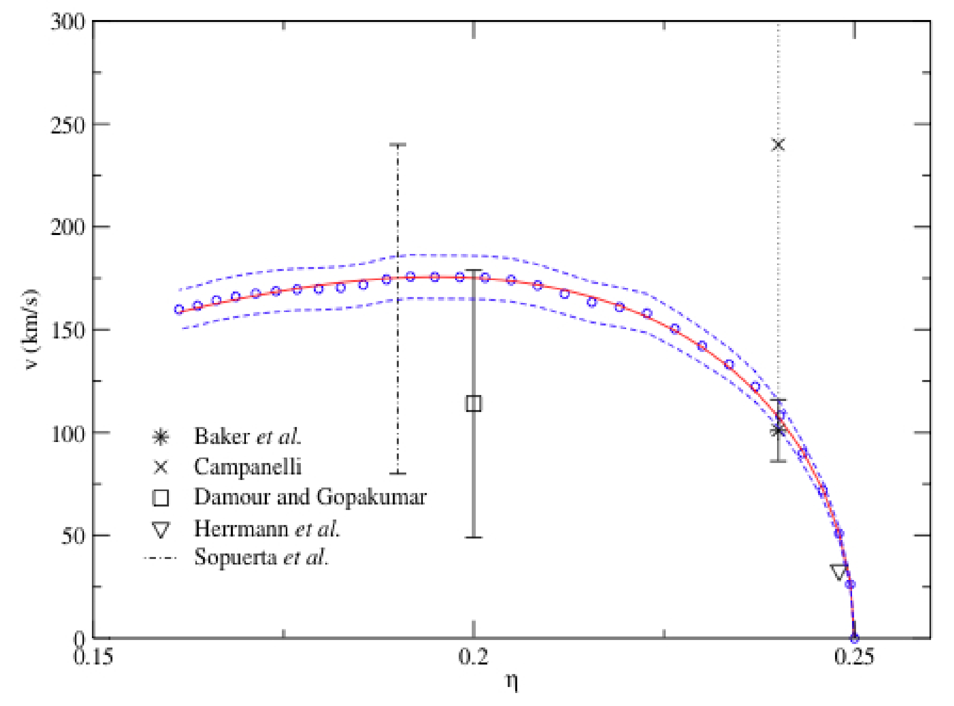


[Fitchett, 1983]

$\Longrightarrow$ analyze numerically!

[Hermann *et.al.*, 2007; Baker *et.al.*, 2006; González *et.al.*, 2007]

Unequal masses: maximum recoil



[González *et.al.*, 2007] (Here $\eta \equiv q/(1+q)^2$)

- maximum recoil for non-spinning black holes $V_{\rm kick}^{\rm mass} \approx 175 \text{ km/s}$
- maximum recoil realized for mass ratio $q = 0.36 \pm 0.03$

Note: recoil speed independent of binary mass

Spinning black holes

Can achieve *much* larger recoil speeds from binary black holes with large, anti-aligned spins in orbital plane

$$V_{\rm kick}^{\rm spin} \lesssim 4000 \text{ km/s}$$

Jan. 29 2007	`gr-qc/0701164v1`	Campanelli *et.al.*	454 km/s
Feb. 8 2007	`gr-qc/0702052v1`	González *et.al.*	2500 km/s
Feb. 25 2007	`gr-qc/0702133v1`	Campanelli *et.al.*	4000 km/s

$\Longrightarrow$ $V_{\rm kick}$ depends strongly on both magnitude and orientation of black-hole spin, but can be very large

$\Longrightarrow$ this raises a host of interesting astrophysical questions...

Astrophysical consequences of black hole recoil

Can we observe recoiling black holes?

