Density growth in Kantowski-Sachs cosmologies with cosmological constant

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1. 1+3 and 1+1+2 covariant formalisms
   - 1+3 covariant formalism
   - Propagation equations and constraints
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3. Density perturbations
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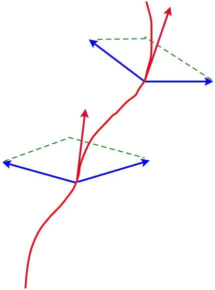
4. Summary and outlook

1+3 covariant formalism

1+3 covariant split of spacetime by Ellis, Bruni, van Elst et.al.

- Prefered timelike vector u^a. Projection operator onto perpendicular 3-space with $h_{ab} = g_{ab} + u_a u_b$.
- Covariant time derivative: $\dot{\psi}_{a..b} \equiv u^c \nabla_c \psi_{a...b}$
- Projected derivative: $\tilde{\nabla}_c \psi_{a...b} \equiv h_c^f h_a^d ... h_b^e \nabla_f \psi_{d...e}$



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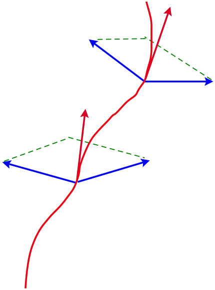

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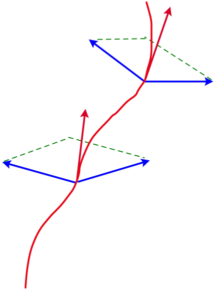

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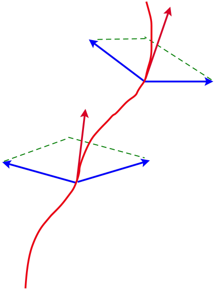

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1+3 covariant formalism

- The covariant derivative of the 4-velocity can be decomposed as

$$\nabla_a u_b = -u_a \dot{u}_b + \tilde{\nabla}_a u_b = -u_a \dot{u}_b + \frac{1}{3}\theta h_{ab} + \omega_{ab} + \sigma_{ab}$$

where $\dot{u}_a \equiv u^b \nabla_b u_a$ is the acceleration, $\theta \equiv \tilde{\nabla}_a u^a$ the expansion, $\sigma_{ab} \equiv \tilde{\nabla}_{<a} u_{b>}$ the shear and $\omega_{ab} \equiv \tilde{\nabla}_{[a} u_{b]}$ the vorticity of u^a.

- Other used varibles: Density μ, pressure $p = p(\mu)$ (barytropic eqution of state), cosmological constant Λ, the electric part of the Weyl tensor $E_{ab} \equiv C_{acbd} u^c u^d$ and the magnetic part of the Weyl tensor $H_{ab} \equiv \frac{1}{2}\eta_{ade} C^{de}{}_{bc} u^c$.

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Propagation equations and constraints

Propagation equations and constraints for the case of perfect fluid with barytropic equation of state, $p = p(\mu)$, and zero vorticity, $\omega_{ab} = 0$

Propagation equations from Ricci identities:

- $$\dot{\theta} - \tilde{\nabla}_a \dot{u}^a = -\frac{1}{3}\theta^2 + \dot{u}_a \dot{u}^a - 2\sigma^2 - \frac{1}{2}(\mu + 3p) + \Lambda\,,$$
  where $\sigma^2 \equiv \frac{1}{2}\sigma^{ab}\sigma_{ab}$.
- $$\dot{\sigma}^{<ab>} - \tilde{\nabla}^{<a}\dot{u}^{b>} = -\frac{2}{3}\theta\sigma^{ab} + \dot{u}^{<a}\dot{u}^{b>} - \sigma^{<a}{}_c\sigma^{b>c} - E^{ab}$$

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Propagation equations and constraints

Constraints from Ricci identities:

- $$\tilde{\nabla}_b \sigma^{ab} - \frac{2}{3}\tilde{\nabla}^a \theta = 0$$

- $$H^{ab} = (\mathrm{curl}\,\sigma)^{ab} \equiv \eta^{cd<a}\tilde{\nabla}_c \sigma^{b>}{}_d \, ,$$

Twice contracted Bianchi identities:

- $$\dot{\mu} = -\theta(\mu + p)$$

- $$\tilde{\nabla}_a p + (\mu + p)\dot{u}_a = 0$$

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C. A. Clarkson & R. Barrett, Class. Quan. Grav. **20**, 3855 (2003)
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- Decomposition of derivatives of n^a:

$$\tilde{\nabla}_a n_b = n_a a_b + \frac{1}{2}\phi N_{ab} + \xi \epsilon_{ab} + \zeta_{ab}$$
$$\dot{n}_a = \mathcal{A} u_a + \alpha_a$$

- where

$$a_a \equiv \hat{n}_a, \quad \phi \equiv \delta_a n^a, \quad \xi \equiv \frac{1}{2}\epsilon^{ab}\delta_a n_b, \quad \zeta_{ab} \equiv \delta_{\{a} n_{b\}},$$
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Kantowski-Sachs

Kantowski-Sachs cosmologies with cosmological constant Λ.

- 4-dimensional isometry group acting multiply transitive on 3-spaces with topology $R \times S_2$. Locally Rotationally Symmetric (LRS).
- $$ds^2 = -dt^2 + a_1^2(t)dz^2 + a_2^2(t)\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$
- The expansion and shear are given by

$$\theta = \frac{\dot{a}_1}{a_1} + 2\frac{\dot{a}_2}{a_2}$$

$$\Sigma \equiv \sigma_{11} = -2\sigma_{22} = -2\sigma_{33} = \frac{2}{3}\left(\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2}\right)$$

- Can undergo bounce.

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- Can undergo bounce.

Vacuum solutions

All vacuum Kantowski-Sachs can be found

- The equilibrium points $\pm X$:

$$ds^2 = -dt^2 + e^{\pm 2\sqrt{\Lambda}t}dz^2 + \frac{1}{\Lambda}\left(d\theta^2 + \sin^2\theta d\varphi^2\right).$$

M. Goliath and G.F.R. Ellis, Phys.Rev. D, **60**, 023502 (1999)

- $$ds^2 = -dt^2 + f^2(t)dz^2 + \frac{1}{\Lambda}\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

where $f(t) = a_0\cosh(\sqrt{\Lambda}t)$ or $f(t) = a_0\sinh(\sqrt{\Lambda}t)$. The first experiences a bounce in the z-direction.

- Schwarzschild-de Sitter:

$$ds^2 = -A^{-1}dT^2 + Adz^2 + T^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right),$$

where $A \equiv \left(\frac{2M}{T} - 1 + \frac{\Lambda}{3}T^2\right)$.

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$$ds^2 = -dt^2 + e^{\pm 2\sqrt{\Lambda}t} dz^2 + \frac{1}{\Lambda}\left(d\theta^2 + \sin^2\theta d\varphi^2\right) .$$

  M. Goliath and G.F.R. Ellis, Phys.Rev. D, **60**, 023502 (1999)

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  where $f(t) = a_0\cosh(\sqrt{\Lambda}t)$ or $f(t) = a_0\sinh(\sqrt{\Lambda}t)$. The first experiences a bounce in the z-direction.

- Schwarzschild-de Sitter:

$$ds^2 = -A^{-1}dT^2 + Adz^2 + T^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right) ,$$

  where $A \equiv \left(\frac{2M}{T} - 1 + \frac{\Lambda}{3}T^2\right)$.

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Density perturbations

Purpose:

To study the time-development of first order density perturbations on Kantowski-Sachs backgrounds and in particular on those undergoing bounces, i.e. those where expansion changes sign in one or several directions.

Inhomogeneity variables

As inhomogeneity variable we use

- The density gradient: $\mathcal{D}_a \equiv \frac{a\tilde{\nabla}_a\mu}{\mu}$.

  Here a is the average scale factor, defined from $\theta = 3\frac{\dot{a}}{a}$.

- The density fluctuations $\frac{\delta\mu}{\mu}$ on a length scale l are related to the quantity $\mathcal{D}_a$ through $\frac{\delta\mu}{\mu} \sim (\mathcal{D}_a\mathcal{D}^a)^{1/2} l/a = (\mathcal{D}_a\mathcal{D}^a)^{1/2} l_0$, where $l_0 = l/a$ is the comoving dimensionless length scale.

