Cosmic magnetic fields and dark energy in extended electromagnetism

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The problems of dark energy and cosmic magnetic fields

- Cosmological constant: simple and accurate description for cosmic acceleration, but...
- ... a more fundamental explanation of its tiny value would be more satisfactory.
- Large-distance modifications of gravity suggeste
- What about electromagnetism on large scales?
- Its behaviour on astrophysical and cosmological scales is far from clear: unknown origin of magnetic fields observed in galaxies and clusters.

EM quantization in flat space

$$S = \int d^4x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\xi}{2} (\partial_\mu A^\mu)^2 + A_\mu J^\mu \right]$$

$$A_\mu \to A_\mu + \partial_\mu \theta$$

$$\Box \theta = 0$$

Only residual gauge symmetry

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$$\Box(\partial_\mu A^\mu) = 0$$

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$$\partial_\mu A^\mu = 0$$

$$\partial_\mu A^{\mu(+)} |\phi\rangle = 0$$

$$\Updownarrow$$

$$n_0(\vec{k}) = n_\parallel(\vec{k})$$

2 physical states with positive energy

EM quantization in an expanding universe

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Free scalar field non-conformally coupled to gravity

$$\Box (\nabla_\nu A^\nu) = 0$$

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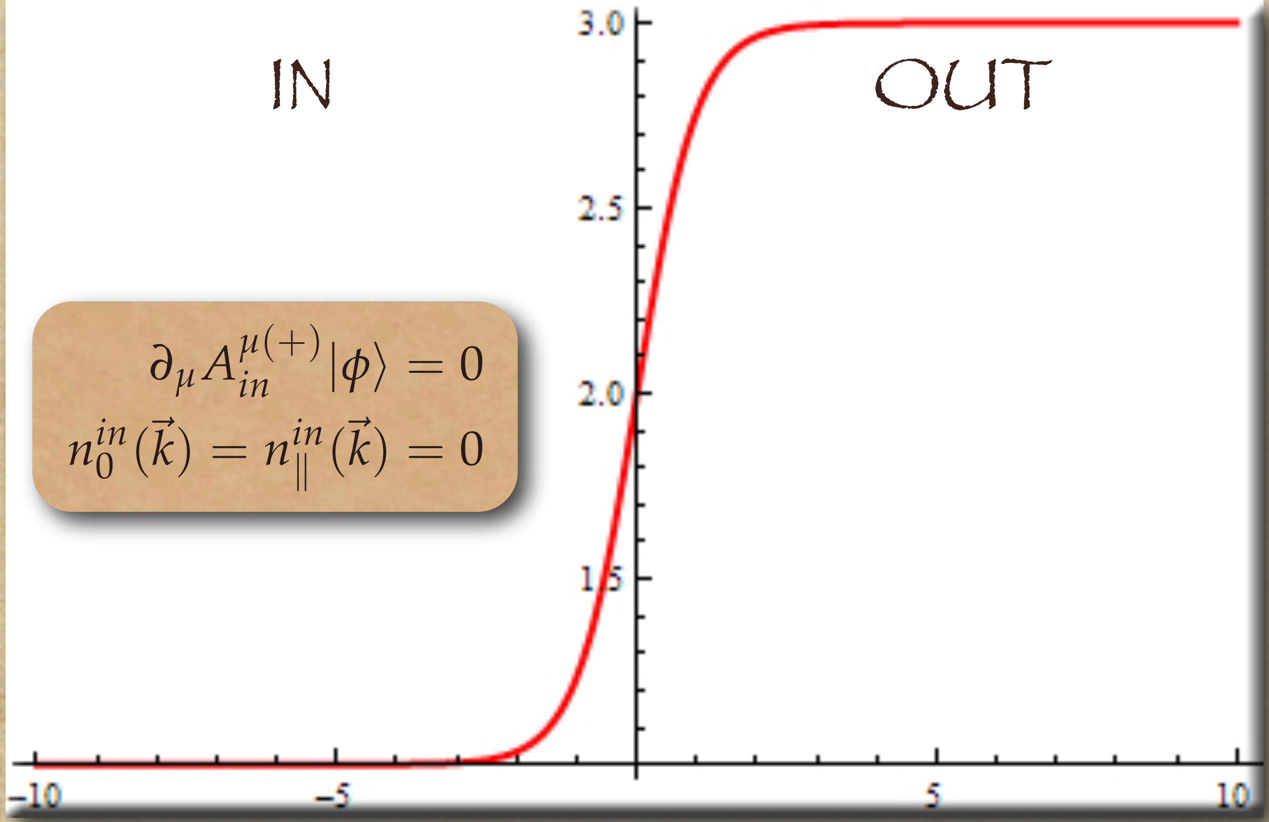
Lorenz condition?

EM quantization in an expanding universe

We consider an expanding universe with two asymptotically Minkowski regions: $a(\eta) = 2 + \tanh(\eta/\eta_0)$