- Coalescence of supermassive black holes triggered by merger of their host galaxies
- Recoiling black hole carries accretion disk

○ displacement of AGN from center of host galaxy [Loeb, 2007, Volonteri & Madau, 2008]

○ shocks in accretion disk [Lippai *et.al.*, 2008]

○ flares from shocks in accretion disk [Shields & Bonning, 2008]

○ "Hypercompact stellar systems" around recoiling black holes [Merritt *et.al.*, 2008]

○ relative shift between narrow emission lines (from galaxy) and broad emission lines (from disk) [Bonning *et.al.*, 2007]
$\Longrightarrow$ possible identification of a candidate: SDSS J092712.65+294344.0 (2,650 km/s) [Komossa *et.al.*, 2008; Shields *et.al.*, 2009]

Escape speed from centers of galaxies

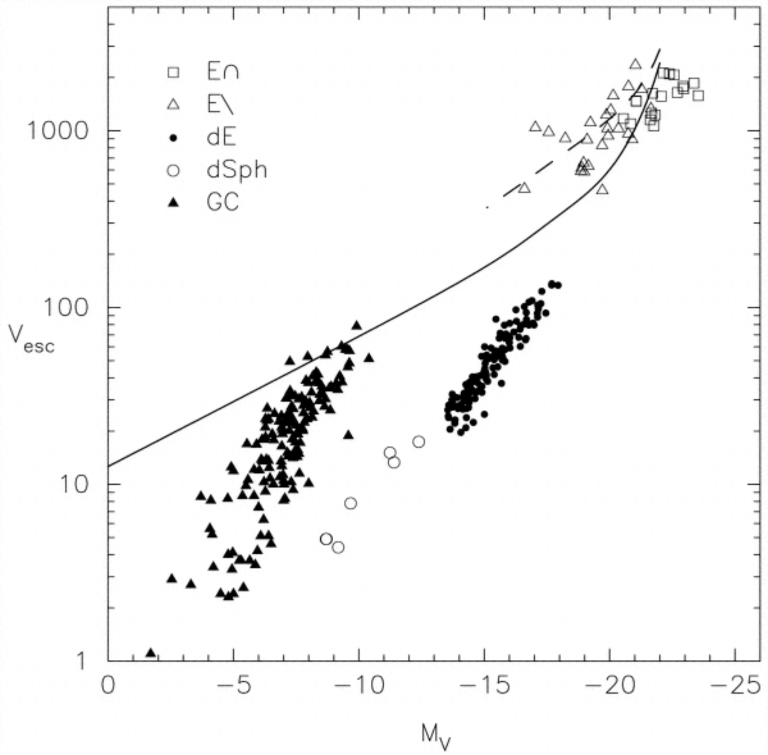
Escape speeds from centers of many galaxies are

$$V_{\rm esc} \lesssim 2000 \text{ km/s}$$

$\Longrightarrow$ recoiling black holes can exceed escape speed

But: galaxies contain massive black holes in their cores

[Richstone *et.al.*, 1998; Ferrarese & Ford, 2005]



[Merritt *et.al.*, 2004]

How come we observe supermassive black holes at cores of galaxies?

What is role of recoil speeds in hierarchical structure formation?

Merger trees

High likelihood of black hole escape may still lead to large fraction of galaxies containing black holes

- probability of galaxy having black hole in $(i+1)$-st generation

$$\begin{aligned} f_{i+1} &= 0 \times (1-f_i)^2 \\ &\quad + f_i(1-f_i) + (1-f_i) f_i \\ &\quad + (1-p_{\rm ej}) f_i^2 \end{aligned}$$

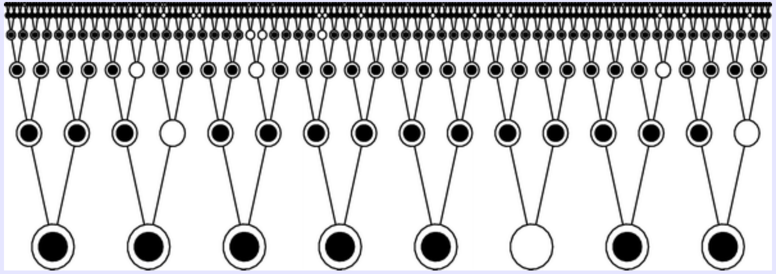
- equilibrium occupation

$$f_\infty = \frac{1}{1+p_{\rm ej}}$$

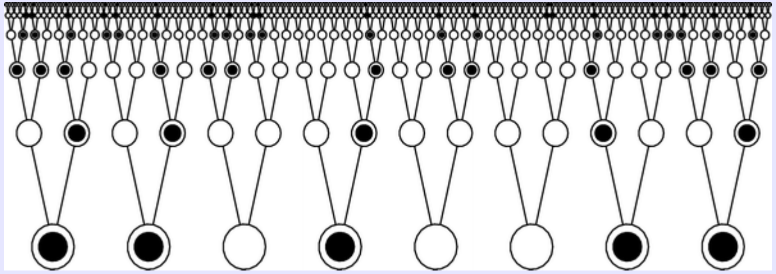
- reached after a few mergers

$p_{\rm ej}$ depends on average recoil speed...

