Models of Relativistic Neutron Stars with surface *Crust*

With applications to giant glitches of Vela Pulsar

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2. Construction of Models of Neutron Star with Surface Crust
3. Study of the Properties of the Model using Realistic Equations of State
4. Application to Giant Glitches of the Vela Pulsar
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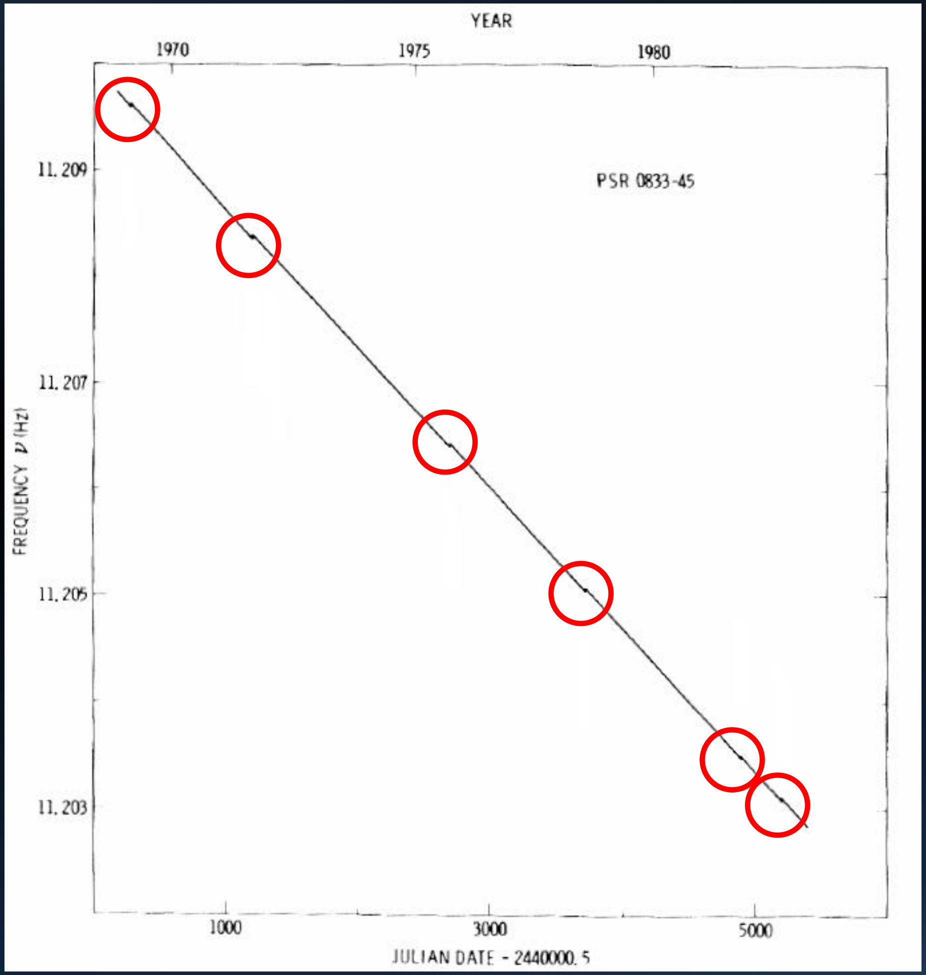
1. Introduction

