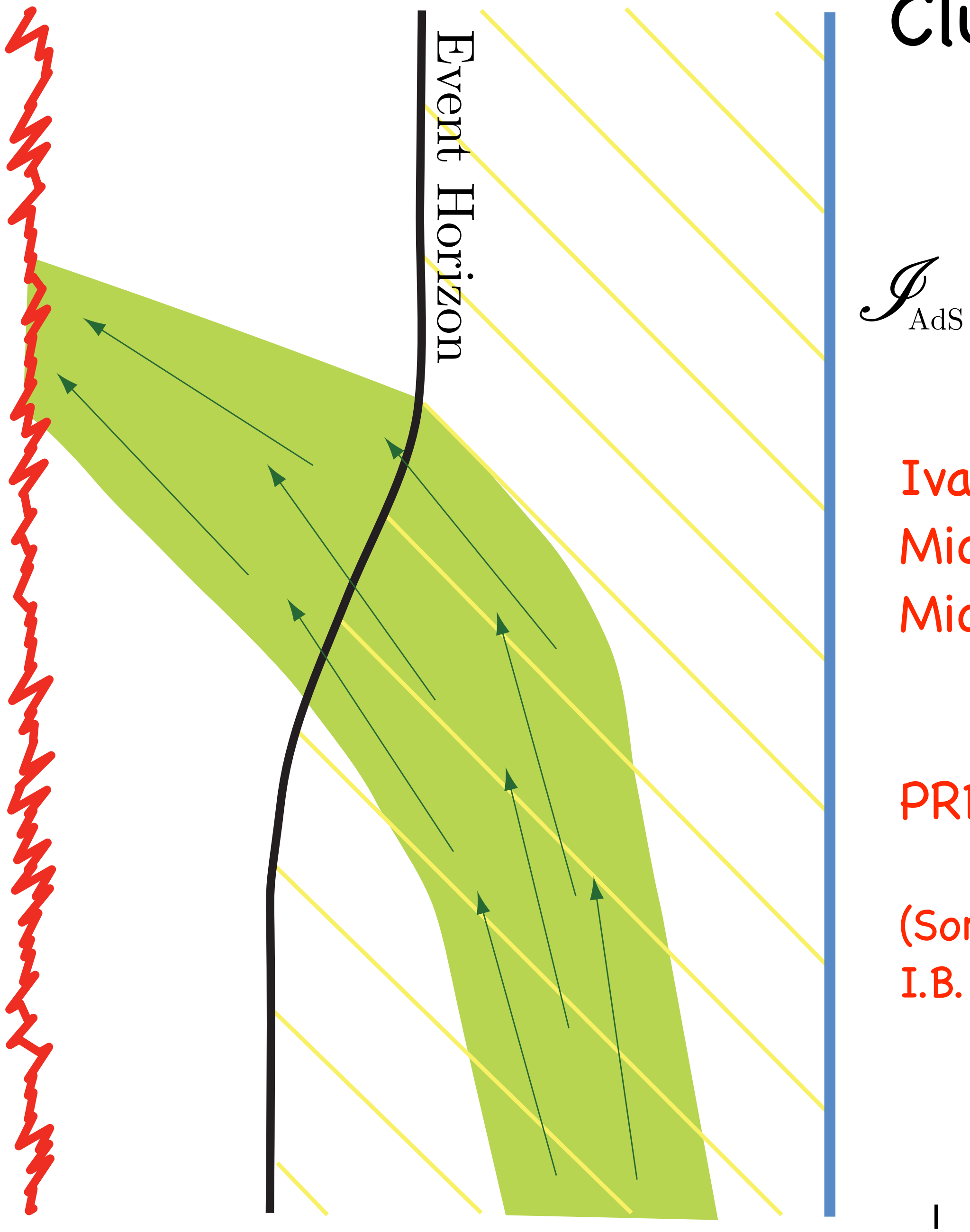
Entropy of an Evolving Black Hole:

Clues from hydrodynamics



ERE 2010 - Granada

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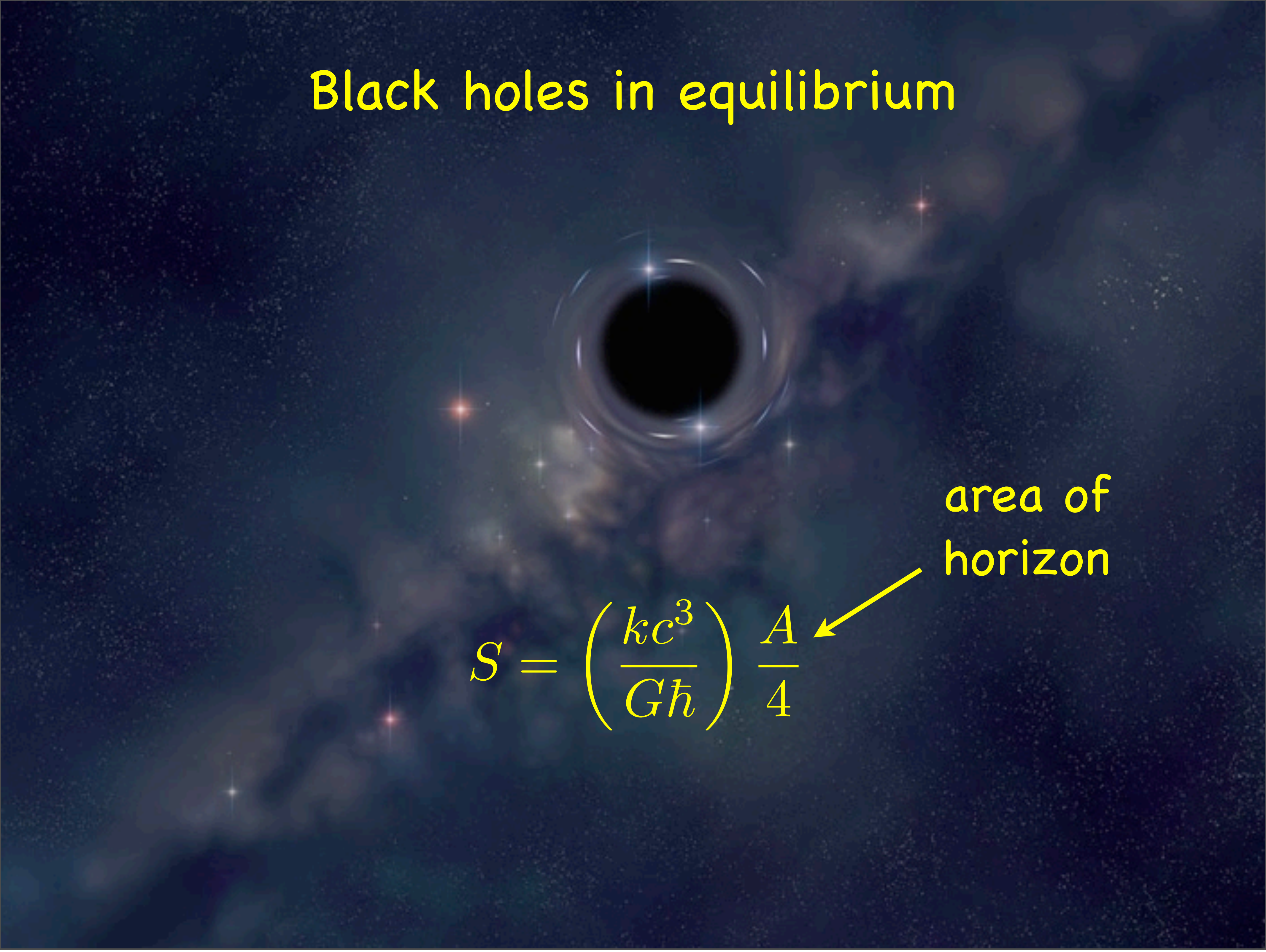
PRD 80, 126013 (2009); arXiv:0910.0748

