Time-domain modeling of Extreme-Mass-Ratio Inspirals for the Laser Interferometer Space Antenna

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Talk based on:

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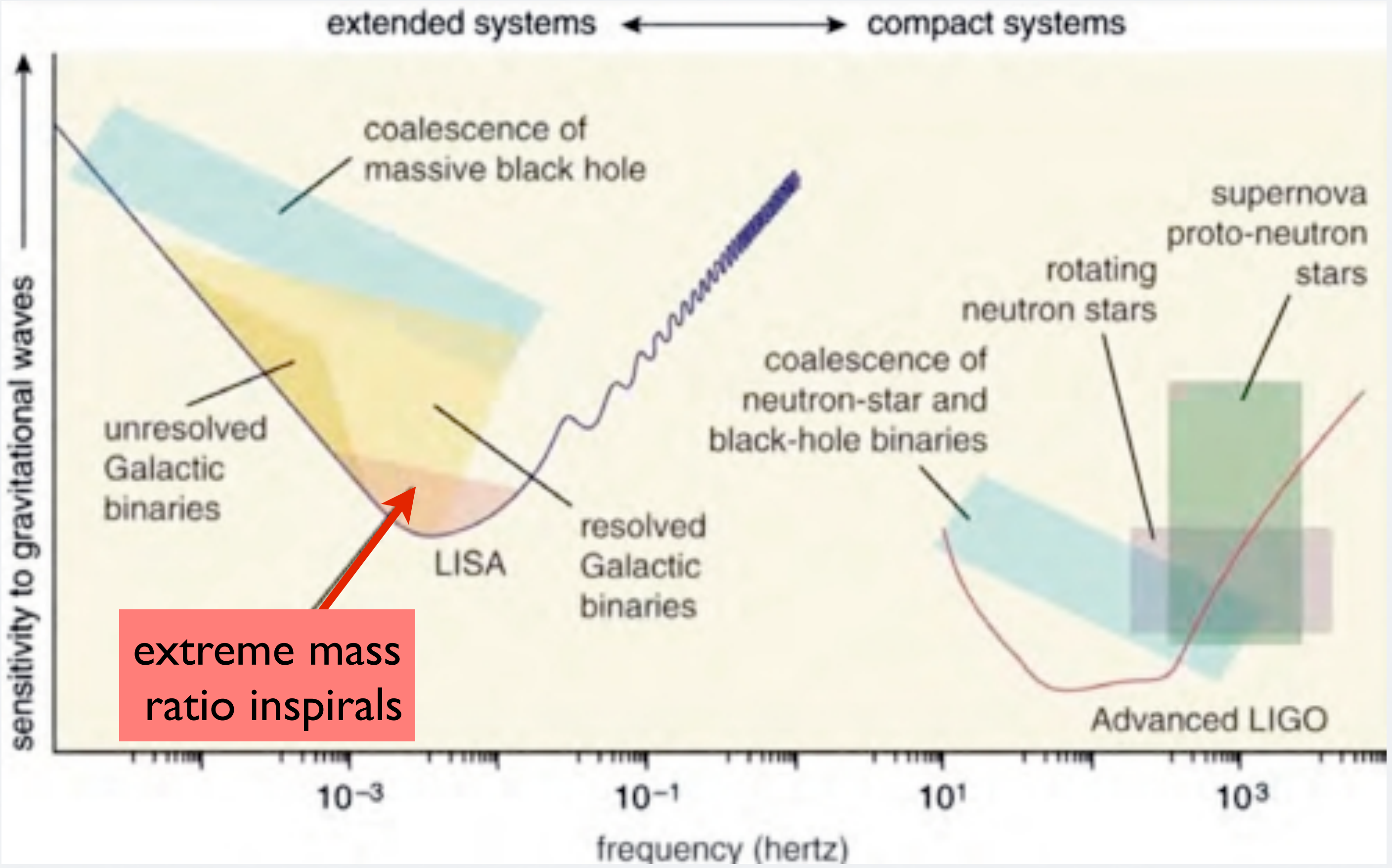
J. Phys. **Conf. Ser.,** 154 012053 (2009), arXiv: **gr-qc 0811.0294v1**

Outlook

- Introduction: Extreme–Mass Ratio Inspiral (EMRI) systems.
- Our computational method: The Particle–without–Particle Scheme.
- Results from the Simulations.
- Conclusions and future work.