$$\partial_\mu A_{in}^{\mu(+)}|\phi\rangle = 0$$
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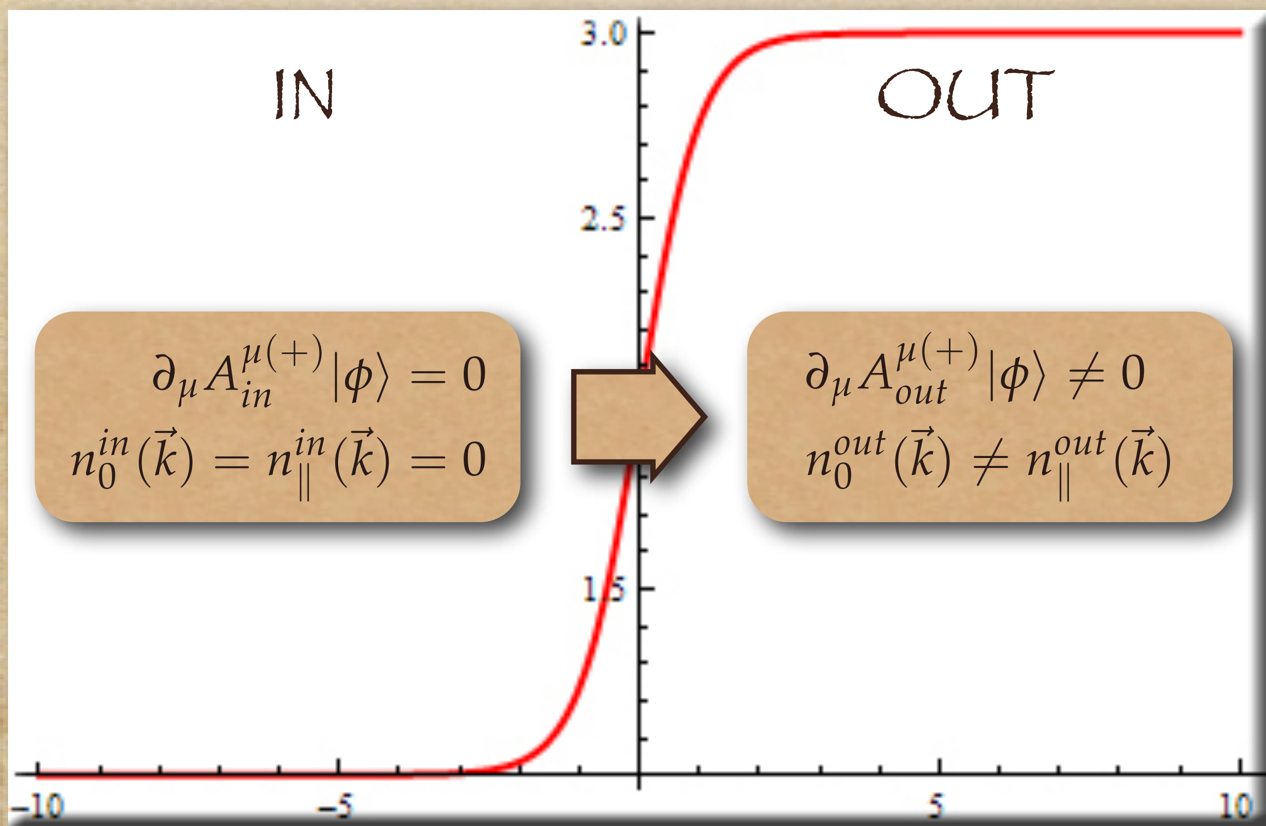

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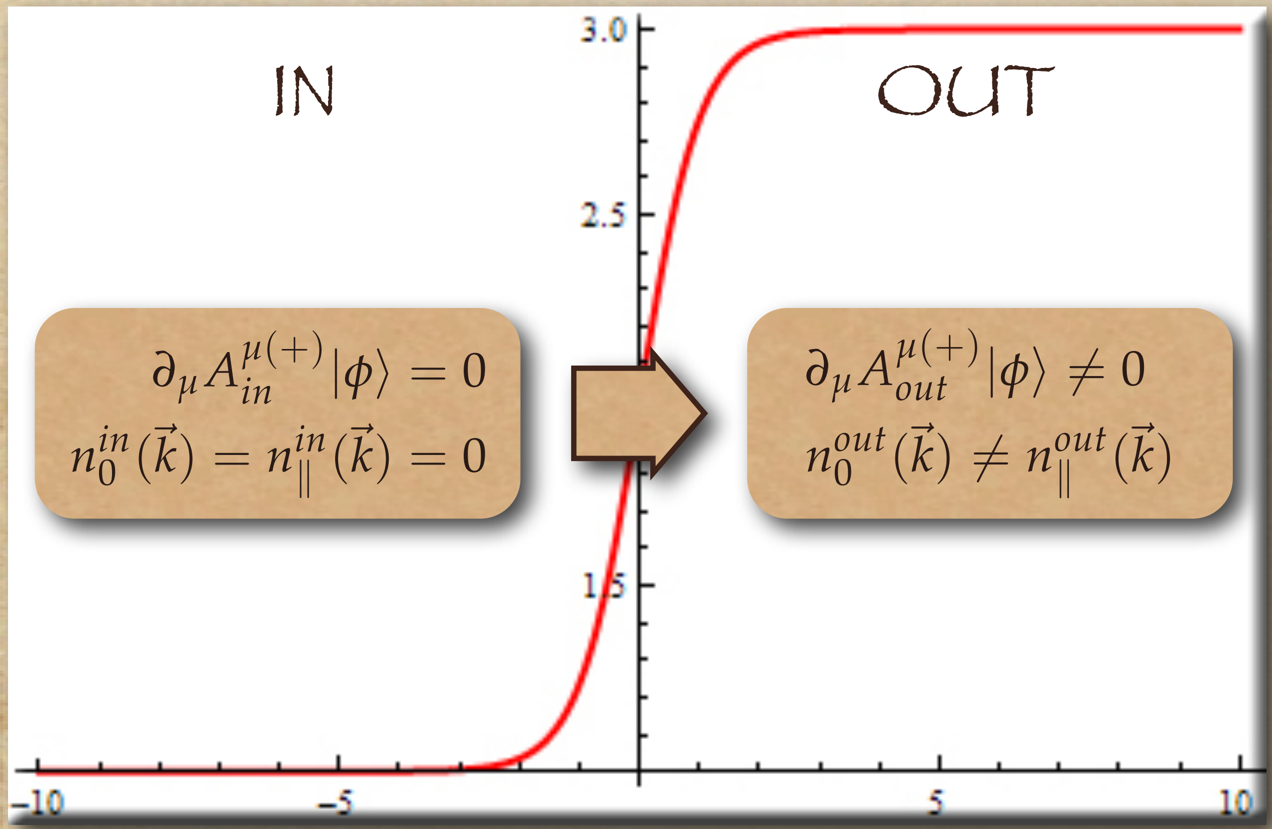