(Some figures and ideas from:
I.B. and Jonathan Martin, arXiv:1007.1642)

Black holes in equilibrium



$$S = \left(\frac{kc^3}{G\hbar}\right)\frac{A}{4}$$

Non-equilibrium black hole

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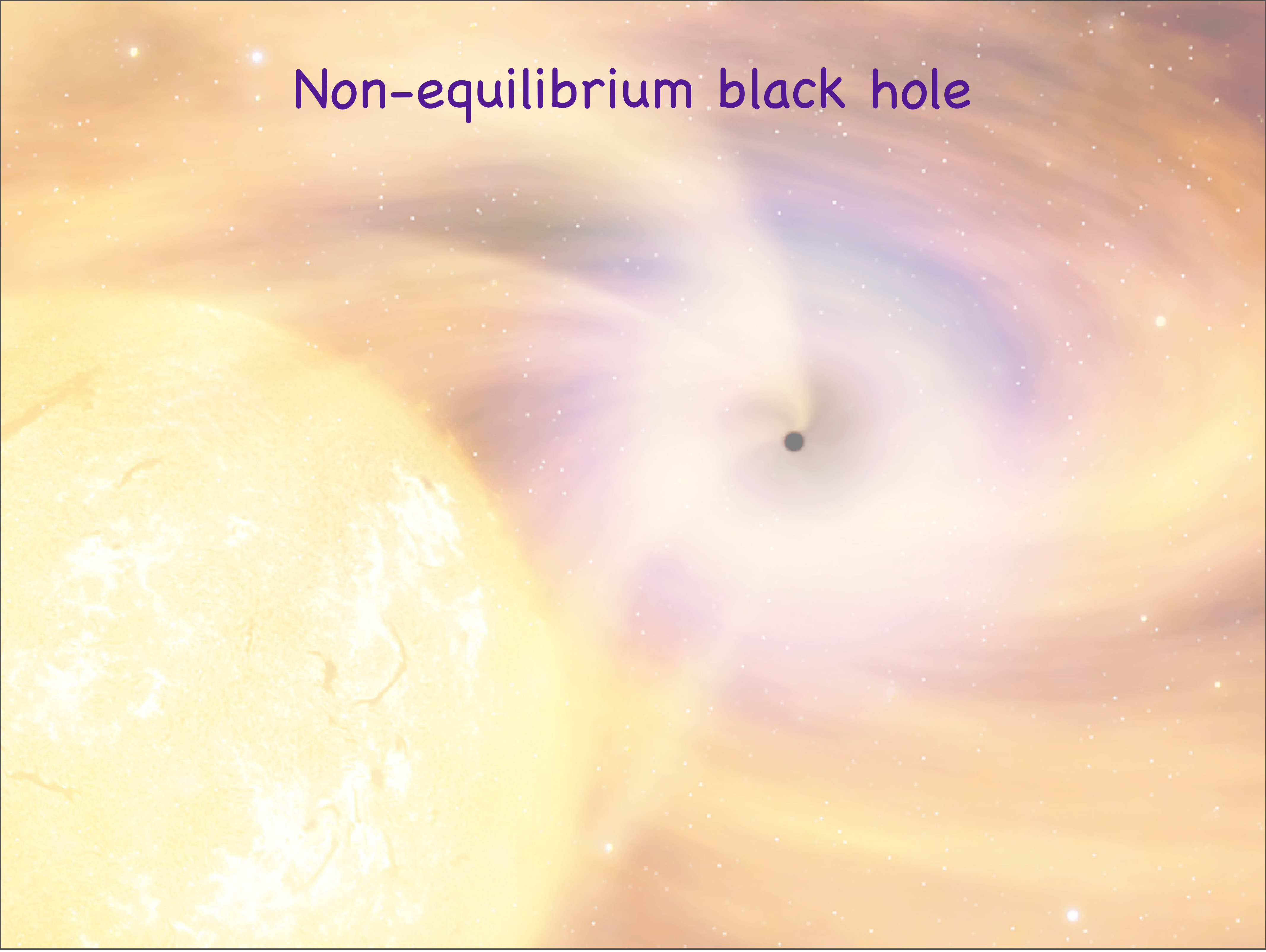