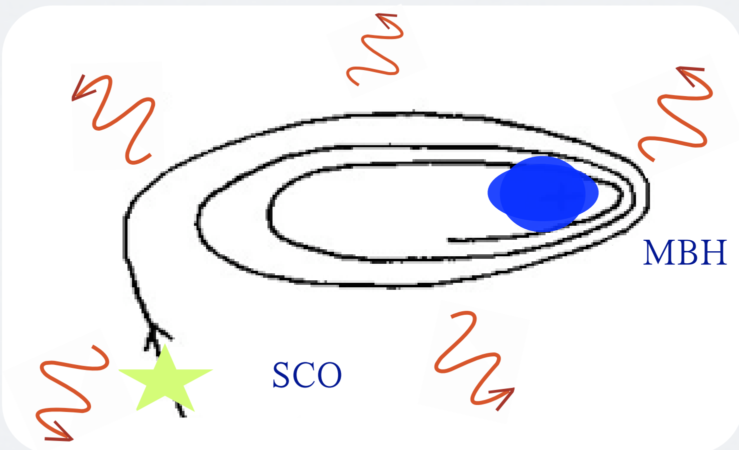
Introduction

- Extreme-Mass-Ratio Inspirals (EMRIs) are astrophysical binary systems which are one of the main sources of GWs for the future Laser Interferometer Space Antenna (LISA).



graphic (c) by LISA Science Case LISA-LIST-RP-436

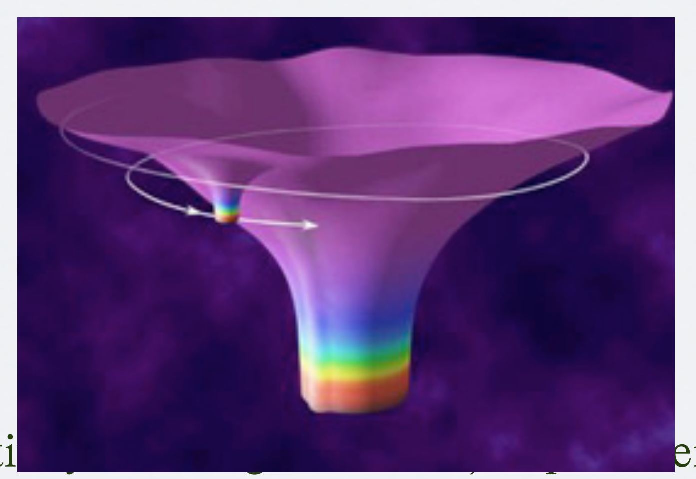
- EMRIs are formed when a massive black hole (MBH), $M \sim 10^4 - 10^7\, M_\odot$, located in a galactic center, captures a stellar-mass compact object (SCO) $m \sim 1 - 50\, M_\odot$, as white dwarfs, neutron stars or stellar BHs. Then the (extreme) mass-ratio for these systems lies in the range: $\mu = m/M_\bullet \sim 10^{-7} - 10^{-3}$