Mixing of positive and negative frequency modes

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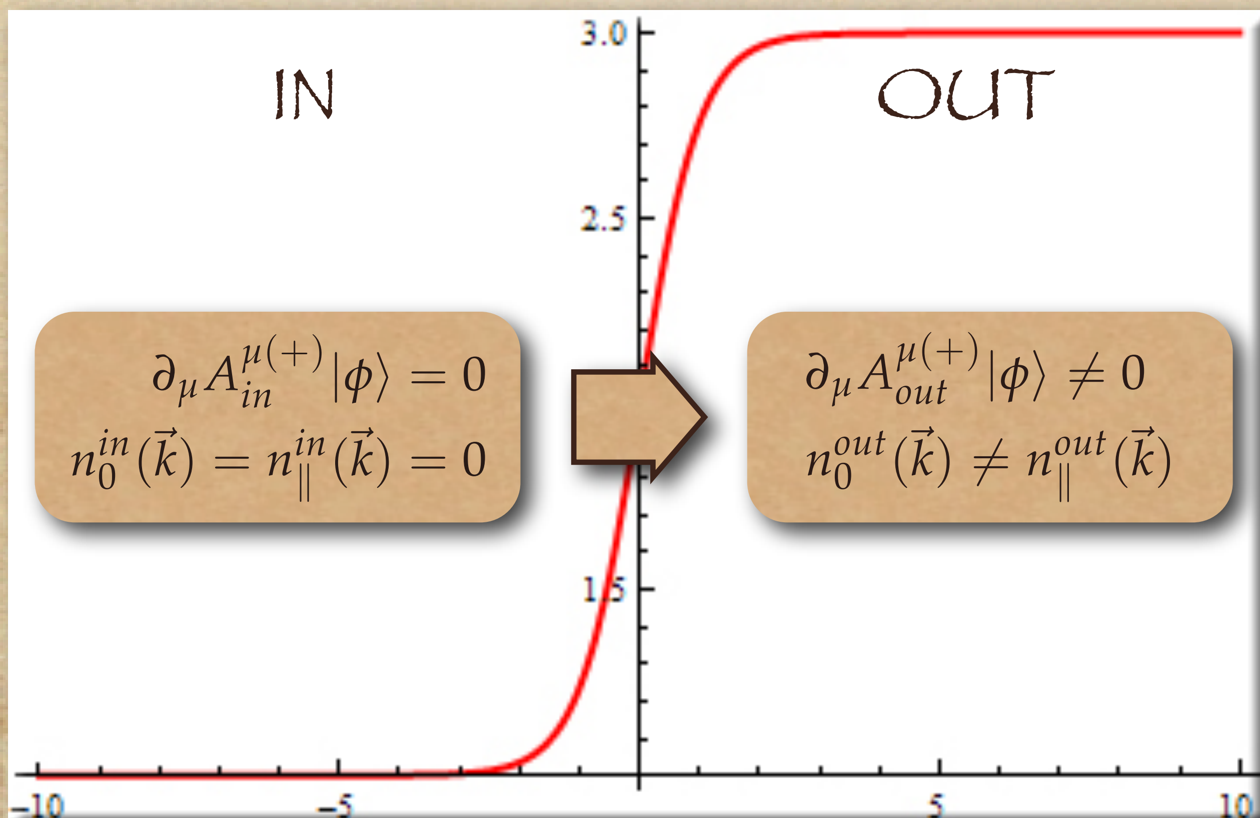

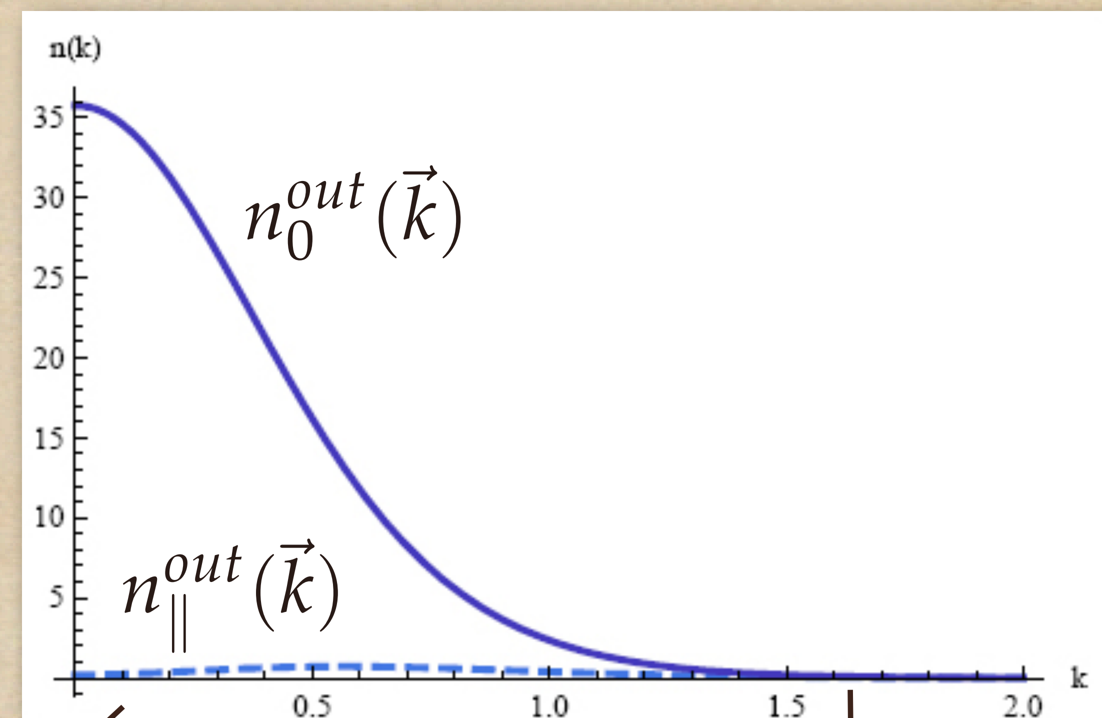

Mixing of positive and negative frequency modes

Breakdown of Lorenz condition on super-Hubble scales

Lorenz condition restored on sub-Hubble scales

Extended EM without the Lorenz condition

Fundamental action for EM

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General solution $A_\mu = A_\mu^{(1)} + A_\mu^{(2)} + A_\mu^{(s)} + \partial_\mu \theta$ Residual gauge mode

Photon New scalar state

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Potential problems

- Modified Maxwell equations
- Unobserved extra polarizations
- Negative energy states
- Conflicts with QED phenomenology