Non-equilibrium black hole

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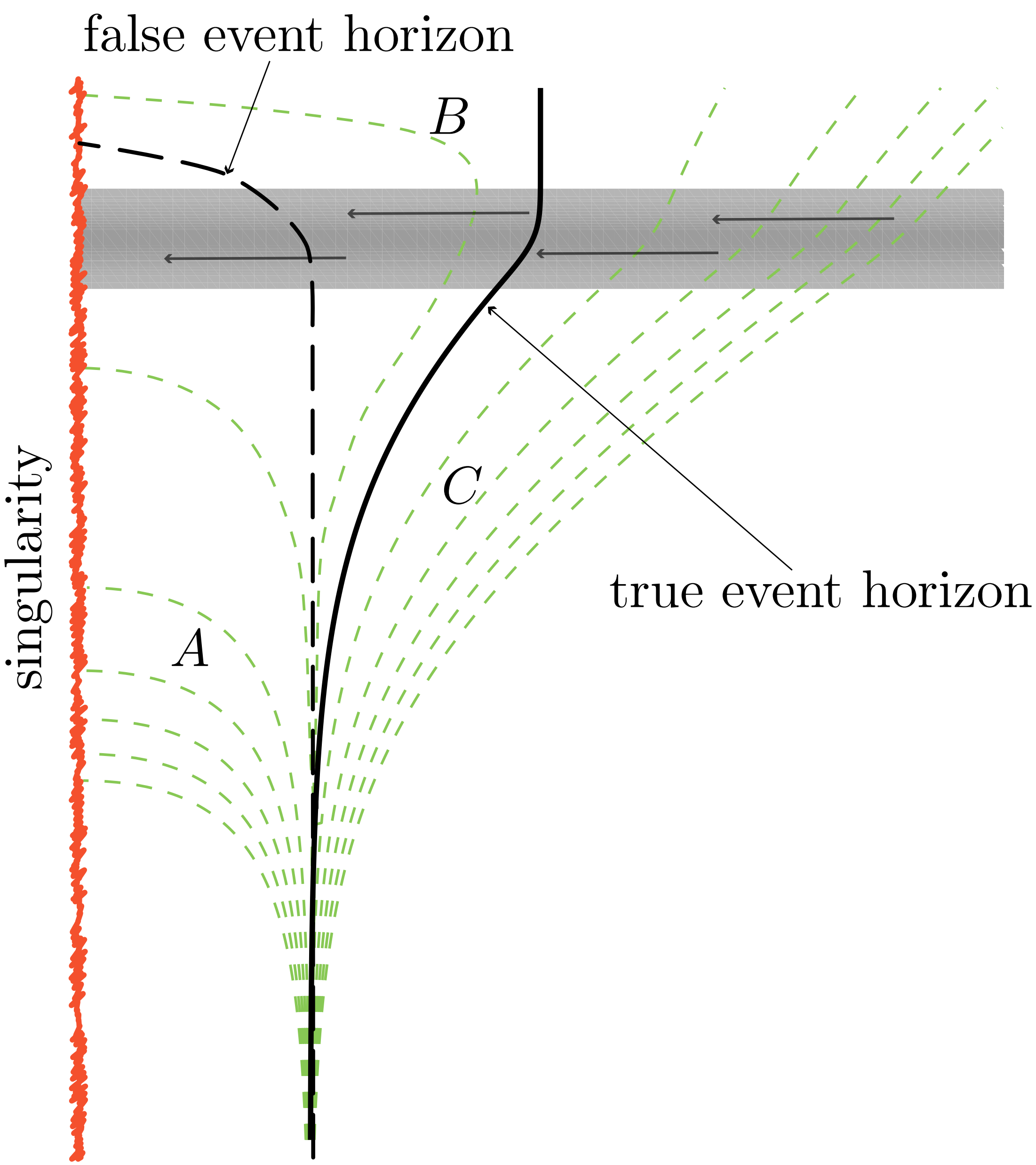
- If so is it still proportional to horizon area?
- If so which horizon?