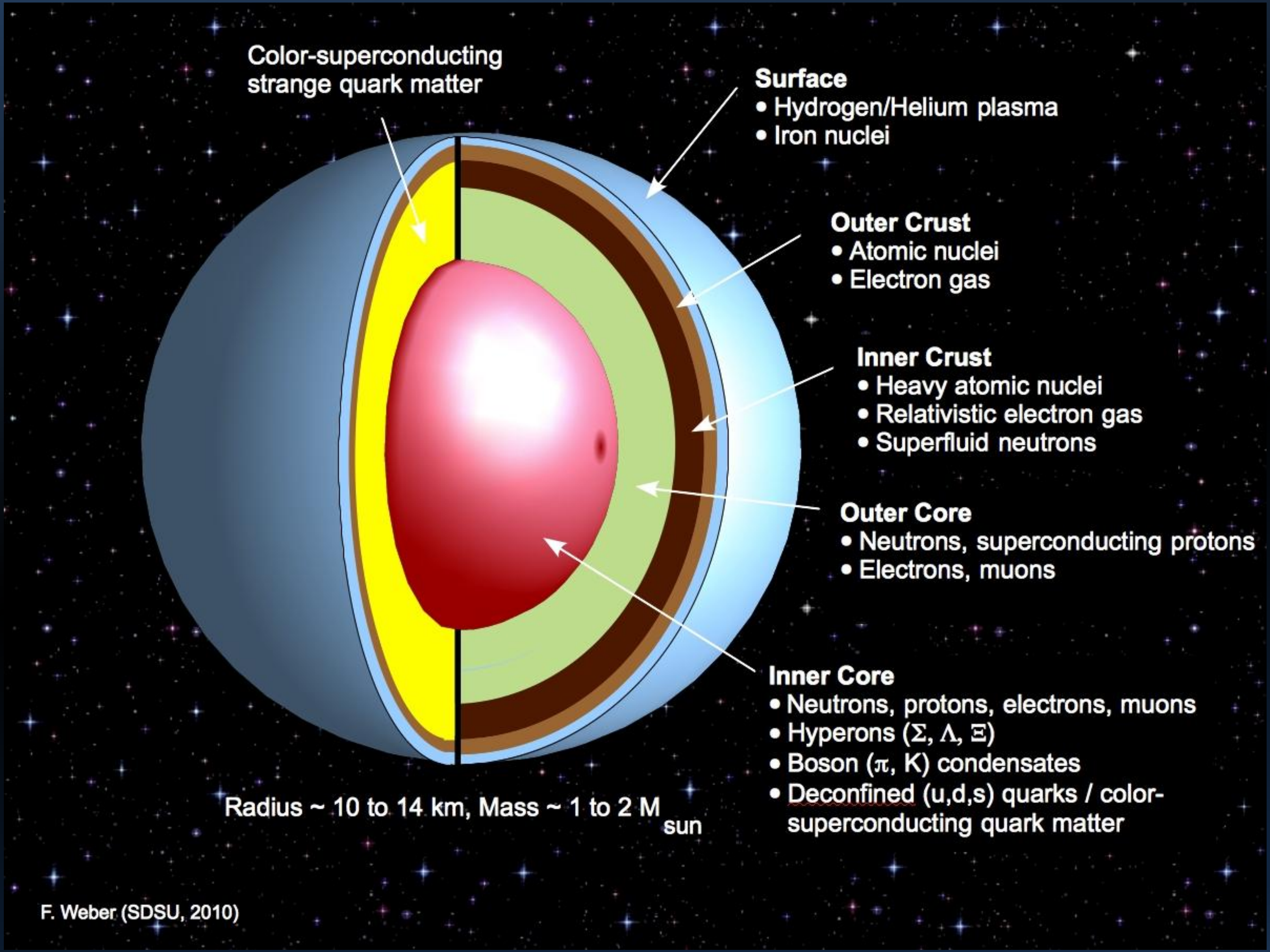
- Neutron stars
  - High density 10^{14} g cm^{-3}
  - Compact objects 1.44 $M_{\odot}$, 10 Km → General Relativity

- Pulsars: Rotating neutron star
  - ➢ Gradual spin down (loss of angular momentum in radiation)
  - ➢ Sudden spin jumps
    Glitches

- Vela pulsar giant glitches: relative period variation of 10^{-6}



- Related to the layer structure of neutron stars:



- The crust rupture releases energy:

  $10^{41} - 10^{43}$ erg ⟶ Heating of the inner crust (10^6 K)

  (Hirano et al. 1997; Van Riper, Epstein & Miler 1997)

- Variation of the equation of state in the inner crust region.