- To close the system, the following auxillary quantities will by used $\mathcal{Z}_a \equiv a\tilde{\nabla}_a\theta\,, \quad \mathcal{T}_a \equiv a\tilde{\nabla}_a\sigma^2\,, \quad \mathcal{S}_a \equiv a\tilde{\nabla}_a(\sigma^{ab}S_{ab})\,.$ where S_{ab} is the traceless part of the 3-Ricci tensor (can be written in a covariant way when $\omega_{ab} = 0$).

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First order equations

The propagation equations for inhomogeneity variables are obtained by taking the gradients of the original propagation equations. The following commutator is then used:

- $$\tilde{\nabla}_a(\dot{f}) - (\tilde{\nabla}_a f)^{\cdot} = -\dot{u}_a \dot{f} + \frac{1}{3}\theta\tilde{\nabla}_a f + \sigma_a{}^c\tilde{\nabla}_c f \,.$$
- The equations are then projected along the prefered direction n^a and onto the perpendicular 2-space with N_{ab} as

$$\mathcal{D} \equiv \mathcal{D}_a n^a \,, \; \mathcal{Z} \equiv \mathcal{Z}_a n^a \,, \; \mathcal{T} \equiv \mathcal{T}_a n^a \,, \; \mathcal{S} \equiv \mathcal{S}_a n^a$$

and

$$\mathcal{D}_{\bar{a}} \equiv \mathcal{D}_b N^{ab} \,, \; \mathcal{Z}_{\bar{a}} \equiv \mathcal{Z}_b N^{ab} \,, \; \mathcal{T}_{\bar{a}} \equiv \mathcal{T}_b N^{ab} \,, \; \mathcal{S}_{\bar{a}} \equiv \mathcal{S}_b N^{ab}$$

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First order equations

- To get spatial derivatives in the form of two Lacplace-like operators $\delta^2 \equiv \delta_a \delta^a$ and $\hat{\Delta} \equiv n^a \tilde{\nabla}_a n^b \tilde{\nabla}_b$ it is suitable to act on the two systems with $n^a \tilde{\nabla}_a$ and δ_a respectively.
- New variables are then defined as

$$\hat{\mathcal{D}} \equiv n^a \tilde{\nabla}_a \mathcal{D} \quad \text{and} \quad \mathcal{D\!\!\!/} \equiv \delta^a \mathcal{D}_{\bar{a}}$$

and similarly for the other variables.

- To remove some singular terms we then redefine $\hat{\mathcal{T}}$ and $\mathcal{T\!\!\!/}$ according to

$$\hat{\mathcal{T}}_{old} = \Sigma^2 \hat{\mathcal{T}}_{new} + \frac{\Sigma}{\tilde{S}} \hat{\mathcal{S}} \quad \text{and} \quad \mathcal{T\!\!\!/}_{old} = \Sigma^2 \mathcal{T\!\!\!/}_{new} + \frac{\Sigma}{\tilde{S}} \mathcal{S\!\!\!/}$$

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First order equations

First order system for hat variables

$$\dot{\hat{\mathcal{D}}} = \left[\theta\left(\frac{p}{\mu} - \frac{1}{3}\right) - 2\Sigma\right]\hat{\mathcal{D}} - \left(1 + \frac{p}{\mu}\right)\hat{\mathcal{Z}}$$

$$\dot{\hat{\mathcal{Z}}} = -\left(\theta + 2\Sigma\right)\hat{\mathcal{Z}} - 2\Sigma^2\hat{\mathcal{T}} + \left[-\frac{1}{2}\mu + \frac{3}{2}\frac{\mu p'}{\mu + p}\left(\tilde{S} + \frac{3}{2}\Sigma^2\right)\right]\hat{\mathcal{D}} - -2\frac{\Sigma}{\tilde{S}}\hat{\mathcal{S}} - \frac{\mu p'}{\mu + p}\hat{\Delta}\left[\hat{\mathcal{D}} + \not{\mathcal{D}}\right]$$

where

$$\tilde{S} = -\frac{2}{3}\mu - \frac{2}{3}\Lambda - \frac{1}{2}\Sigma^2 + \frac{2}{9}\theta^2 = -\frac{2}{3}K < 0$$