Difficulties with event horizons

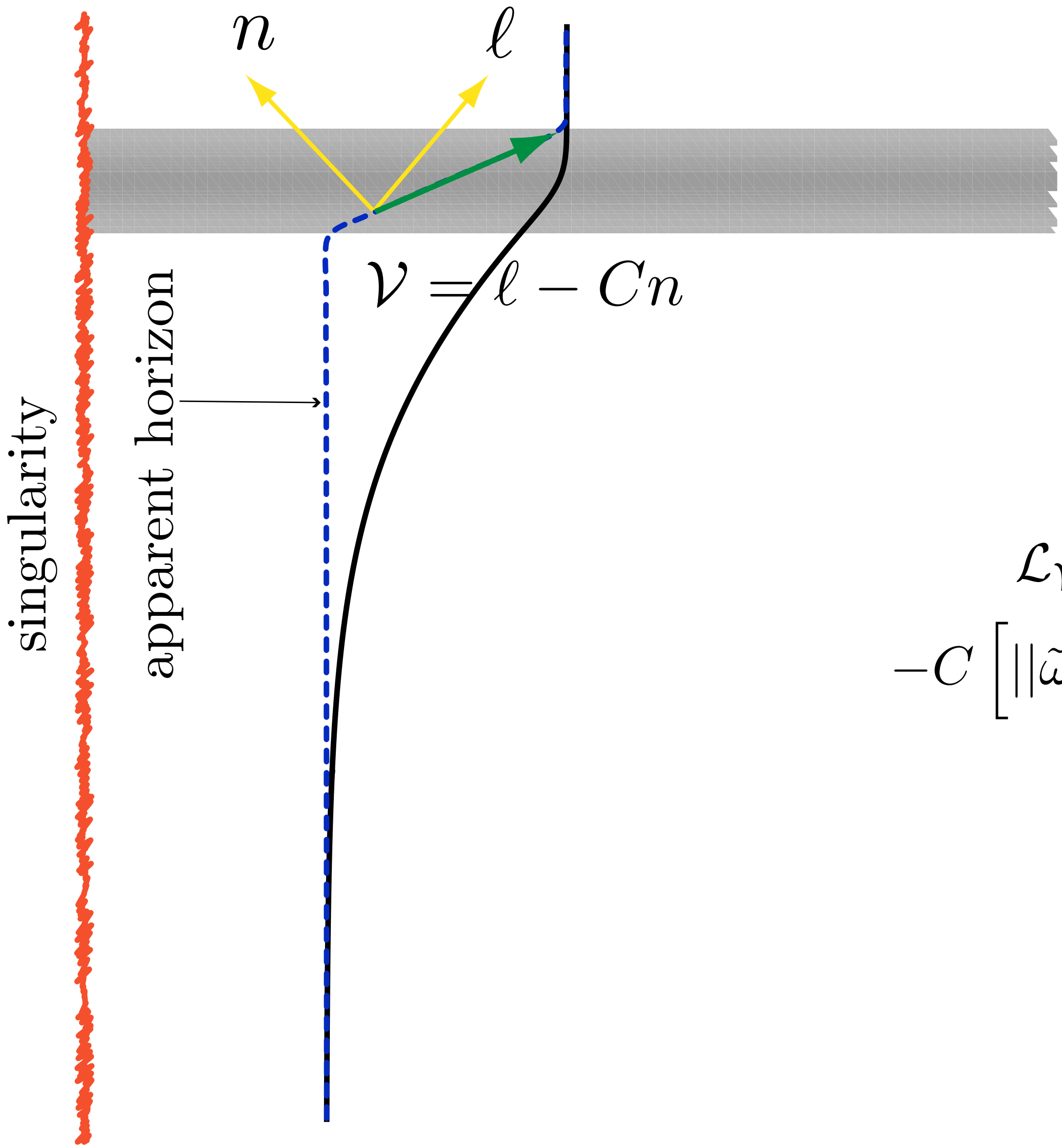


- Locally they are null hypersurfaces ruled by null geodesics

$$\mathcal{L}_\ell \theta_{(\ell)} = \kappa_\ell \theta_{(\ell)} - \|\sigma^{(\ell)}\|^2 - G_{ab}\ell^a \ell^b - \frac{\theta_{(\ell)}^2}{(n-1)}$$

- But exactly which hypersurface is defined by future boundary conditions

Difficulties with apparent horizons



- Hypersurface foliated with surfaces of vanishing outward null expansion:

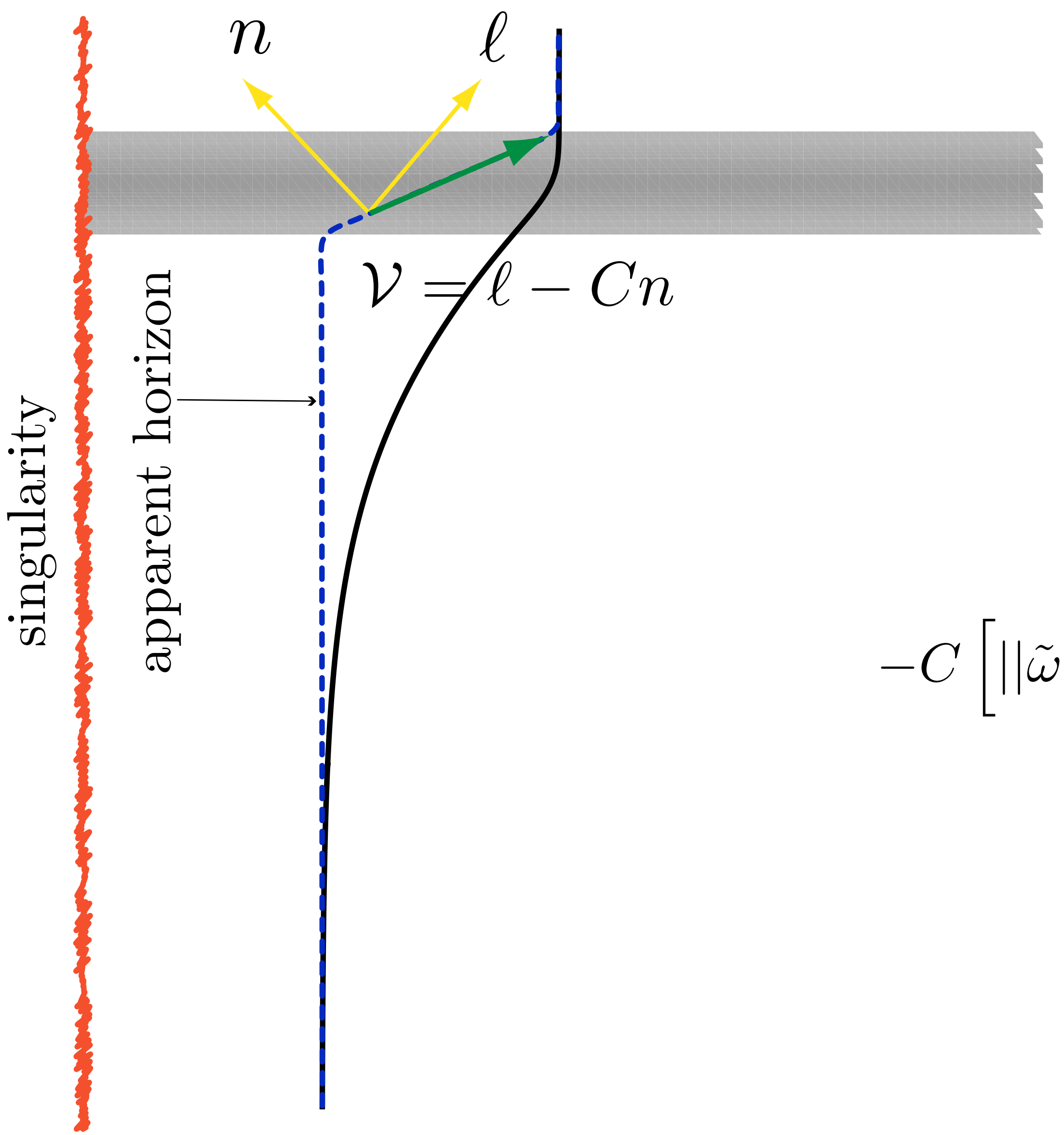
$$\theta_{(\ell)} = \tilde{q}^{ab} \nabla_a \ell_b = 0$$