  Increase in the core-crust transition pressure order 10^{-10}

✓ Most of the crustal matter is in the inner crust

(Pethick, Lorenz & Ravenhall 1995)

✓ Dynamical properties depend strongly on the transition pressure

(Lattimer & Prakash 2001, Cheng, Yuan & Zhang 2002)

↓

We can approximate the crust to a surface energy layer

In this model, the core-crust transition pressure arise as an essential parameter of a configuration

2. Construction of models of Neutron Star with surface crust

2. Construction of models of Neutron Stars with surface crust

- Neutron Star in permanent rigid rotation (constant angular velocity Ω)
  Stationary space-time with axial symmetry
- For typical second and millisecond pulsars $\Omega < \sqrt{\frac{M}{a^3}} \equiv \Omega_S$
- Hartle-Thorne perturbative solution (Hartle 1967; Hartle & Thorne 1968) up to second order in Ω:

$$ds^2 = -e^{2\psi(r)}(1+2h(r,\theta))dt^2 + e^{2\lambda(r)}\left(1+\frac{2m(r,\theta)}{r-2M(r)}\right)dr^2 + r^2(1+2k(r,\theta))\left[d\theta^2 - \sin^2\theta(d\varphi - \omega(r,\theta)dt)^2\right]$$

Order zero — Order Ω — Order Ω^2

- Matter in the core of the star → perfect fluid with realistic equation of state

$$p = p(\rho)$$

2. Construction of models of Neutron Stars with surface crust

- In the surface of the star (R=a) we introduce a surface layer of energy:

$$S_c^{\mu\nu}(a) = -\rho_c(\theta) u_c^{\mu} u_c^{\nu}$$

$$\rho_c(\theta) = \varepsilon + \delta\varepsilon(\theta) = \varepsilon + \delta\varepsilon_0 + \delta\varepsilon_2 P_2(\theta)$$

Zero order surface energy density (ε)

Ω^2 order surface energy density ($\delta\varepsilon(\theta)$)

- Surface layer of energy → perfect fluid in rigid rotation

Angular velocity Ω_c

2. Construction of models of Neutron Stars with surface crust

- Matching between the interior and exterior solutions:

Border of the star Σ (R=a) → usual junction conditions (intrinsic formulation)

1) Continuity of the first fundamental form:

$$\Delta[h_{\mu\nu}(a)] = h_{\mu\nu}|_{\Sigma_{ext}} - h_{\mu\nu}|_{\Sigma_{int}} = 0$$

2) Discontinuity in the second fundamental form:

$$(n^{\rho} n_{\rho})(\Delta[\chi_{\nu}^{\mu}(a)] - \Delta[\chi_{\rho}^{\rho}(a)]\delta_{\nu}^{\mu}) = 8\pi S_{c\nu}^{\mu}$$

- Expansion up to second order in Ω of these two conditions :

- Zero order

  - Mass function condition:

$$M_{ext} = M_{int} + 4\pi a^2 \varepsilon \sqrt{1 - \frac{2M_{int}}{a}} - 8\pi^2 a^2 \varepsilon^2$$

Zero order mass of the interior of the star

$$M_{int} = 4\pi \int_0^a \rho(R) R^2 dR$$

Surface crust mass $4\pi a^2 \varepsilon$ with relativistic correction $\sqrt{1 - \frac{2M_{int}}{a}}$

Negative contribution.

Bounding energy term proportional to ε^2

❖ Zero order

- Transtition pressure condition:

$$\frac{M_{ext}}{a^2\sqrt{1-\frac{2M_{ext}}{a}}}-\frac{M_{int}+4\pi a^3 p_{int}}{a^2\sqrt{1-\frac{2M_{int}}{a}}}=4\pi\varepsilon$$