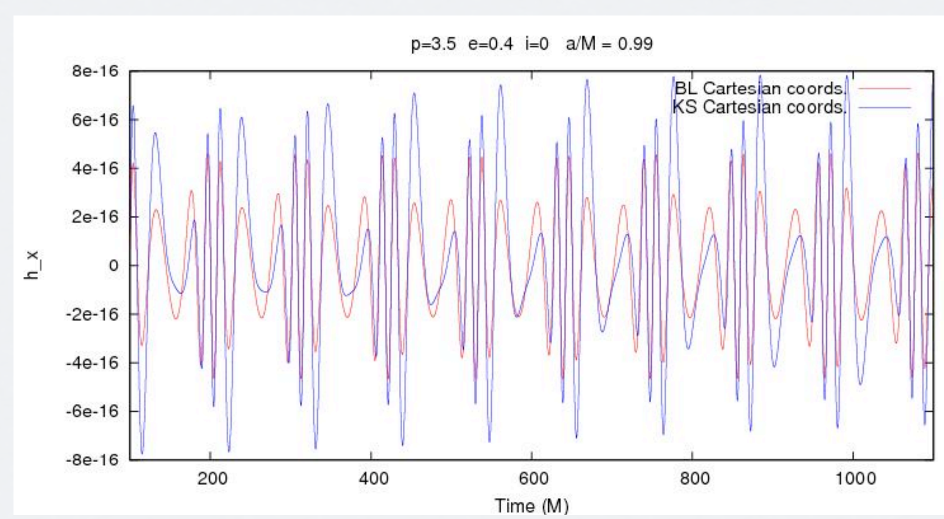


- Once the SCO becomes gravitationally bounded to the MBH, it performs an eccentric relativistic orbit which shrinks and circularizes due to the loss of energy and angular momentum via the emission of gravitational waves (GWs).

- The SCO inspiral is due to the interaction of the SCO with its own gravitational field: the gravitational back-reaction.

- During the last year before plunge, an EMRI will spend about $\sim 10^5$ cycles inside the LISA band, tracking much of the geometry of the MBH spacetime. This information will be encoded in the structure of the gravitational waves emitted.





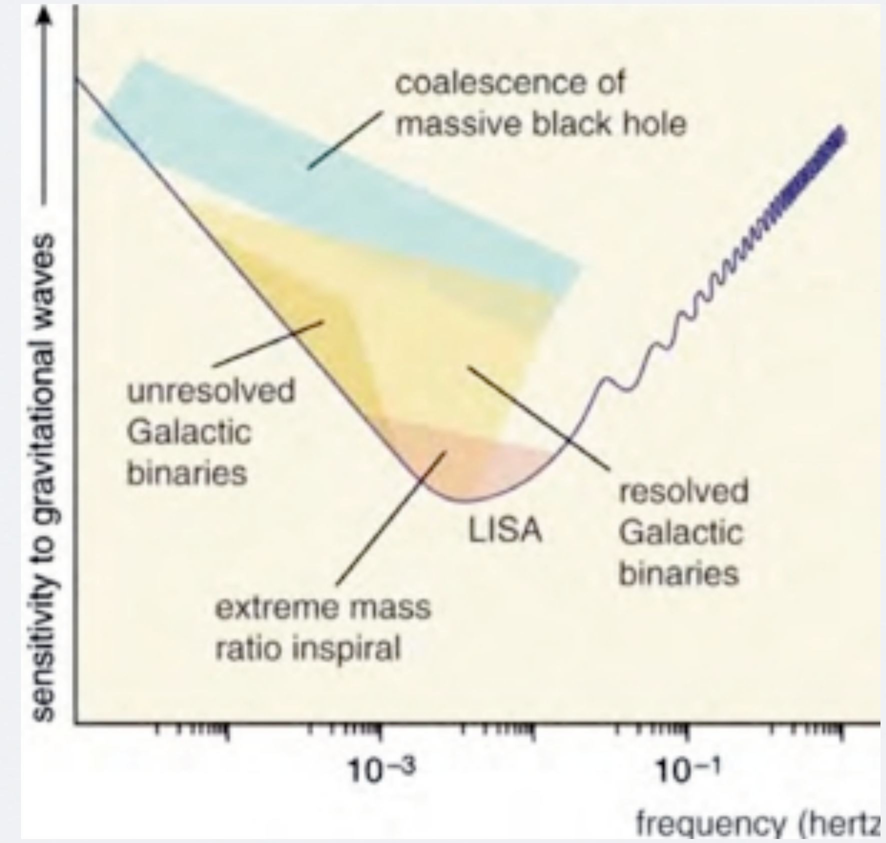
- The EMRI data will unveil unknown features of the universe which will have implications on **Astrophysics, Cosmology and Fundamental Physics**. This will let us:
  - Understand better the dynamics around galactic centers, the formation history of MBHs and its implications for galaxy formation models.
  - Obtain information about the mass spectrum of stellar black holes in galactic centers.
  - Perform precise measurements of cosmological parameters.
  - Test alternative theories of Gravity.
  - etc.