- Can be (quasi)locally identified. Local dynamics.

$$\mathcal{L}_{\mathcal{V}} \theta_{(\ell)} = \mathcal{L}_{\ell} \theta_{(\ell)} + \kappa_{\mathcal{V}} \theta_{(\ell)} - d^2 C + 2 \tilde{\omega}^a d_a C$$
$$-C \left[||\tilde{\omega}||^2 - d_a \tilde{\omega}^a - \tilde{R}/2 + G_{ab} \ell^a n^b - \theta_{(\ell)} \theta_{(n)} \right]$$

- Are NOT unique.

Difficulties with apparent horizons



- Hypersurface foliated with surfaces of vanishing outward null expansion:

$$\theta_{(\ell)} = \tilde{q}^{ab} \nabla_a \ell_b = 0$$

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$$0 = \mathcal{L}_\ell \theta_{(\ell)} - d^2 C + 2\tilde{\omega}^a d_a C$$
$$-C \left[||\tilde{\omega}||^2 - d_a \tilde{\omega}^a - \tilde{R}/2 + G_{ab} \ell^a n^b \right]$$

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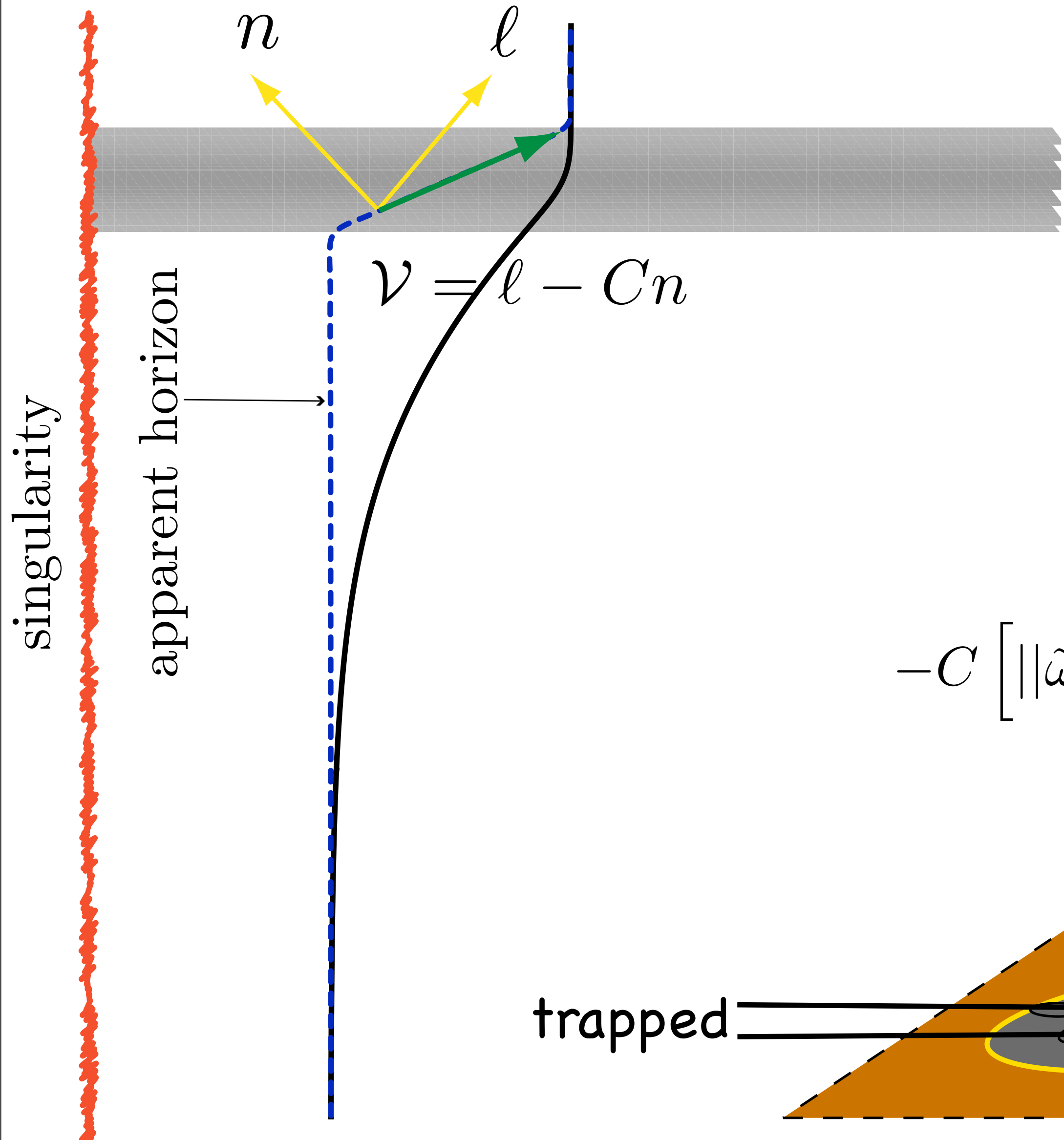