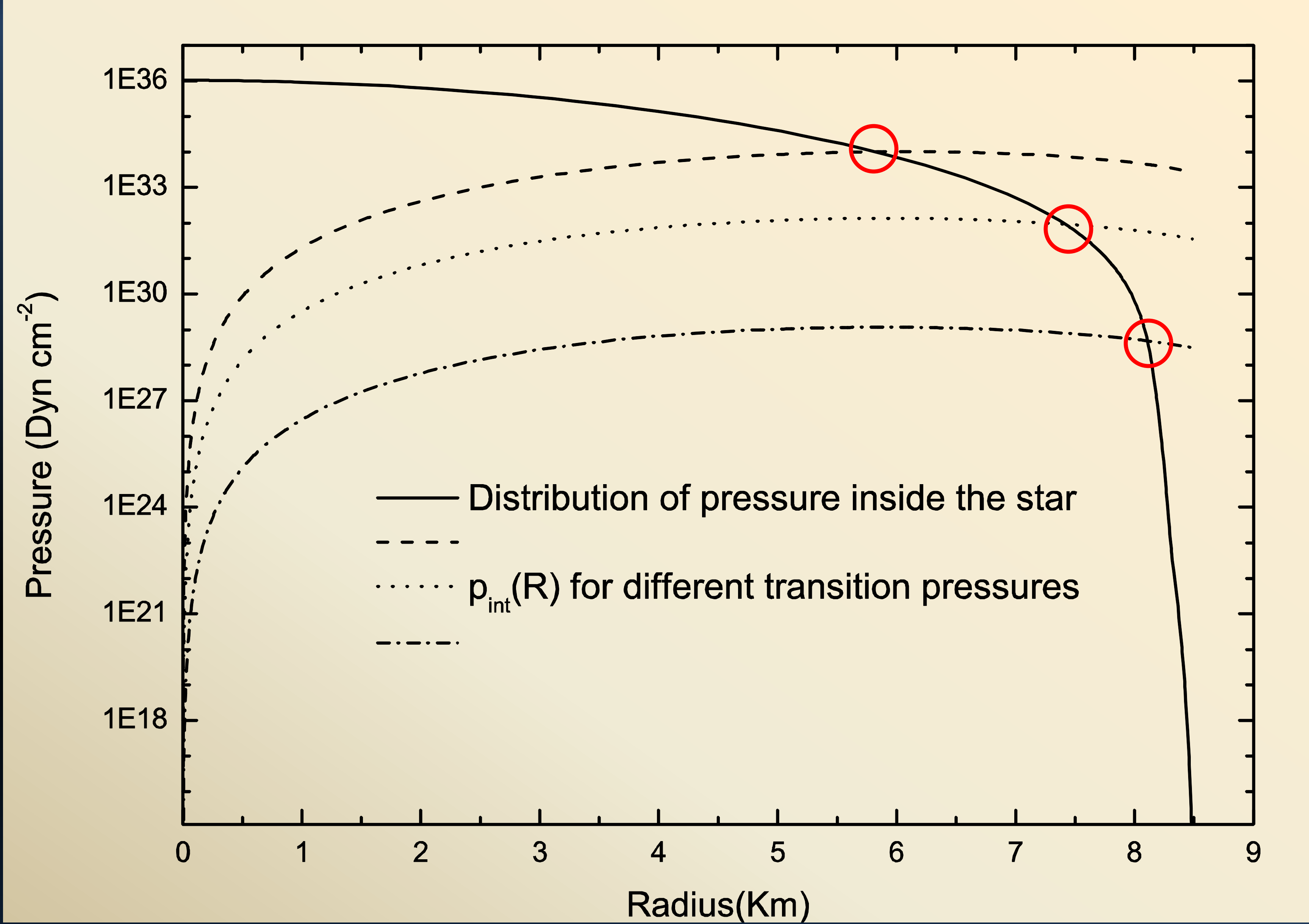
Zero order surface energy density

↓

Discontinuity of the pressure in the surface of the star.

We interpret this pressure as the core-crust transtition pressure.
Determines the radius of the neutron star with surface crust configuration.