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Photon New scalar state

$$\Box \left(\nabla^\nu A_\nu^{(s)} \right) = 0$$

sub-Hubble: $\nabla_\nu A_{\vec{k}}^{\nu(s)} \simeq \frac{C}{a} e^{-ik\eta}$ ⇒ Maxwell equations ok

super-Hubble: $\nabla_\nu A_{\vec{k}}^{\nu(s)} \simeq$ constant ⇒ Dark energy

Cosmological evolution

$$\ddot{A}_0 + 3H\dot{A}_0 + 3\dot{H}A_0 = 0$$

$$\ddot{\vec{A}} + H\dot{\vec{A}} = 0$$

$$\rho_{A_0} = \frac{\xi}{2}\left(\dot{A}_0 + 3HA_0\right)^2 = \text{constant}$$

$$\rho_{\vec{A}} = \frac{1}{2a^2}(\dot{\vec{A}})^2 \propto \frac{1}{a^4}$$

$$\frac{d}{dt}(\nabla_\mu A^\mu) = \frac{d}{dt}(\dot{A}_0 + 3HA_0) = 0$$

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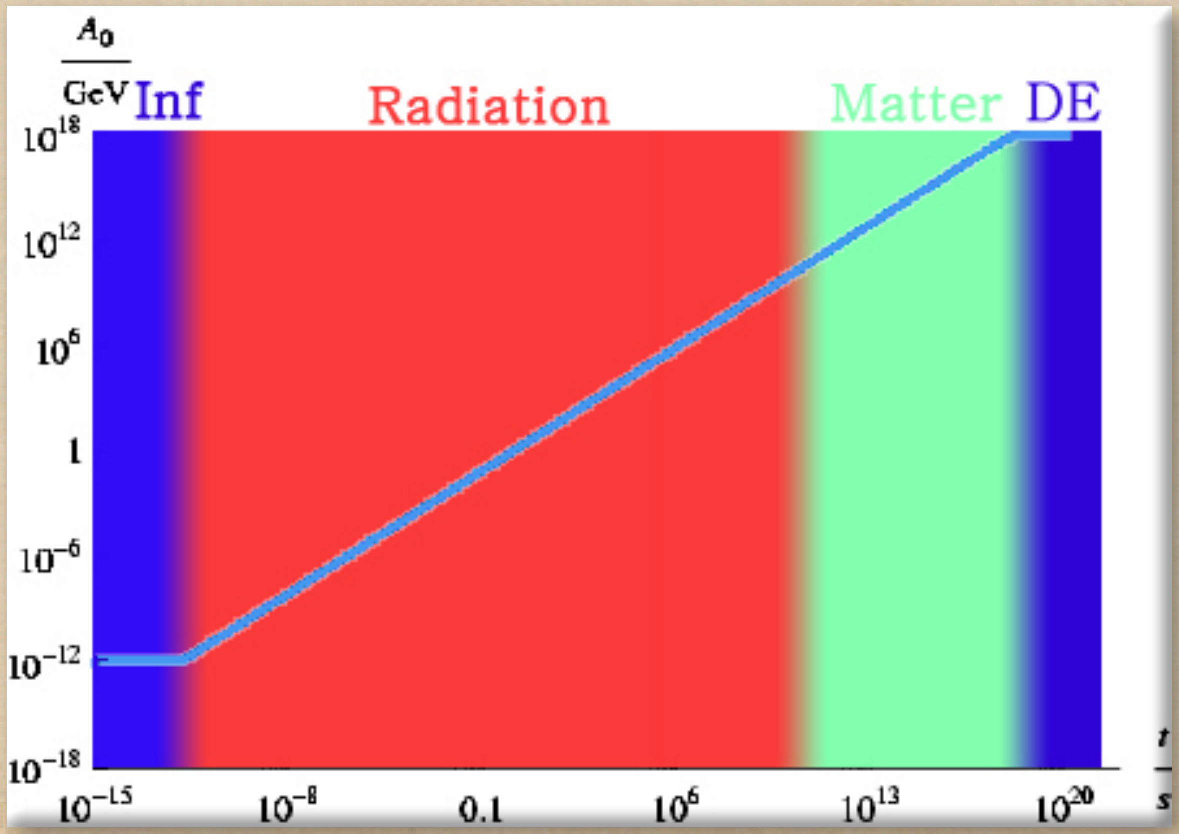

$\Omega_\Lambda \simeq 0.7$

$A_0 \simeq 0.3\ \mathrm{M_P}$

Initial amplitude?

Initial conditions during inflation

Power spectrum generated during inflation from quantum fluctuations

$$\mathcal{P}_{A_0} \equiv 4\pi k^3 |A_{0k}|^2 = \frac{H_I^2}{16\pi^2}$$

Initial amplitude set by Hubble constant during inflation

$$A_{0I}^2 \sim H_I^2$$

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$$\rho_\Lambda \sim (10^{-3}\mathrm{eV})^4$$

$$\rho_{A_0} \sim H^2 A_0^2 \sim H_I^2 A_{0I}^2 \sim H_I^4 \simeq \left(\frac{M_I^2}{M_P}\right)^4$$

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⇒

$$M_I \sim 1\ \mathrm{TeV}$$

Electroweak scale

Initial conditions during inflation

Power spectrum generated during inflation from quantum fluctuations

$$\mathcal{P}_{A_0} \equiv 4\pi k^3 |A_{0k}|^2 = \frac{H_I^2}{16\pi^2}$$

Initial amplitude set by Hubble constant during inflation $A_{0I}^2 \sim H_I^2$

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$$\rho_{A_0} \sim H^2 A_0^2 \sim H_I^2 A_{0I}^2 \sim H_I^4 \simeq \left(\frac{M_I^2}{M_P}\right)^4$$

⇒

$M_I \sim 1\ TeV$

Electroweak scale

Dark energy from physics at the EW scale
Arkani-Hamed et al. PRL 85 (2000) 4434

$$\rho_\Lambda \simeq \left(\frac{M_{EW}^2}{M_P}\right)^4$$

Viability and consistency

Local gravity tests

PPN parameters exactly the same as GR for any value of A_0, so it has the same small scales behavior.