$p_{\rm ej} = 0.1$



$p_{\rm ej} = 0.9$



[Schnittman, 2007]

What is average recoil speed?

... depends on astrophysical conditions (see Monica Colpi's talk)

- mass-ratio
  [Volonteri & Madau, 2008]
- spin
  - ○ magnitude:
    affected by accretion, mergers (minor/major), initial conditions...
    [Gammie *et.al.*, 2004; Berti & Volonteri, 2010]
  - ○ orientation:
    gas-poor galaxies: isotropic distribution
    gas-rich galaxies: spins align
    [Bogdanović *et.al.*, 2007]

For isotropic spin distribution, average speed probably several hundred km/s...

[Schnittman & Buonanno, 2007; Lousto *et.al.*, 2010]

... but much smaller for mergers in gas-rich galaxies

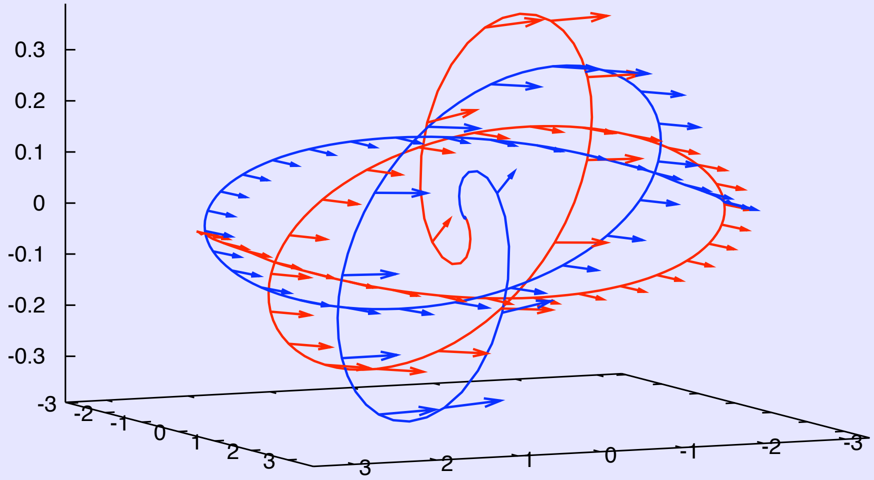
[Bogdanović *et.al.*, 2007; Dotti *et.al.*, 2010; Sijacki *et.al.*, 2010]

Other effects of black hole spin

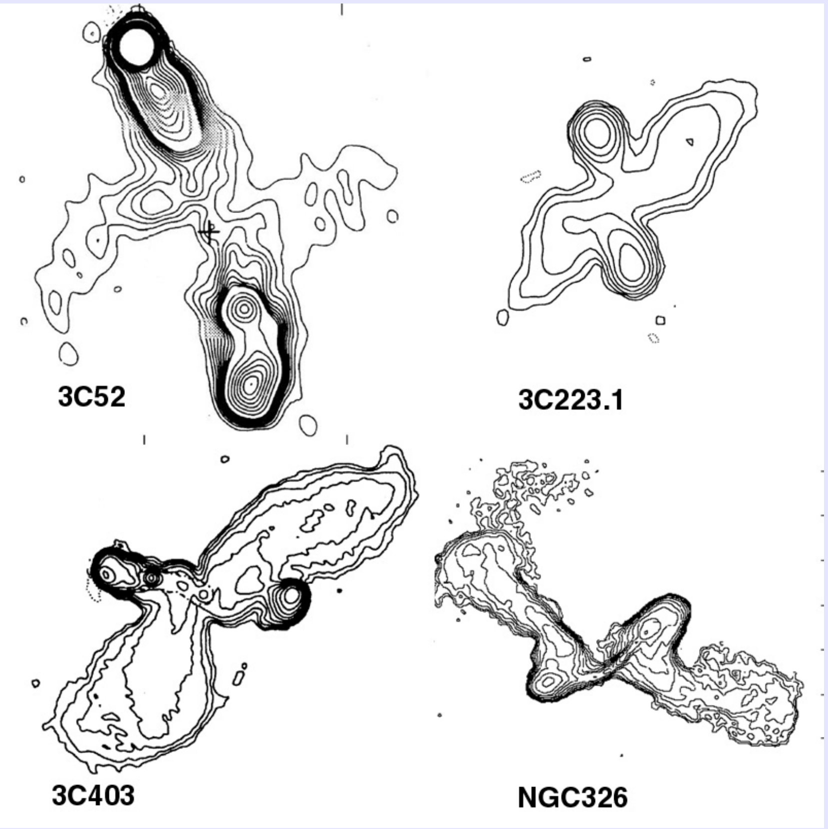
- If black hole spins are aligned with orbital angular momentum, immediate merger would lead remnant spin exceeding Kerr limit