to zeroth order and

First order equations

$$
\begin{aligned}
\dot{\hat{\mathcal{T}}} \;=\; & -\left(\frac{1}{3}\theta + 2\Sigma + \frac{\Sigma^3}{\tilde{S}}\right)\hat{\mathcal{T}} - \left(\frac{\Sigma^2}{\tilde{S}^2} + \frac{1}{\tilde{S}}\right)\hat{\mathcal{S}} \\
& - \left[\frac{\Sigma\mu}{\tilde{S}} + \frac{\mu p'}{\mu + p}\left(\theta - \frac{3}{2}\Sigma\right)\right]\hat{\mathcal{D}} + \left(1 + \frac{2}{3}\frac{\Sigma\theta}{\tilde{S}}\right)\hat{\mathcal{Z}} \\
& + \frac{\mu p'}{\mu + p}\frac{1}{\tilde{S}}\left[\left(\frac{1}{2}\Sigma - \frac{1}{3}\theta\right)\hat{\Delta}\hat{\mathcal{D}} - \left(\Sigma - \frac{1}{6}\theta\right)\hat{\Delta}\left(\not{\mathcal{D}}\right)\right] \\
& - \frac{1}{\tilde{S}}\hat{\Delta}\left(\hat{\mathcal{Z}} - \frac{1}{2}\not{\mathcal{Z}}\right) + \frac{\Sigma}{\tilde{S}}\hat{\Delta}\left(\hat{\mathcal{T}} + \not{\mathcal{T}}\right) + \frac{1}{\tilde{S}^2}\hat{\Delta}\left(\hat{\mathcal{S}} + \not{\mathcal{S}}\right)
\end{aligned}
$$

First order equations

$$\begin{aligned}\dot{\hat{\mathcal{S}}} =& \left[\mu\Sigma^2 + \frac{\mu p'}{\mu+p}\tilde{S}\left(\frac{5}{2}\theta\Sigma + \frac{3}{2}\tilde{S} - \frac{3}{2}\Sigma^2\right)\right]\hat{\mathcal{D}} - \left(\frac{2}{3}\theta\Sigma + \frac{5}{2}\tilde{S}\right)\Sigma\hat{\mathcal{Z}} \\ &+ \left(\Sigma^4 + 2\tilde{S}^2\right)\hat{\mathcal{T}} + \left(\frac{\Sigma^3}{\tilde{S}} - 2\theta\right)\hat{\mathcal{S}} + \Sigma\hat{\Delta}\hat{\mathcal{Z}} - \frac{1}{2}\Sigma\hat{\Delta}\left(\not{\mathcal{Z}}\right) - \Sigma^2\hat{\Delta}\hat{\mathcal{T}} + \\ &\frac{\mu p'}{\mu+p}\left[\left(\frac{1}{3}\theta\Sigma - \tilde{S} - \frac{1}{2}\Sigma^2\right)\hat{\Delta}\hat{\mathcal{D}} + \frac{1}{2}\left(\tilde{S} - \frac{1}{3}\theta\Sigma + 2\Sigma^2\right)\hat{\Delta}\left(\not{\mathcal{D}}\right)\right] \\ &- \frac{\Sigma}{\tilde{S}}\hat{\Delta}(\hat{\mathcal{S}} + \not{\mathcal{S}}) - \Sigma^2\hat{\Delta}\left(\not{\mathcal{T}}\right)\end{aligned}$$

First order equations

First order system for slashed variables

$$
\begin{aligned}
\dot{\not{\mathcal{D}}} &= \left[\theta\left(\frac{p}{\mu}-\frac{1}{3}\right)+\Sigma\right]\not{\mathcal{D}}-\left(1+\frac{p}{\mu}\right)\not{\mathcal{Z}} \\
\dot{\not{\mathcal{Z}}} &= \left(\Sigma-\theta\right)\not{\mathcal{Z}}-2\Sigma^2\not{\mathcal{T}}+\left[-\frac{1}{2}\mu+\frac{3}{2}\frac{\mu p'}{\mu+p}\left(\tilde{S}+\frac{3}{2}\Sigma^2\right)\right]\not{\mathcal{D}}- \\
&\quad -2\frac{\Sigma}{\tilde{S}}\not{\mathcal{S}}-\frac{\mu p'}{\mu+p}\delta^2\left[\hat{\mathcal{D}}+\not{\mathcal{D}}\right]
\end{aligned}
$$