2. Construction of models of Neutron Stars with surface crust



- <u>Ω order</u>

  - Continuity in the inertial dragging

$$\omega(a)_{ext} = \omega(a)_{int} \equiv \omega(a)$$

  - Expression for total angular momentum:

$$J = \frac{1}{6} a^4 e^{\lambda(a)_{ext}} \left(e^{-\lambda(a)_{int}} \left[\partial_R \overline{\omega}(a) \right]_{int} + 16\pi\varepsilon \overline{\omega_c}(a) \right)$$

Always positive

$$\overline{\omega_c} = \Omega_c - \omega(a)$$

$\omega_c < 0$ if $\Omega_c < 0$

There are configurations with null total angular momentum

J=0

❖ Ω^2 order

- Continuity in the mean radius and surface eccentricity
- Conditions for second order mass perturbation and surface mass distribution
- Determination of the second order surface energy density
- Condition for the angular velocity of the surface crust:

$$\Omega_c = \begin{Bmatrix} \Omega \\ 2\omega(a) - \Omega \end{Bmatrix}$$

→ Core-crust co-rotating configuration

→ Core-crust contra-rotating configuration

3. Study of the Properties of the Model using Realistic Equations of State

3. Study of the properties of the model using realistic EoS

- Model of neutron star with surface crust:

    - For a fixed Equation of State:
        - Central density ρ_c
        - Core angular velocity Ω

    - The Equation of State is fixed choosing:
        - Core of the star $p=p(\rho)$
        - Transition pressure p_{int}

→ 3 parameters

- Equations of state used in this work

I. Numerical equation of state
High density (Glendening) + Low density (BPS)

II. Analytical fit to SLy Equation of State
P. Haensel and A. Y. Potekhin

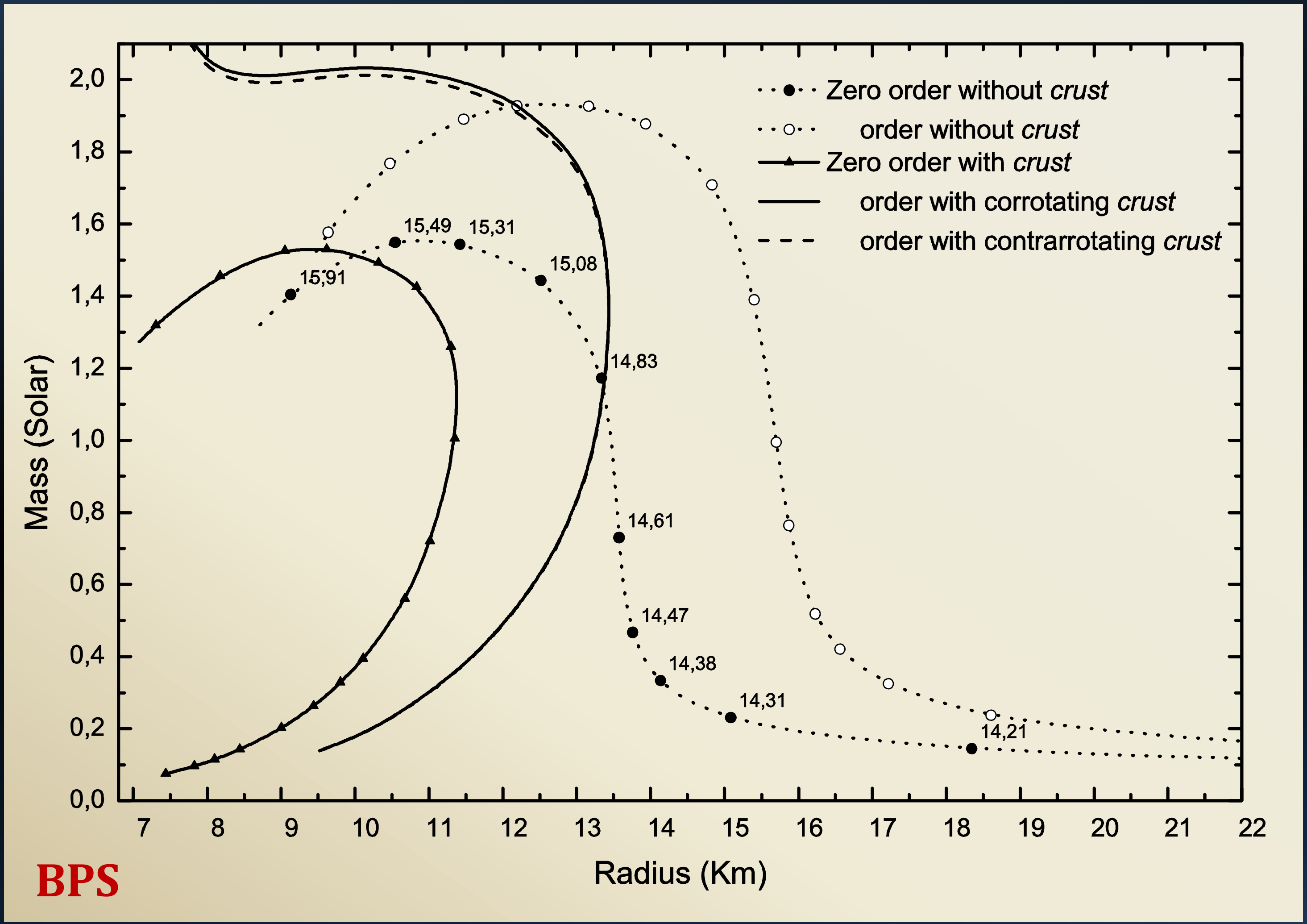
Effect of the surface density over configurations with different central densities