$\Longrightarrow$ "orbital hang-up" [Campanelli $et.al.$, 2006]

- If black hole spins are not aligned, merger may lead to spin flip



[Campanelli $et.al.$, 2007]



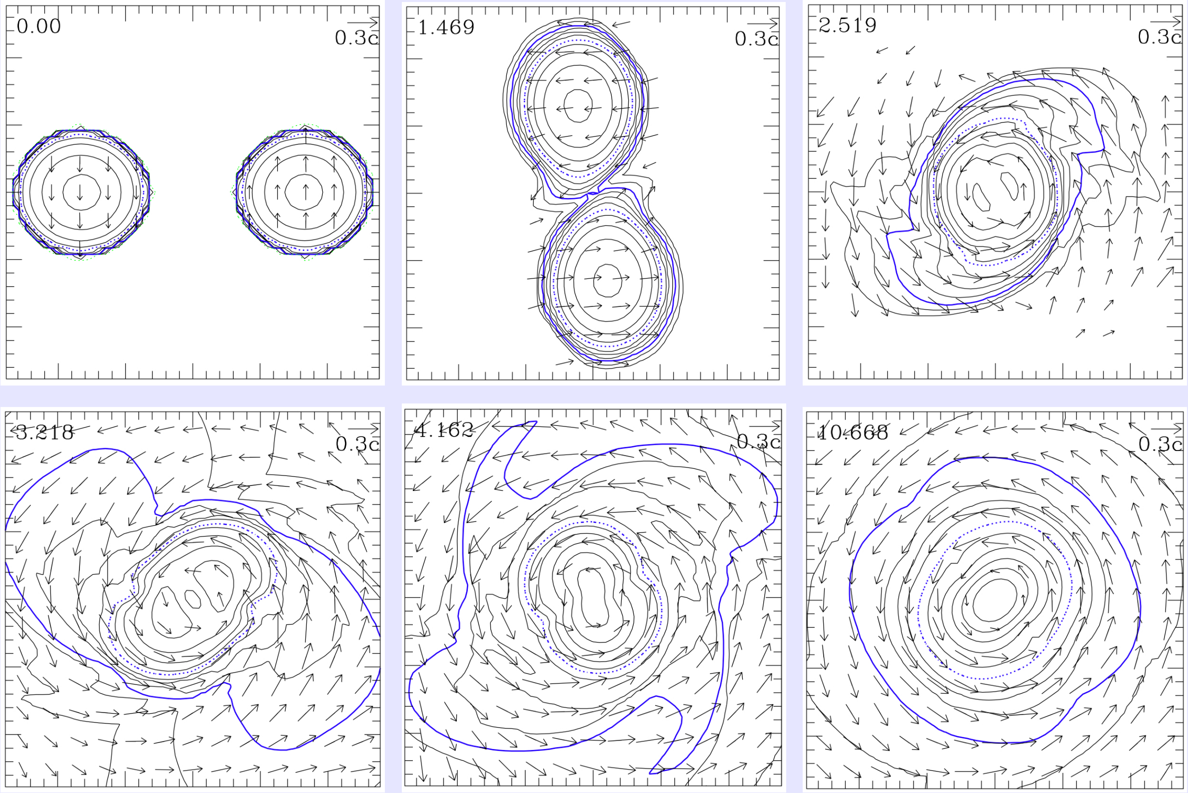
[Merritt and Ekers, 2002]

$\Longrightarrow$ possible explanation for "X-shaped" radio jets?

Before we leave binary black holes...