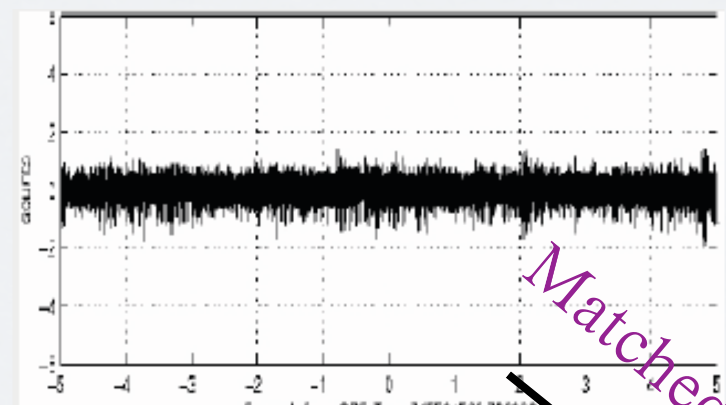
Studying EMRIs

- The **EMRI signals will be hidden** in the LISA instrumental noise and the stellar compact binaries signals in the LISA band.

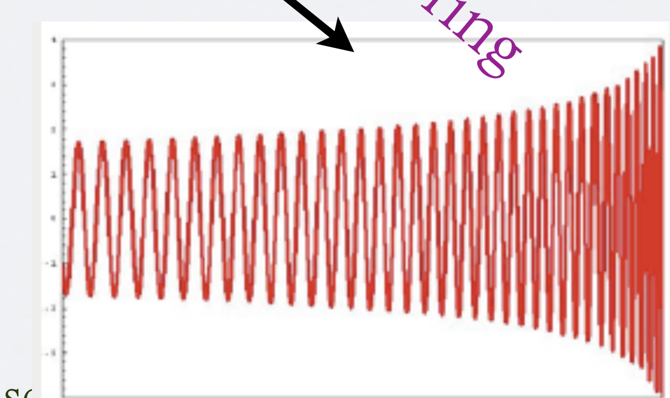
- This means that **we need accurate theoretical templates** in order to compare them with the detected signals and extract the physical parameters: **Matched filtering technique.**

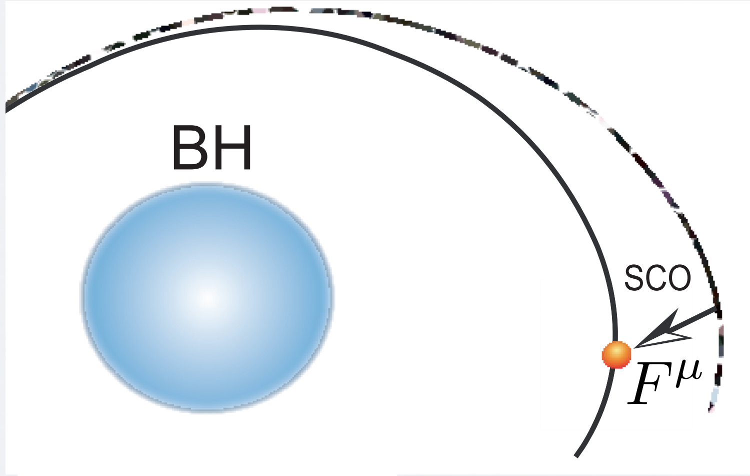
> Due to the complexity of the EMRI signals, it is more suitable to use **time-domain techniques** to obtain the templates.





Matched filtering





$$g_{\mu\nu} = g^{BH}_{\mu\nu} + h_{\mu\nu}$$

- Due to their extreme mass–ratio, EMRIs can be treated in the framework of **perturbation theory,** where the backreaction is pictured as the action of a **local self-force.**

- An analogous EMRI problem consists in a **scalar point particle** endowed with a scalar charge q falling in a **geodesic of a non-rotating MBH spacetime (Schwarzschild).** This is a testbed for numerical codes to compute the gravitational self–force.

$$-\rho 4\pi = g^{\mu\nu}\nabla_\mu\nabla_\nu\Phi^{ret} = \Box\Phi^{ret} \longrightarrow F^\mu = m\frac{du^\mu}{d\tau} = q(g^{\mu\nu} + u^\mu u^\nu)\nabla_\nu\Phi^{ret}$$

$$\rho = -4\pi q\int \delta_4[x - z(\tau)]d\tau \qquad u^\mu = \frac{dz^\mu}{d\tau}$$

- Due to the spherical symmetry of this system the retarded field can be decomposed into spherical harmonics:

$$\Phi^{ret} = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \Phi^{lm}(t,r) Y^{lm}(\theta,\varphi)$$

The point-like character of the particle makes the **retarded filed**, and thus the self-force, **diverge at the particle location**. Hence, the **self-force must be regularized** and we use the **mode-sum regularization scheme**[1]: provides an analytic expression for the field singularities.