- Fixed crust mass to 10% of the core mass
- Mass shedding limit case

We change the central density of the star

3. Study of the properties of the model using realistic EoS

Effect of the surface density over configurations with different central densities



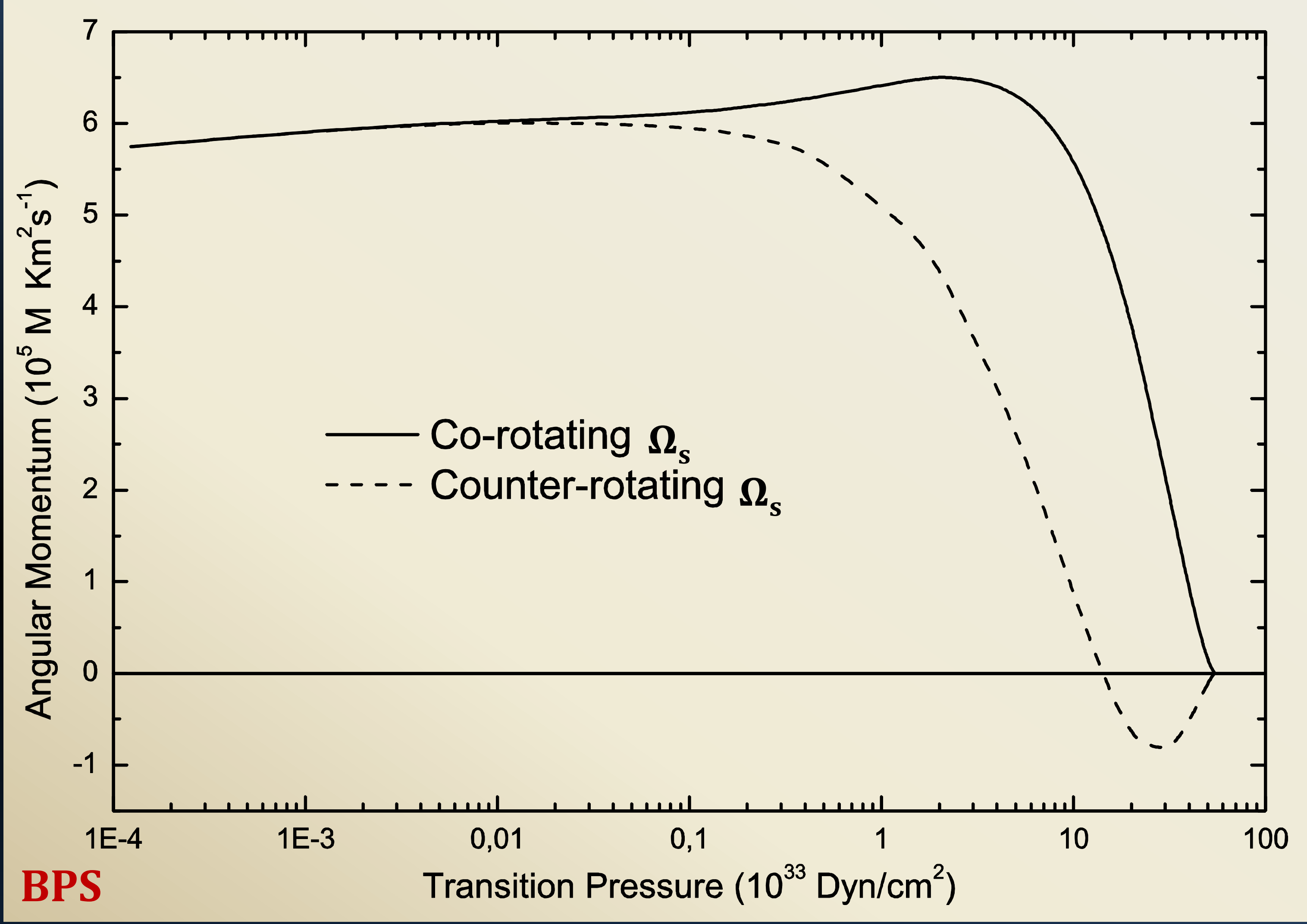
Effect of the variation of the transition pressure in neutron stars with fixed central density