Stability

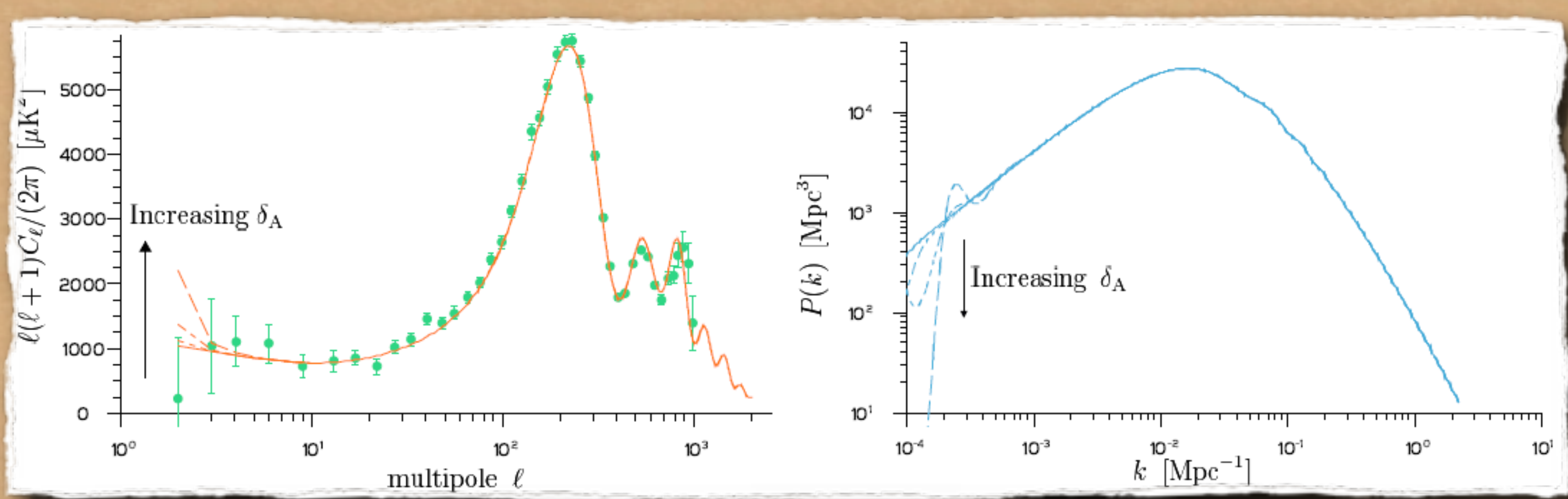
Classical

All the modes propagate at the speed of light.

Quantum

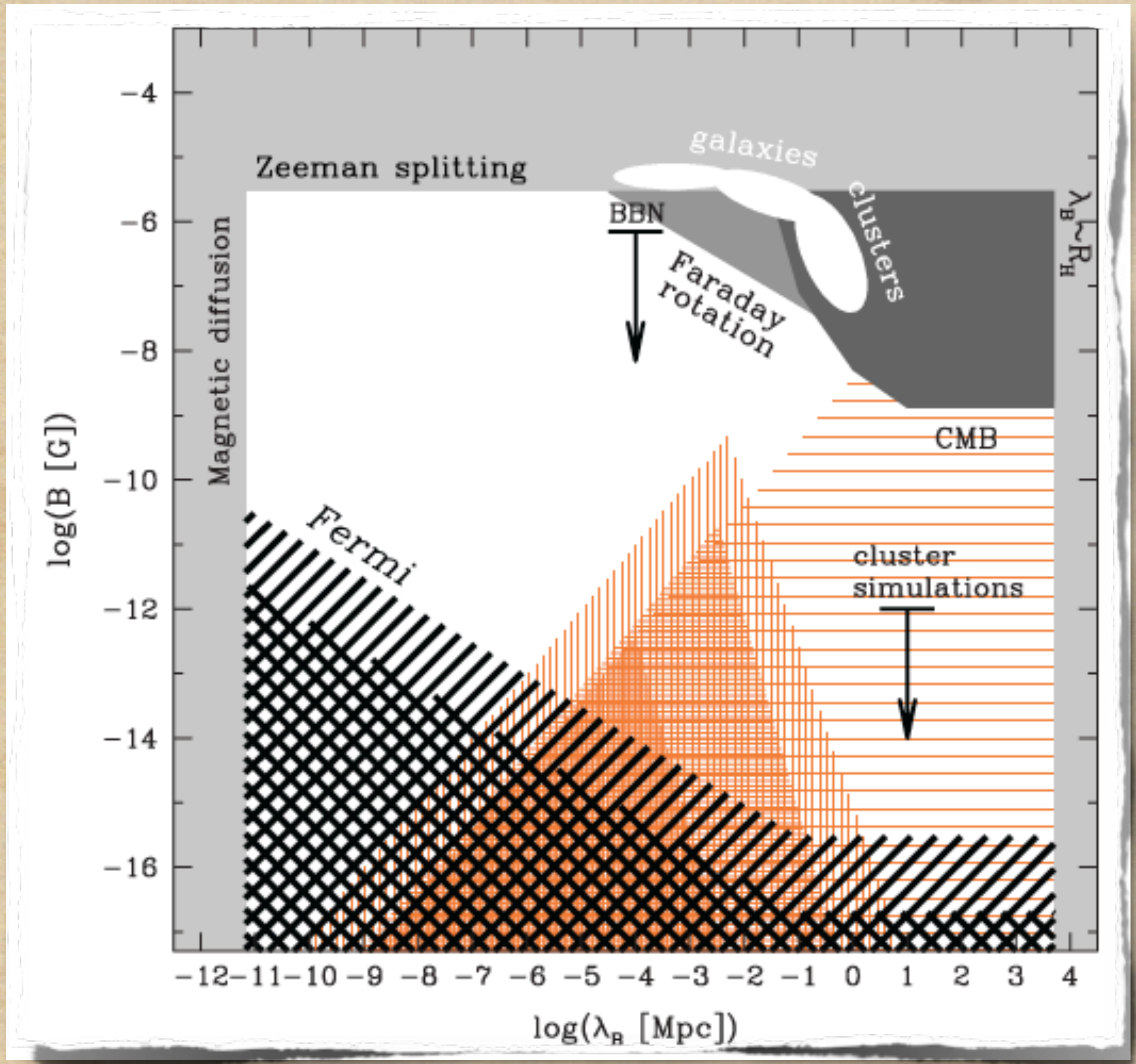
The three physical states carry positive energy.

CMB and LSS



Compatible as long as the initial perturbation is not too large, implying a reduction in the inflation scale by a factor of 15.

Cosmic magnetic fields



A. Neronov & I. Vovk. Science 328, 73 (2010)

- Astrophysical mechanisms: Difficulties to explain intergalactic magnetic fields.
- Inflation-based models: generate super-Hubble modes that are severely constrained by BBN.
- Phase transitions: strongly constrained by causality. Very blue power spectrum leading to weak magnetic fields on large scales.
- Second order perturbations: very weak magnetic fields.