First order equations

$$
\begin{aligned}
\dot{\mathcal{T}} \;=\; & -\left(\frac{1}{3}\theta - \Sigma + \frac{\Sigma^3}{\tilde{S}}\right)\mathcal{T} - \left(\frac{\Sigma^2}{\tilde{S}^2} + \frac{1}{\tilde{S}}\right)\not{\mathcal{S}} - \\
& \left[\frac{\Sigma\mu}{\tilde{S}} + \frac{\mu p'}{\mu + p}\left(\theta - \frac{3}{2}\Sigma\right)\right]\not{\mathcal{D}} + \left(1 + \frac{2}{3}\frac{\Sigma\theta}{\tilde{S}}\right)\not{\mathcal{Z}} \\
& + \frac{\mu p'}{\mu + p}\frac{1}{\tilde{S}}\left[\left(\frac{1}{2}\Sigma - \frac{1}{3}\theta\right)\delta^2\hat{\mathcal{D}} - \left(\Sigma - \frac{1}{6}\theta\right)\delta^2\left(\not{\mathcal{D}}\right)\right] \\
& - \frac{1}{\tilde{S}}\delta^2(\hat{\mathcal{Z}} - \frac{1}{2}\not{\mathcal{Z}}) + \frac{\Sigma}{\tilde{S}}\delta^2(\hat{\mathcal{T}} + \mathcal{T}) + \frac{1}{\tilde{S}^2}\delta^2(\hat{\mathcal{S}} + \not{\mathcal{S}})
\end{aligned}
$$

First order equations

$$\begin{aligned}\dot{\not{S}} \;=\;& \left[\mu\Sigma^2 + \frac{\mu p'}{\mu+p}\tilde{S}\left(\frac{5}{2}\theta\Sigma + \frac{3}{2}\tilde{S} - \frac{3}{2}\Sigma^2\right)\right]\not{\mathcal{D}} - \left(\frac{2}{3}\theta\Sigma + \frac{5}{2}\tilde{S}\right)\Sigma\not{Z} \\ &+ \left(\Sigma^4 + 2\tilde{S}^2\right)\not{\mathcal{T}} + \left(\frac{\Sigma^3}{\tilde{S}} - 2\theta + 3\Sigma\right)\not{S} + \Sigma\delta^2\left(\hat{\mathcal{Z}} - \frac{1}{2}\not{Z}\right) + \\ &\frac{\mu p'}{\mu+p}\left[\left(\frac{1}{3}\theta\Sigma - \frac{1}{2}\Sigma^2 - \tilde{S}\right)\delta^2\hat{\mathcal{D}} + \frac{1}{2}\left(\tilde{S} - \frac{1}{3}\theta\Sigma + 2\Sigma^2\right)\delta^2\left(\not{\mathcal{D}}\right)\right] \\ &- \frac{\Sigma}{\tilde{S}}\delta^2(\hat{\mathcal{S}} + \not{S}) - \Sigma^2\delta^2\left(\hat{\mathcal{T}} + \not{\mathcal{T}}\right).\end{aligned}$$

Harmonic decomposition

Harmonic decomposition in terms of comoving wavenumbers $k_{\|}$ and $k_{\perp}$

- $$\Psi = \sum_{k_{\|},k_{\perp}} \Psi_{k_{\|}k_{\perp}} P_{k_{\|}} Q_{k_{\perp}}$$
- $$\hat{\Delta} P_{k_{\|}} = -\frac{k_{\|}^2}{a_1^2} P_{k_{\|}}, \quad \delta_a P_{k_{\|}} = \dot{P}_{k_{\|}} = 0$$
  Can be chosen as $P_{k_{\|}} = e^{ik_{\|}z}$ where z in 1-direction.
- $$\delta^2 Q_{\perp} = -\frac{k_{\perp}^2}{a_2^2} Q_{\perp}, \quad \hat{Q}_{\perp} = \dot{Q}_{\perp} = 0$$
  C.A. Clarkson, Phys. Rev. D, **76**, 104034 (2007)

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Analytical solutions

Exact solutions to the perturbed equations can be found around some of the vacuum solutions for the limit $k_{\parallel} = k_{\perp} = 0$ (i.e. infinite wavelenght). Could approximate the growth/decay of long wave density perturbations for the case $p << \mu << \Lambda$.