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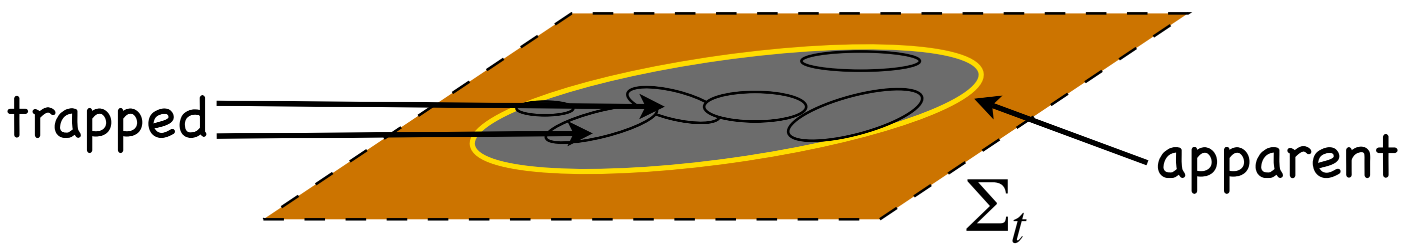
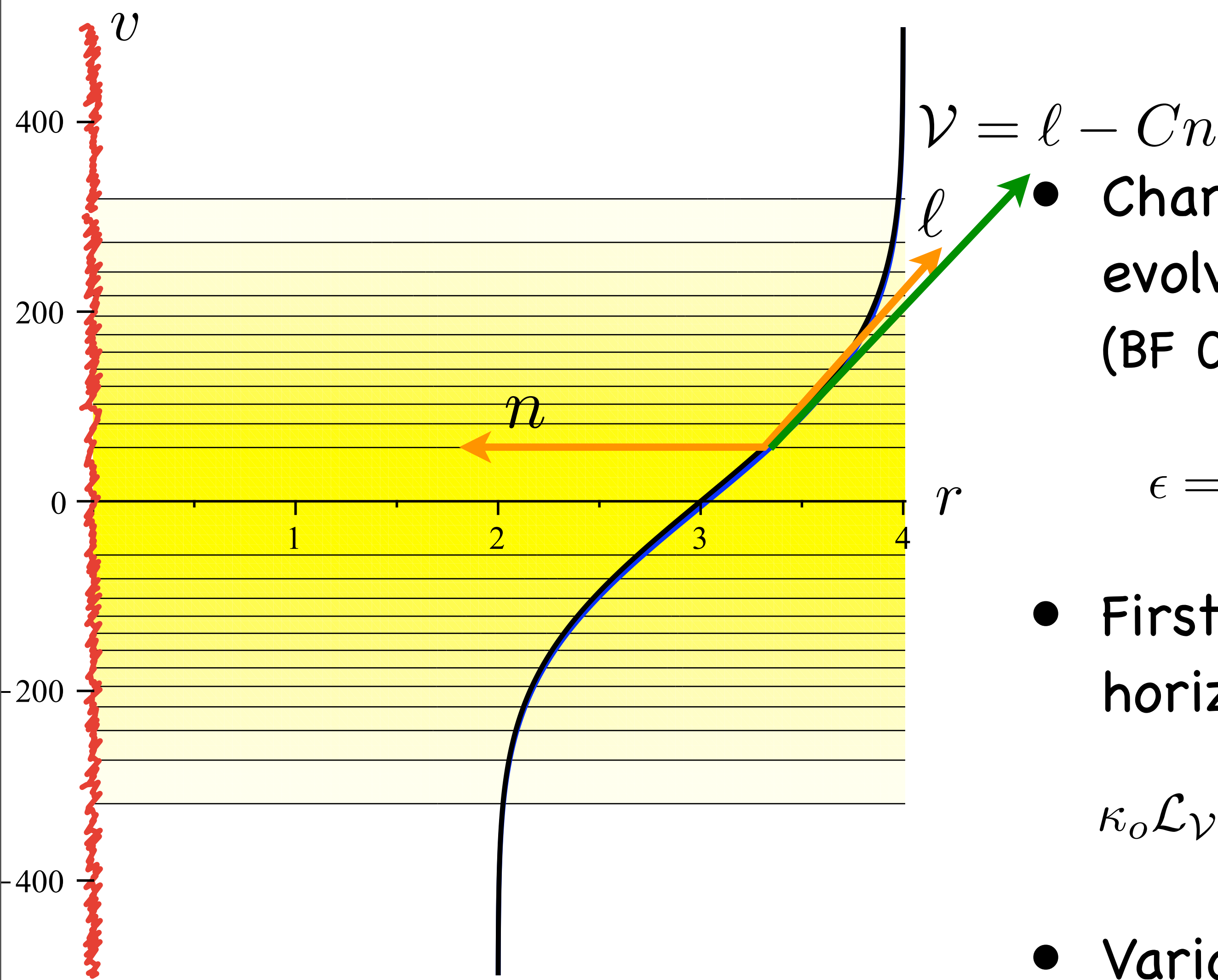


Quasi-equilbrium regime



- Characterized by slowly evolving apparent horizons (BF 03,07)

$$\epsilon = \frac{\mathcal{L}_{\hat{\mathcal{V}}}\sqrt{\tilde{q}}}{\sqrt{\tilde{q}}} = -\sqrt{\frac{C}{2}}\theta_{(n)} \ll 1$$

- First law holds (probably all horizons)

$$\kappa_o \mathcal{L}_{\mathcal{V}}\sqrt{\tilde{q}} \approx \sqrt{\tilde{q}}\left(||\sigma^{(\ell)}||^2 + G_{ab}\ell^a\ell^b\right)$$

- Various horizons are close (BM 10, Nielsen 10)