1 Barack et al. PRL, 88, 2002.

$$\Box\Phi^{ret} = \Box(\Phi^S + \Phi^R) \begin{cases} \Box\Phi^S = -4\pi q \delta(z) \\ \Box\Phi^R = 0 \end{cases} \longrightarrow \mathcal{F}_\alpha = q(\nabla_\alpha \Phi^{ret} - \nabla_\alpha \Phi^S) = q\nabla_\alpha \Phi^R$$

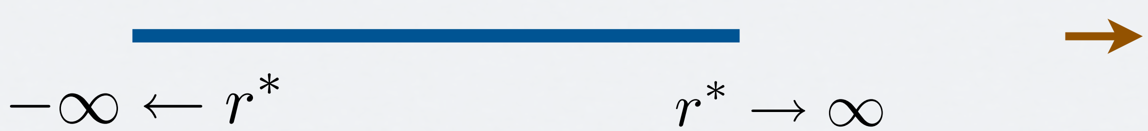
> We need a **numerical method to compute the full retarded field** and by applying the mode-sum regularization scheme obtain the self-force.

- We have developed a multidomain numerical code which avoids working with the singularity associated with the SCO (q).

- It employs time-domain techniques which allows us to deal easily with eccentric orbits.

Solving the Field Equation:
The Particle-without-Particle Scheme

We perform a **division of the spatial computational domain** into two disjoints regions or subdomains, one at the left of the particle $r^* < r^*_p$ and the other at the right of the particle $r^* > r^*_p$ $\left(r^* = r + 2M\ln\left(\frac{r}{2M} - 1\right)\right)$:



Locating **the particle at the interface** between subdomains we avoid the problems associated with the singularity of the source.

> We evolve independent **homogeneous wave equations**, with **smooth solutions**, inside each region: very convenient for **convergence** purposes.

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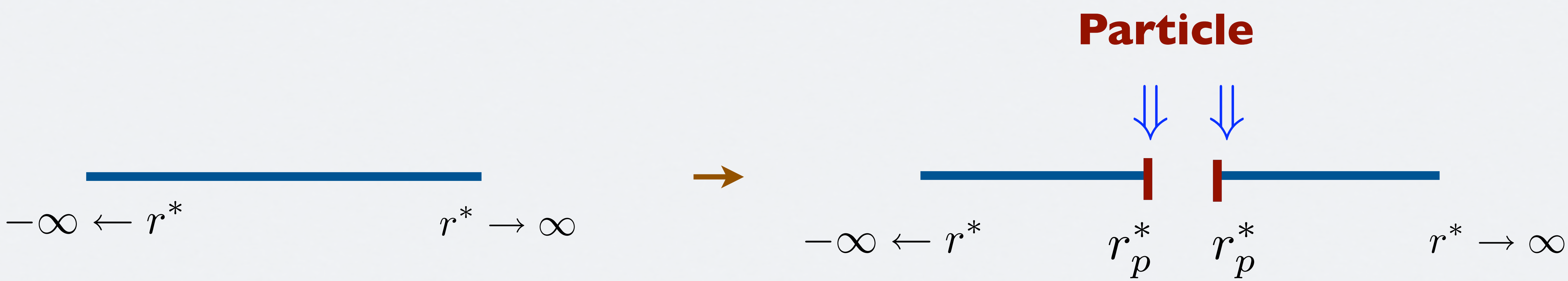


Locating the particle at the interface between subdomains we avoid the problems associated with the singularity of the source.

We evolve independent homogeneous wave equations, with smooth solutions, inside each region: very convenient for convergence purposes.