- Fixed central density $\rho_c = 1.2 \cdot 10^{15}$ g cm^{-3}
- Mass shedding limit case

We change the transition pressure of the star

3. Study of the properties of the model using realistic EoS

Effect of the variation of the transition pressure in neutron stars with fixed central density



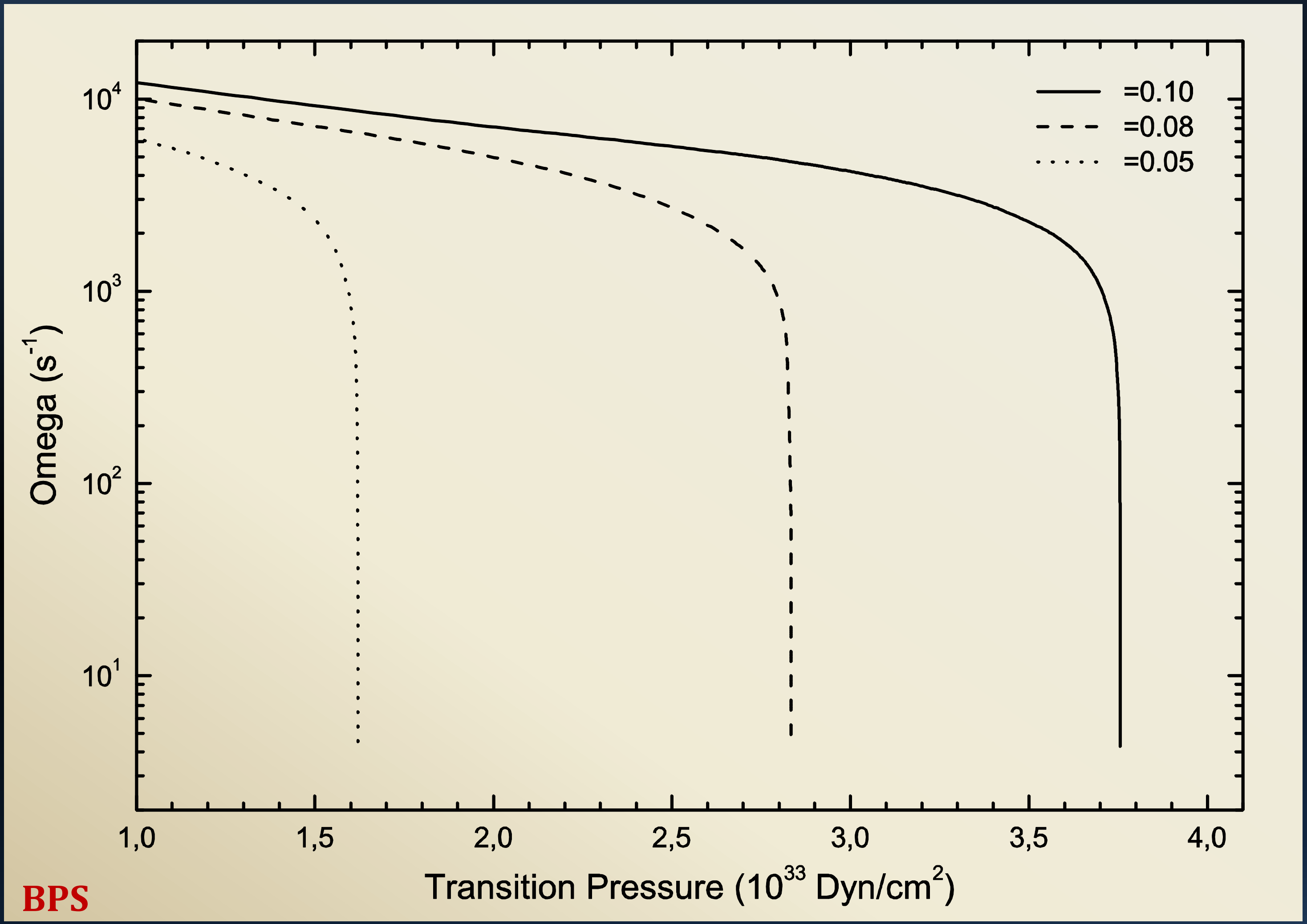
Effect of the variation of the transition pressure in neutron stars with fixed total mass and crust mass percentage