Effective electromagnetic current

$$\nabla_\nu F^{\mu\nu} + \xi \nabla^\mu \left(\nabla_\nu A^\nu \right) = J^\mu$$

Effective electromagnetic current

$$\nabla_\nu F^{\mu\nu} + \xi \nabla^\mu \left(\nabla_\nu A^\nu\right) = J^\mu \quad \longrightarrow \quad \nabla_\nu F^{\mu\nu} = J^\mu_T$$

$$J^\mu_T = J^\mu - \xi \nabla^\mu (\nabla_\nu A^\nu)$$

Effective electromagnetic current

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$\Rightarrow$

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$$\nabla_\mu J^\mu_T = 0$$

Effective electromagnetic current

$$\nabla_\nu F^{\mu\nu} + \xi \nabla^\mu (\nabla_\nu A^\nu) = J^\mu$$

$$\nabla_\nu F^{\mu\nu} = J_T^\mu$$

$$J_T^\mu = J^\mu - \xi \nabla^\mu (\nabla_\nu A^\nu)$$

$$\nabla_\mu J_T^\mu = 0$$

Even if the primordial plasma is electrically neutral, the universe acquires an effective stochastic distribution of charge density given by

$$\rho_g = -\xi \partial_0 (\nabla_\mu A^\mu)$$

Spectrum of effective electric charge

$$P_{\nabla A}(k) = \frac{9H_{k_0}^4}{16\pi^2}\left(\frac{k}{k_0}\right)^{-4\epsilon}$$

$$\Box(\nabla_\nu A^\nu) = 0$$

$$P_\rho(k) = \begin{cases} 0, & k < H_0 \\ \frac{\Omega_M^2 H_0^2 H_{k0}^4}{16\pi^2}\left(\frac{k}{k_0}\right)^{-4\epsilon-2}, & H_0 < k < k_{eq} \\ \frac{2\Omega_M H_0^2 H_{k0}^4}{16\pi^2(1+z_{eq})}\left(\frac{k}{k_0}\right)^{-4\epsilon}, & k > k_{eq}. \end{cases}$$

Super-Hubble modes

Modes entering in the matter era

Modes entering in the radiation era

Cosmic magnetic fields

Ohm's law $J^\mu - u^\mu u_\nu J^\nu = \sigma F^{\mu\nu} u_\nu$ → $E_\mu = 0$

$$F^{\mu\nu}{}_{;\nu} u_\mu = \frac{\epsilon^{\mu\nu\rho\sigma}}{\sqrt{g}} B_\rho u_{\sigma;\nu} u_\mu = J^\mu_{\nabla\cdot A} u_\mu$$ → $\vec{\omega} \cdot \vec{B} = \rho_g^0$

($\vec{\omega}$: $\propto a$)

$$\begin{aligned}\langle B_i(\vec{k}) B_j^*(\vec{h})\rangle &= \frac{(2\pi)^3}{2}(\delta_{ij} - \hat{k}_i\hat{k}_j)\delta(\vec{k}-\vec{h}) B k^n \\ \langle \omega_i(\vec{k}) \omega_j^*(\vec{h})\rangle &= \frac{(2\pi)^3}{2}(\delta_{ij} - \hat{k}_i\hat{k}_j)\delta(\vec{k}-\vec{h}) \Omega k^m\end{aligned}$$

Cosmic magnetic fields

Amplitude of the magnetic field at a scale λ

$$B_\lambda^2 \simeq \frac{4\pi\rho_\lambda^2 G(\lambda,n)G(\lambda,m)}{\omega_\lambda^2 S(\lambda,n,m)}.$$

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CMB constraints on the vorticity

$$\omega_\lambda^2 \lesssim 10^{-10}\frac{z_{rec}^2 G(\lambda,m)}{8l^3(l+1)R(l,m)}$$