- Development of codes for binary black holes primarily motivated by prospect of gravitational wave detection
- immediate astrophysical payoffs quite unexpected

Simulations of binary neutron stars

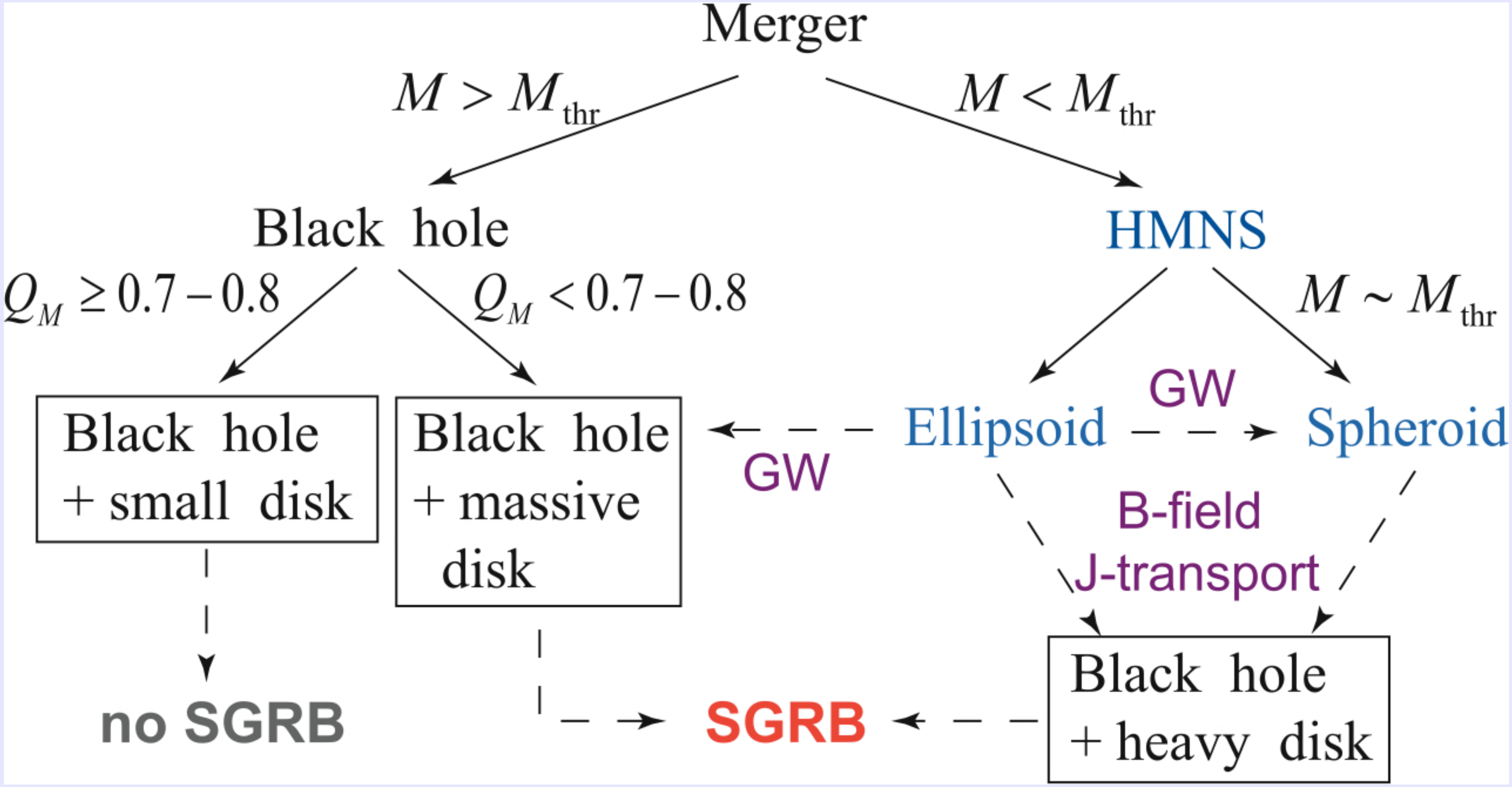


$M_0/M_0^{\max} = 1.075$ [Shibata & Taniguchi, 2006]

- prompt black hole formation only for masses $M > M_{\rm thr} > M_{\rm max}$
- for $M_{\rm thr} > M > M_{\rm max}$ "hypermassive" neutron star supported by differential rotation [Baumgarte *et.al.*, 2000]

Central engines of short gamma-ray bursts?