- Across the boundaries of the subdomains we have to **enforce the continuity** of the **scalar field and of the jumps** of its time and radial derivatives.

- We use the **matching conditions** dictated by the **master equation.**

The **source term** is introduced in the equations as **boundary conditions** at the interface between subdomains.

Solving the set of PDEs numerically:

- The pseudospectral collocation (**PSC**) method to **discretize in space.**
- We use a **Runge-Kutta** method for the **time evolution (method of lines).**

▸ With **PSC methods** the solutions are approximated by an expansion in a basis of Chebyshev polynomials $\{T_n(X)\}$:

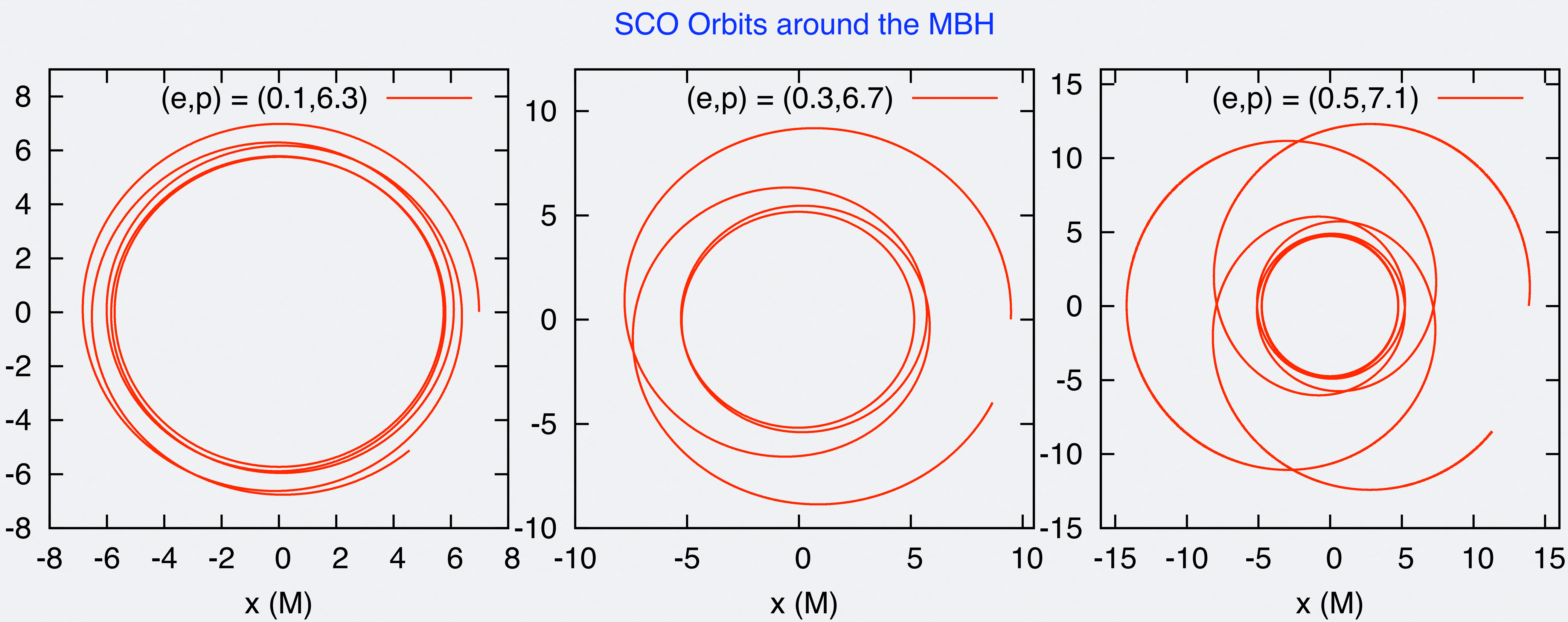
$$\mathcal{U}_N(t, r^*) = \sum_n^N a_n(t) T_n(r^*)$$

where $a_n(t)$ are the spectral coefficients.

> The **PSC method** has the property that it provides **exponential convergence with N** for smooth functions.