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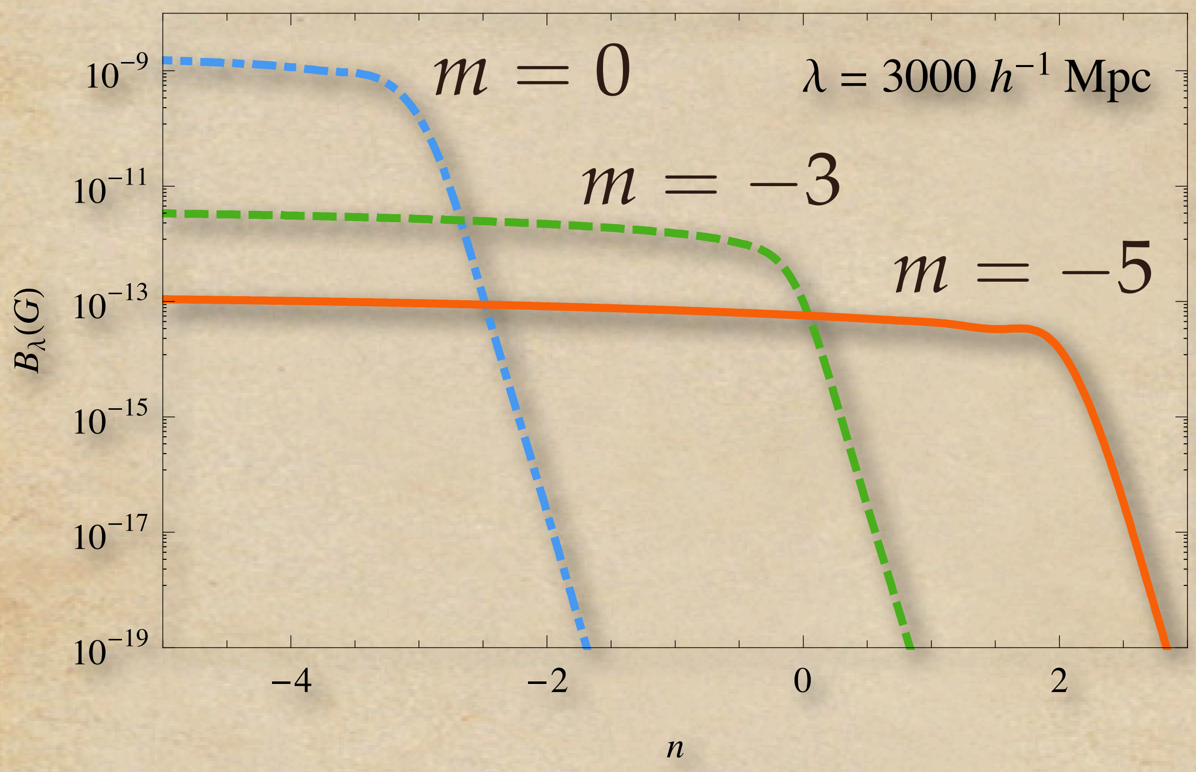
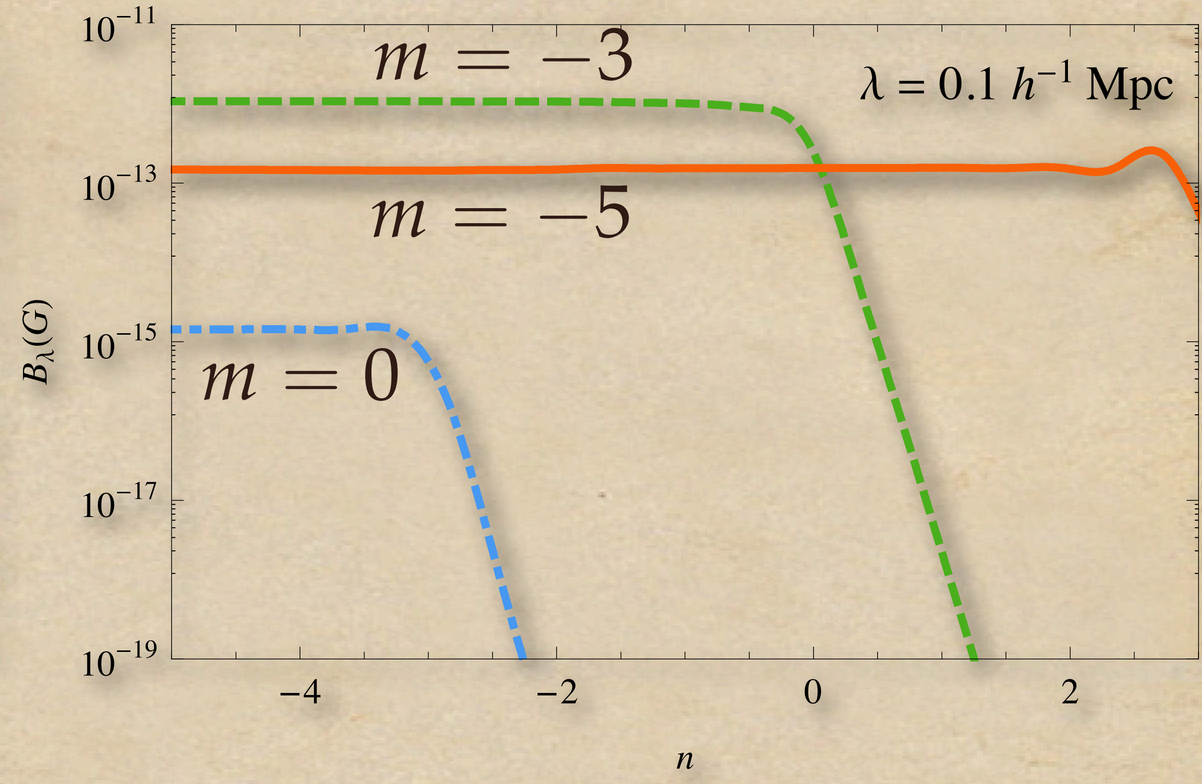
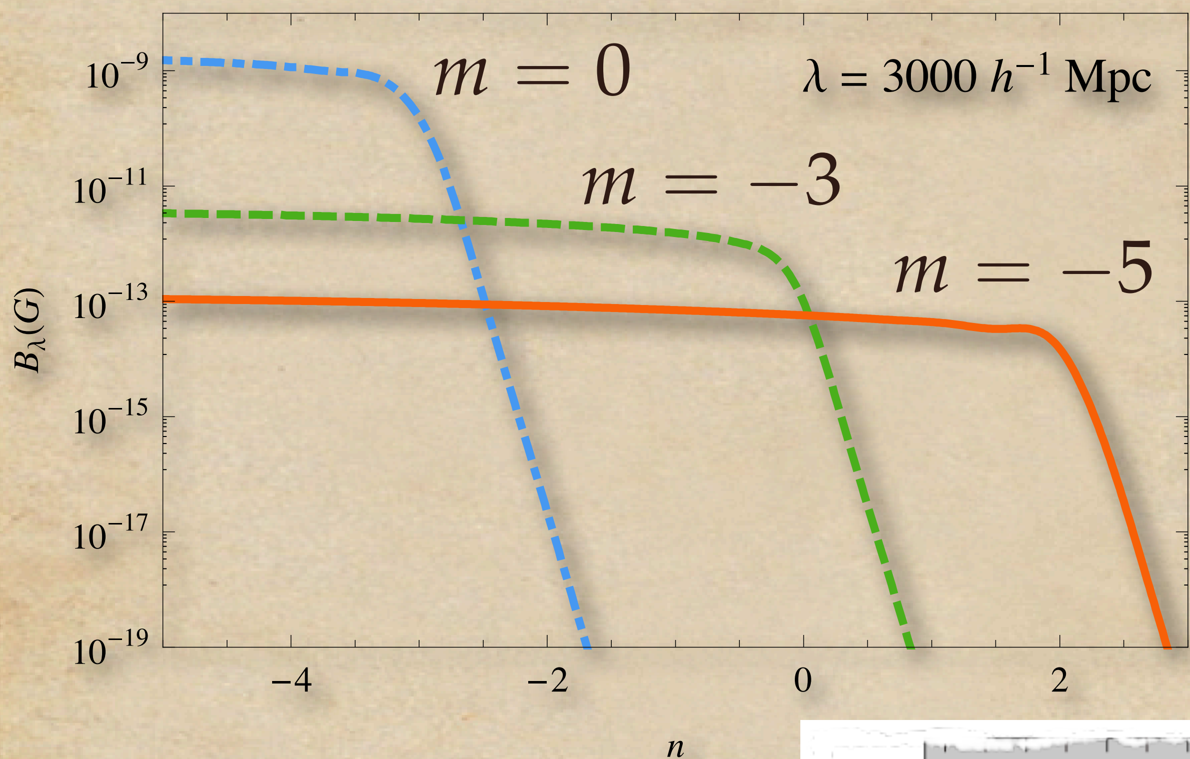
Upper limits on vorticity impose “lower” limits on B