To launch gamma-ray burst, need sizable accretion disk around black hole...



[Shibata & Taniguchi, 2006]

$\Longrightarrow$ see José Font's talk

- Other possible route to SGRB: mixed black hole-neutron star binaries

New developments

Realistic treatment of many astrophysical processes requires

- general relativistic gravitational fields
- relativistic hydrodynamics
- realistic equation of state
- electromagnetic fields
- neutrino transport
- . . .

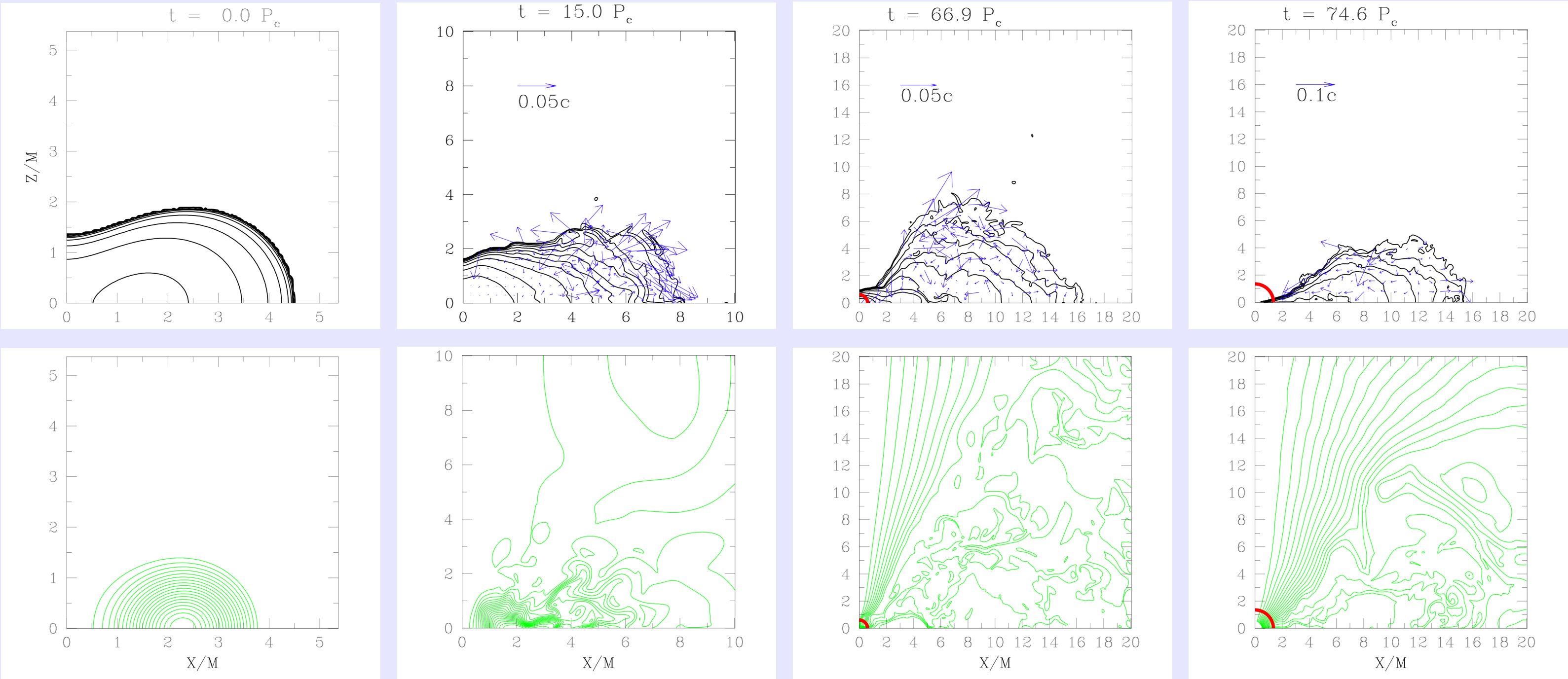
Until recently:

- numerical relativity:
  focus on treatment of relativistic gravitational fields and ideal fluids
- computational astrophysics:
  focus on realistic treatment of many physical processes