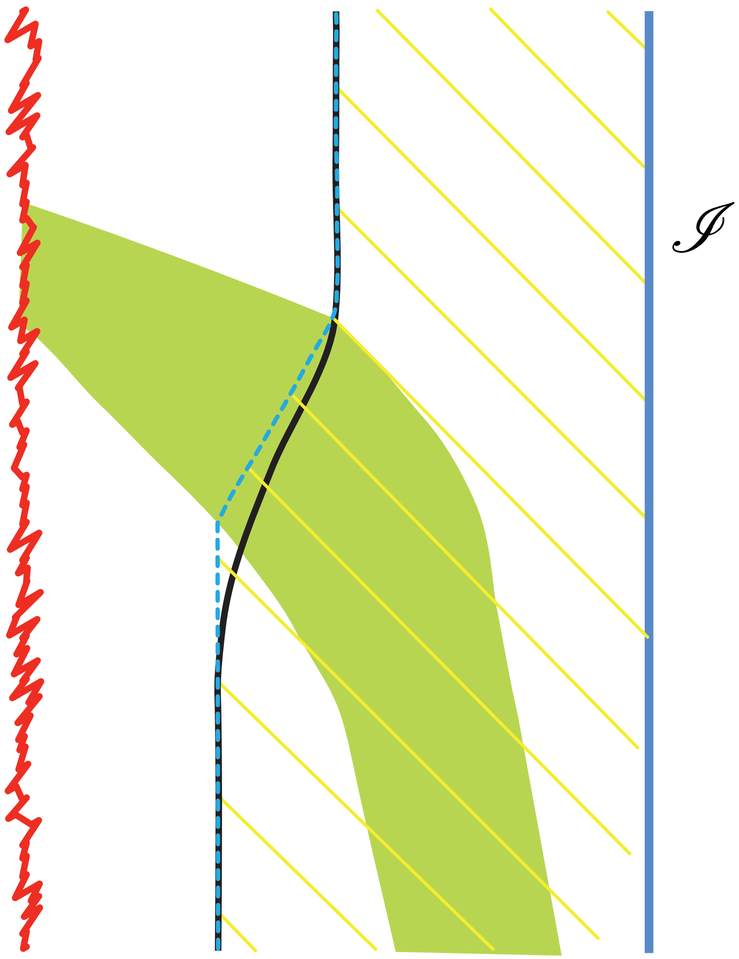
- Most black hole physics is in this regime - including some black hole formations (BK 05)

AdS-CFT - The hydrodynamic regime

- In the long-wavelength limit the CFT becomes a theory of conformally invariant fluid mechanics:

$$\nabla_\mu T^{\mu\nu}_{\mathscr{I}} = 0 \quad \text{and} \quad \nabla_\mu J^\mu_I = 0$$

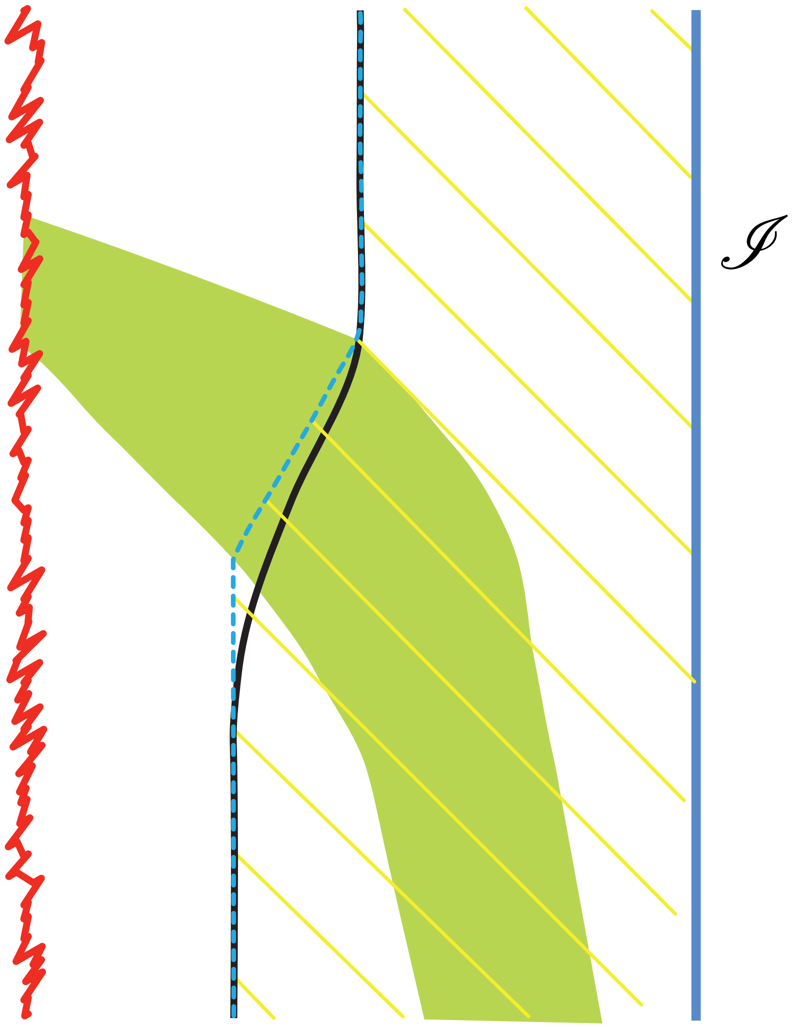
+ invariance under $g_{\mu\nu} \rightarrow e^{2\phi}\tilde{g}_{\mu\nu}$



- Fluid flows on boundary are dual to black brane/hole solutions
- Can be derived from a perturbed GR solution, independent of the original conjecture. (Bhattacharya, Hubeny, Minwalla, Rangamani 07)
- Quasi-equilibrium regime (on both sides)
- Thermodynamics should match

BUT.....

- The boundary is dual to the FULL spacetime.