- Fixed total mass $M_T = 1.44\ M_\odot$
- Fixed crust mass to 5%, 8% and 10% of the core mass
- Co-rotating surface crust

We change the transition pressure of the star

3. Study of the properties of the model using realistic EoS

Effect of the variation of the transition pressure in neutron stars with fixed total mass and crust mass percentage



4. Application to giant glitches of the Vela pulsar

Giant glitch mechanism of the Vela Pulsar:

- Crust rupture
- Energy liberation in the inner crust
- Temperature rising on the outer layers of the star
- Modification of the equation of state of the transition region
- Increasing of the core-crust transition pressure
- Modification of the neutron star dynamical properties
- Increasing of the angular velocity of rotation

4. Application to giant glitches of the Vela pulsar

Total Mass: 1.44
Central Density: $1.279 \cdot 10^{15}$ g cm^{-3}
Initial core-crust transition Pressure: $3.751 \cdot 10^{33}$ dyn cm^{-2}

Date (MJD)	$\delta\Omega(10^{-6})$	$\delta p_{int}(10^{-10})$	$\delta ecc(10^{-6})$	$\delta Q(10^{-6})$	$\delta T(10^{6}K)$
40289	2.34	0.70	2.36	4.73	4.11
41192	2.05	0.58	1.97	3.94	3.75
43693	3.06	0.93	3.15	6.31	4.75
45192	2.05	0.58	1.96	3.93	3.74
48457	2.72	0.81	2.76	5.51	4.43
51559	3.09	0.93	3.14	6.28	4.75
53959	2.62	0.82	2.74	5.48	4.46

Core-crust Transition pressure relative changes order 10^{-10} for the Vela giant glitches

Thermal pressure variations are order 10^{6} K for the obtained core-crust transition changes

In agreement with heating due to energy depositions of 10^{42} erg

Post-glitch epoch: last for hundreds of days

↓

Pulsar tends to recover its usual rythm of spin down

For the glitches considered, the fraction of angular velocity recovered during the post-glitch epoch is 1/100

↓

Initial core-crust transition pressure is not recovered

Permanent changes in the transition region equation of state after every giant glitch

5. Conclusions

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- ✓ Intrinsic formulation of the junction conditions in our model of neutron star with surface crust
- ✓ The core-crust transition pressure arise as an essential parameter of the configurations of our model
- ✓ Configurations with core-crust contra-rotation are found
- ✓ The increase of the transition pressure (due to changes in the equation of state because of thermal energy deposition after the crust rupture) explains the angular velocity increase of glitches
- ✓ The equation of state is permamently modified after every giant glitch of the Vela Pulsar