$\Longrightarrow$ time to merge efforts

General relativistic magnetohydrodynamics (GRMHD)

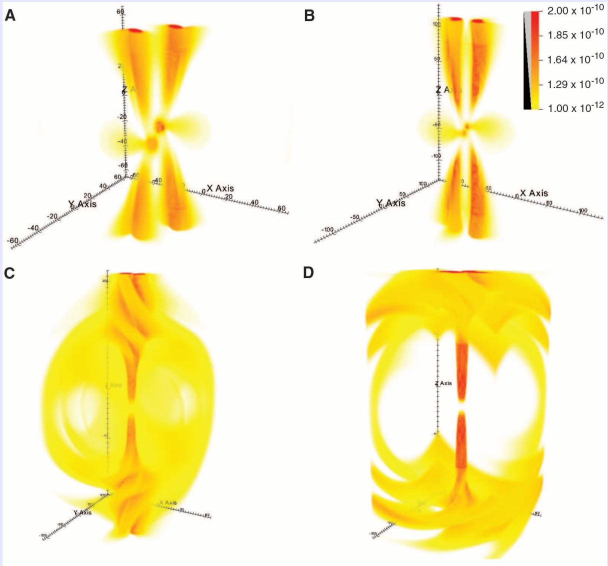
Example: magnetically induced delayed collapse of hypermassive neutron star

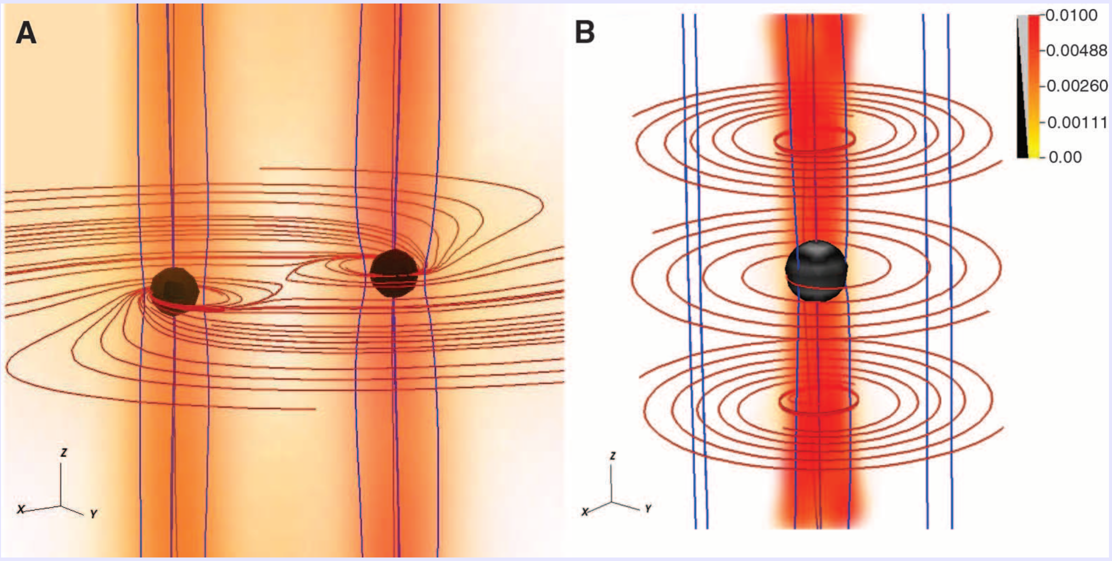


[Duez *et.al.*, 2006]

Maxwell's equations

Example: merger of binary black holes surrounded by magnetized plasma





[Palenzuela $et. al.$, 2010]

$\Longrightarrow$ Blandford-Znajek jets even for non-spinning (but orbiting) black holes

$\Longrightarrow$ gravitational wave signal accompanied by powerful electromagnetic counterpart

$\Longrightarrow$ see Luis Lehner's talk

Future developments

- “numerical relativity” simulations will incorporate more “astrophysics”
  - for example, sophisticated neutrino transport
- “computational astrophysics” will incorporate more “numerical relativity”
  - e.g., efforts of Garching group

⟹ important application: supernovae calculations

Conclusions

Relativistic astrophysics and numerical relativity have always benefitted from each other

- many numerical relativity simulations motivated by astrophysical questions
- many astrophysical questions triggered by numerical relativity results