- Perturbations around vacuum bounce solution:

  $ds^2 = -dt^2 + a_0^2 \cosh^2(\sqrt{\Lambda}t)dz^2 + \frac{1}{\Lambda}\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$:
- $\hat{\mathcal{D}} = (A_1 + A_2\theta)\left(\Lambda - \theta^2\right)^{5/6} + A_3\theta\left(\Lambda - \theta^2\right)^{1/3} + A_4\left(\Lambda - \theta^2\right)^{5/6} \times \left(\frac{1}{2}\ln\left(1 - \frac{\theta^2}{\Lambda}\right) - \frac{\theta}{4\sqrt{\Lambda}}\ln\left(\frac{\sqrt{\Lambda}+\theta}{\sqrt{\Lambda}-\theta}\right) + \frac{\theta}{\sqrt{\Lambda-\theta^2}}\arcsin\left(\frac{\theta}{\sqrt{\Lambda}}\right)\right)$ and $\mathcal{D} = \left(\frac{a_1}{a_2}\right)^2\hat{\mathcal{D}} = \hat{\mathcal{D}}/(\Lambda - \theta^2)$, where $\theta = \sqrt{\Lambda}\tanh(\sqrt{\Lambda}t)$.

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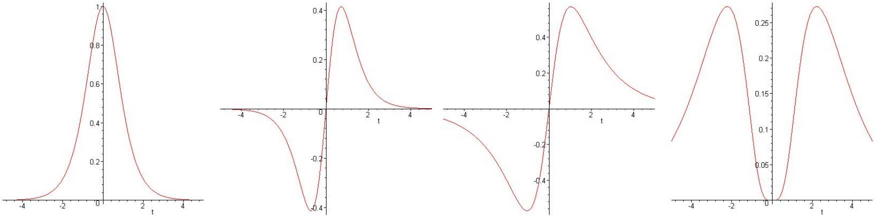
$\hat{\mathcal{D}}$-modes



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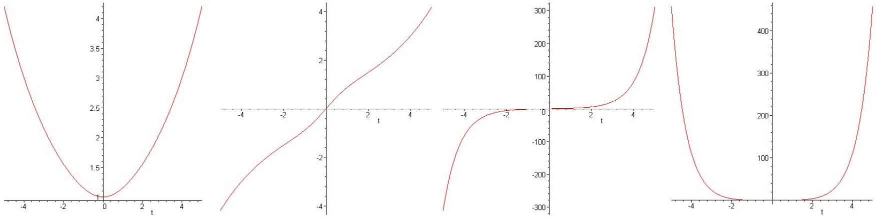

Numerical solutions

Example of a numerical solution. Properties of background solution:

- Radiation $p = \frac{1}{3}\mu$.
- The anisotropy direction n_a starts contracting, goes through a bounce and then expands forever. Asymptotically $\theta_{||} \to \sqrt{\Lambda/3}$
- In the perpendicular directions the initial expansion is small and becomes almost negligible for some time before it starts expanding again. Asymptotically $\theta_{\perp} \to \sqrt{\Lambda/3}$, so that $\theta \to \sqrt{3\Lambda}$.
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- It starts close to the critical point ${}_{-}X$, passes through a bounce, is close to the critical point ${}_{+}X$ for an intermediate period and then eventually approaches de Sitter.

Numerical solutions

Example of a numerical solution. Properties of background solution:

- Radiation $p = \frac{1}{3}\mu$.
- The anisotropy direction n_a starts contracting, goes through a bounce and then expands forever. Asymptotically $\theta_{\|} \to \sqrt{\Lambda/3}$
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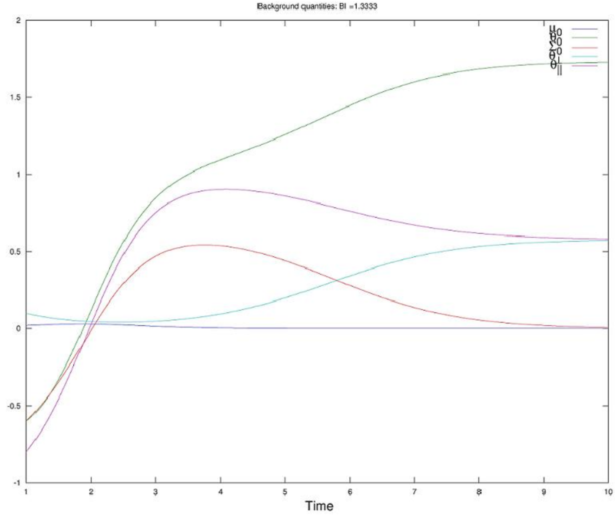
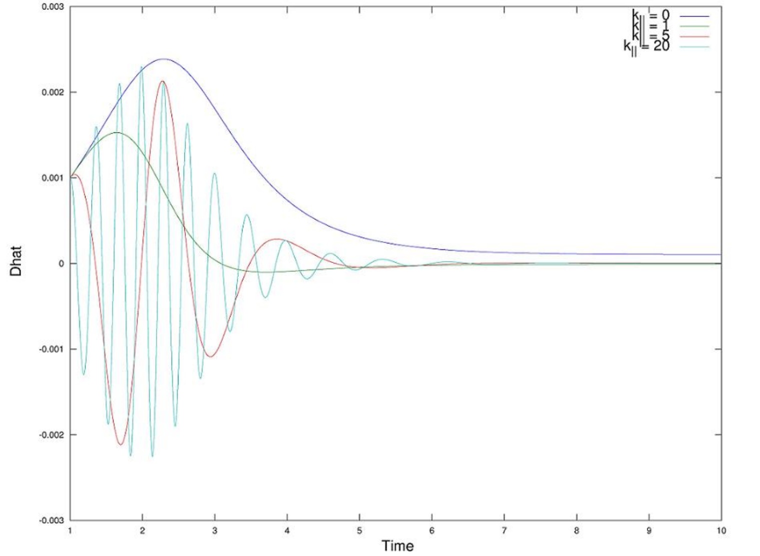
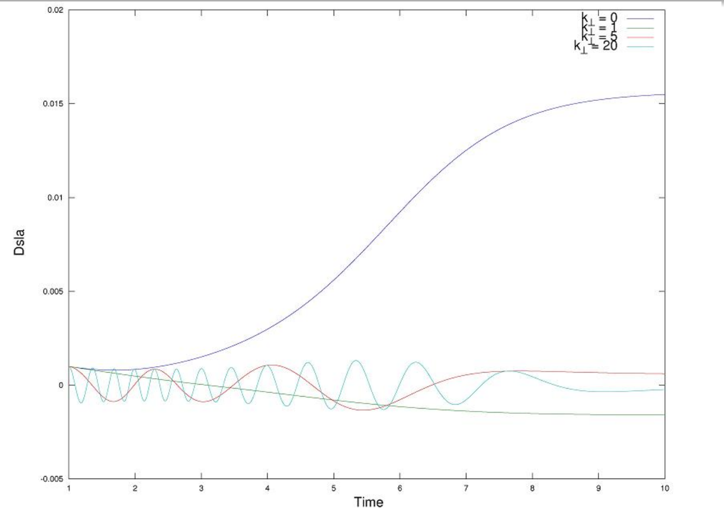

Background quantities: BI =1.3333
Time

Numerical solutions

The growth of the density perturbations $\hat{\mathcal{D}}$ and $\not{\mathcal{D}}$ for the wave numbers $k_{\parallel}/a_{10} = k_{\perp}/a_{20} = 0, 1, 5$ and 20. Initially, at $t_0 = 1$, $\hat{\mathcal{D}} = \not{\mathcal{D}} = 0.001$.


$k_{||} = 0$
$k_{||} = 1$
$k_{||} = 5$
$k_{||} = 20$
Dhat
Time
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
1
2
3
4
5
6
7
8
9
10


0.02
0.015
0.01
0.005
0
-0.005
Dsla
1
2
3
4
5
6
7
8
9
10
Time
k⊥ = 0
k⊥ = 1
k⊥ = 5
k⊥ = 20

Summary and outlook

- Closed system for scalar perturbations on the background of Kantowski-Sachs cosmologies with cosmological constant obtained.
- The growth or decay of density gradients have been studied numerically for different wavelenghts and initial perturbations on a number of backgrounds.
- Can be solved analytically for some vacuum backgrounds in the long wavelength limit. Agree well with some numerical dust solutions with $\mu << \Lambda$.
- Future work: Tensor perturbations, including generation and propagation of gravitational waves.
- Second order perturbations.

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In memory of Brian Edgar