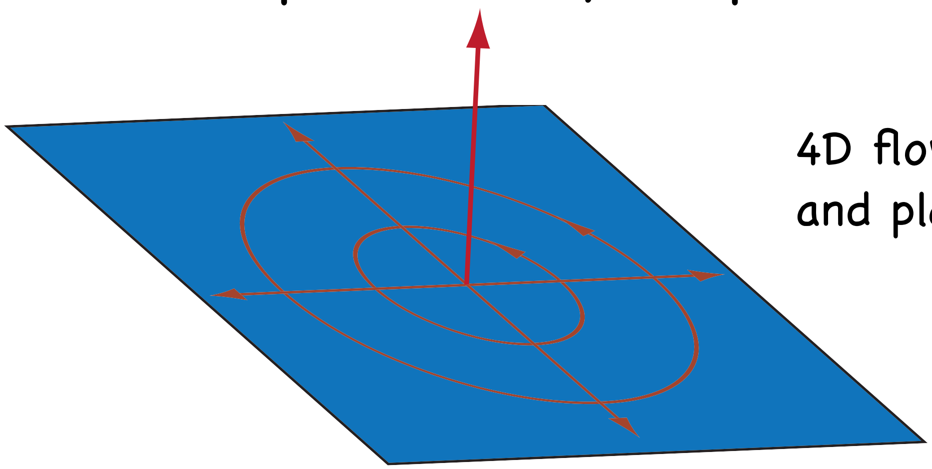
- How do you match bulk to boundary?
- Thermodynamics isn't uniquely defined on the boundary either...
  - thermodynamic quantities must be consistent with the symmetries
  - entropy flow is a vector field with non-negative divergence

$$\nabla_\mu S^\mu \geq 0$$

  - do the uncertainties match?

Bjorken flow

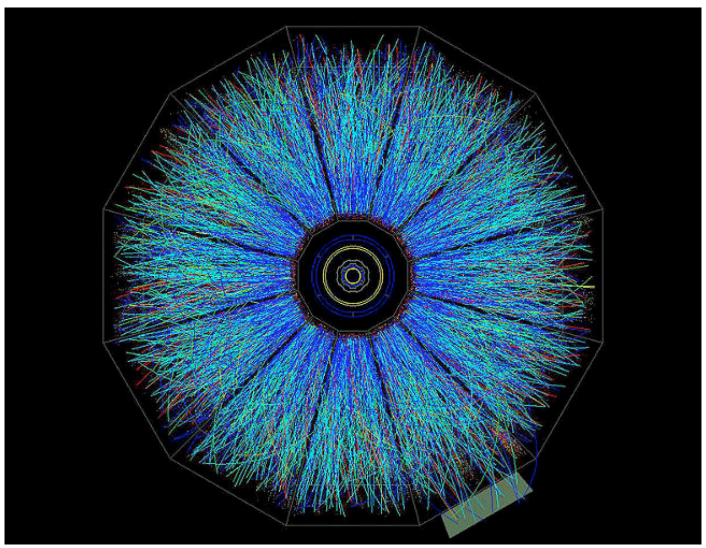
- A particularly simple example is boost-invariant flow:



4D flow with boost invariance in direction of motion and planar symmetry perpendicular to motion

- All variables depend on proper time alone (large τ)

$$\varepsilon = e_o T^4 \quad , \quad s = \frac{4e_o}{3} T^3 \ \text{ and } \ T \approx \frac{\Lambda}{\tau^{1/3}}$$



Quark-gluon plasma at RHIC (2000)

- Dual is a (perturbed) boosted black brane

$$\begin{aligned} ds^2 \quad &= \quad -r^2 A(\tau,r) d\tau^2 + 2 d\tau dr \\ &\quad +(1+\tau r)^2 e^{b(\tau,r)} dy^2 + r^2 e^{c(\tau,r)} dx_\perp^2 \end{aligned}$$

Bjorken flow - gravity vs fluid

- For apparent horizons, the formalisms nicely match:

  i) natural expansion parameter is $\epsilon = \frac{1}{\tau^{2/3}}$

  ii) surface gravity is $\kappa = \frac{2\pi\Lambda}{\tau^{1/3}}$

  iii) first law: $\kappa_{\mathcal{V}}\mathcal{L}_{\mathcal{V}}\sqrt{\tilde{q}} = \sqrt{\tilde{q}}||\sigma_{(\ell)}||^2 \approx \frac{2\pi^2\Lambda^3}{3} \cdot \frac{1}{\tau^2}$

- Entropy:

  i) fluid: $s_{bnd} \propto \frac{\Lambda^3}{\tau}\left\{1 - \frac{1}{2\pi\Lambda} \cdot \frac{1}{\tau^{2/3}} + \frac{0.008\beta}{\Lambda^2} \cdot \frac{1}{\tau^{4/3}} + O\left(\frac{1}{\tau^2}\right)\right\}$

  ii) AH: $s_{AH} = \frac{N_c^2\pi^2}{2} \cdot \frac{\Lambda^3}{\tau}\left\{1 - \frac{1}{2\pi\Lambda} \cdot \frac{1}{\tau^{2/3}} + \frac{0.039}{\Lambda^2} \cdot \frac{1}{\tau^{4/3}} + O\left(\frac{1}{\tau^2}\right)\right\}$

  iii) EH: $s_{EH} = \frac{N_c^2\pi^2}{2} \cdot \frac{\Lambda^3}{\tau}\left\{1 - \frac{1}{2\pi\Lambda} \cdot \frac{1}{\tau^{2/3}} + \frac{0.056}{\Lambda^2} \cdot \frac{1}{\tau^{4/3}} + O\left(\frac{1}{\tau^2}\right)\right\}$

Why the discrepancy?

- Answer #1: (practical entropy) Either horizon is fine. For quasi-equilibrium thermodynamics, don't worry about second order. The horizons are arbitrarily close (BM 10).
- Answer #2: (hydro dynamic unknowns) There is some (unknown) way to fix the fluid entropy flow. Then it would match that of one horizon or the other.
- Answer #3: (bulk corrections) There are other corrections needed in the bulk...
- Answer #3: (phenomenological entropy) Ambiguity on the boundary matches an ambiguity in the bulk. One possibility is to associate entropy with ``almost" null, ``almost" vanishing $\theta_{(\ell)}$, expanding surfaces (with correct limits).

Conclusions

- There is an excellent match between the bulk gravity and boundary fluid thermodynamics.
- The uncertainty on the boundary does NOT match the ambiguity in AH location.
- That uncertainty may suggest a rethinking of entropy in the bulk.
- A more general calculation is on its way...