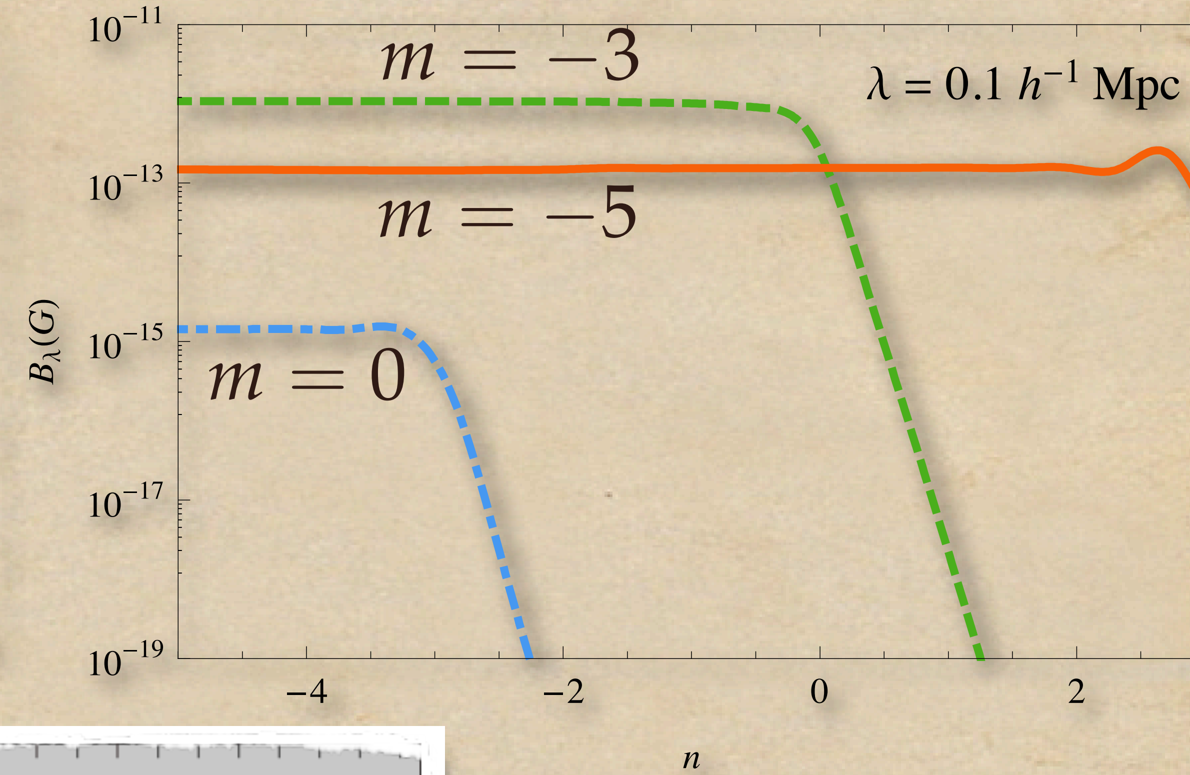
Cosmic magnetic fields

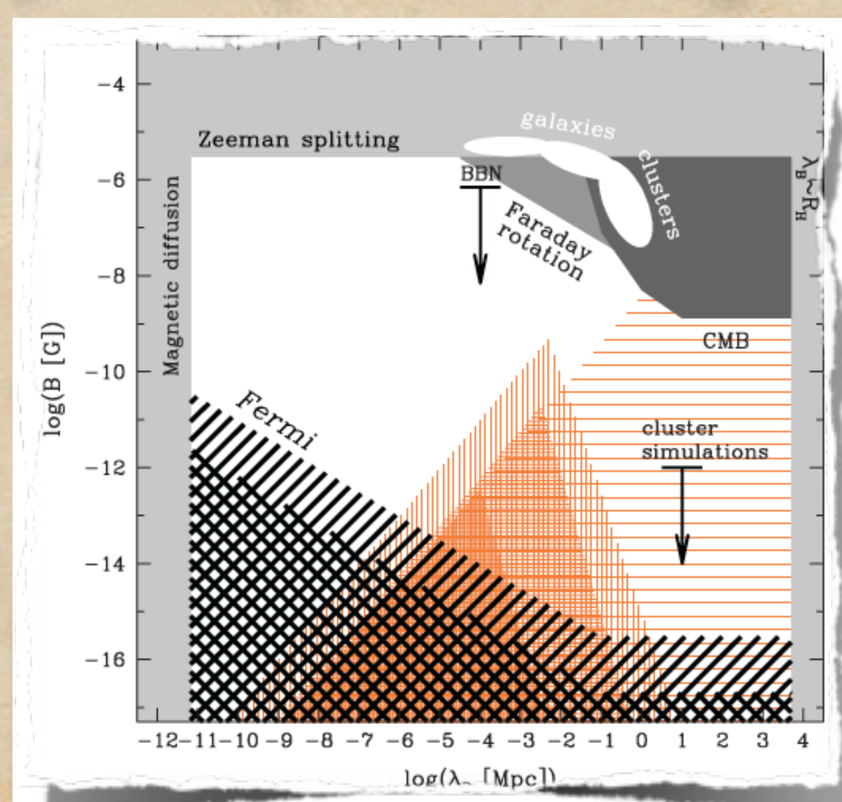




Cosmic magnetic fields







Conclusions

- EM field can be consistently quantized with three physical states without the need of Lorenz condition.
- Quantum fluctuations of the new state during an inflationary epoch at the electroweak scale give rise to an effective cosmological constant on large scales with the correct value.
- The model satisfies all the viability conditions and it is in agreement with CMB and LSS measurements.
- The true nature of dark energy can be established without resorting to new physics.
- Strong cosmic magnetic fields can be generated on large scales.

EM quantization without Lorenz condition

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Photon New scalar state

The gauge-fixed QED effective action in the path-integral formalism in flat spacetime is:

$$e^{iW} = \int [dA][dc][d\bar{c}][d\psi][d\bar{\psi}] e^{i \int d^4x \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\xi}{2} (\partial_\mu A^\mu)^2 + \partial_\mu \bar{c}\, \partial^\mu c + \mathcal{L}_F \right)}$$

$$\propto \int [dA][d\psi][d\bar{\psi}] e^{i \int d^4x \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\xi}{2} (\partial_\mu A^\mu)^2 + \mathcal{L}_F \right)}$$

which coincides with the considered action for flat spacetime.

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Potential problems

- Modified Maxwell equations ✔
- Charge conservation ✔
- Unobserved extra polarizations ✔
- Modified interactions with charged particles ✔