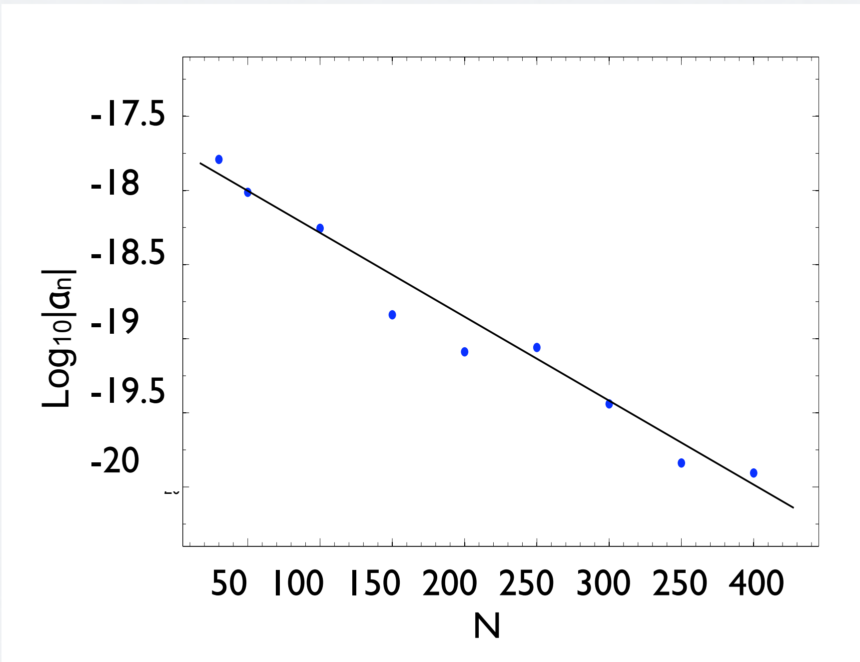
Results from the simulations

- We have computed the self-force components for eccentric orbits with different eccentricity (e) and semilatus rectum (p)



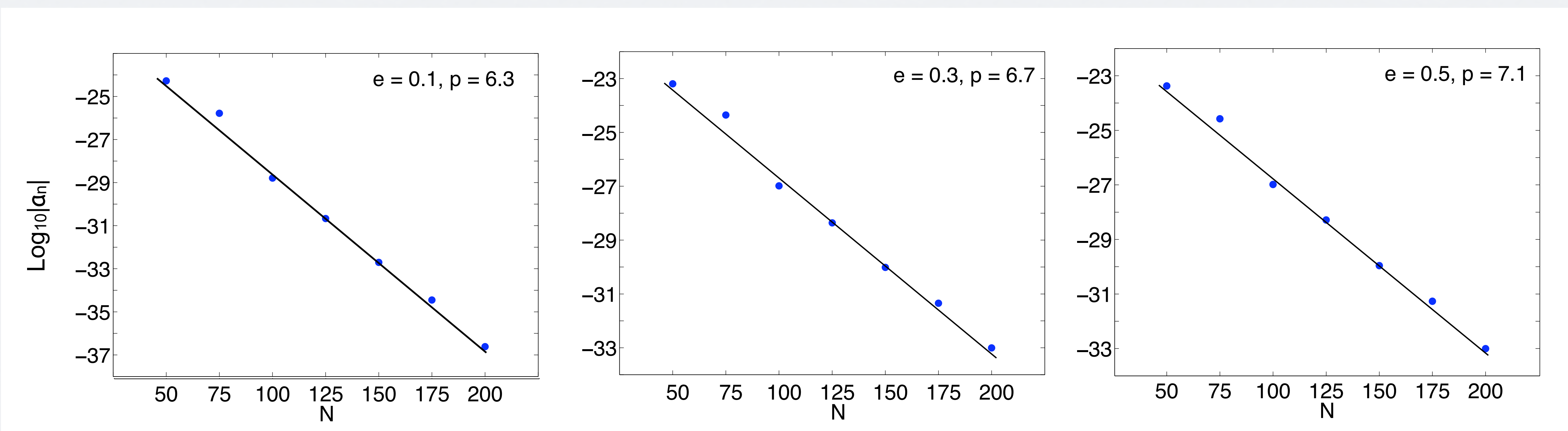
Convergence tests

Circular Orbit

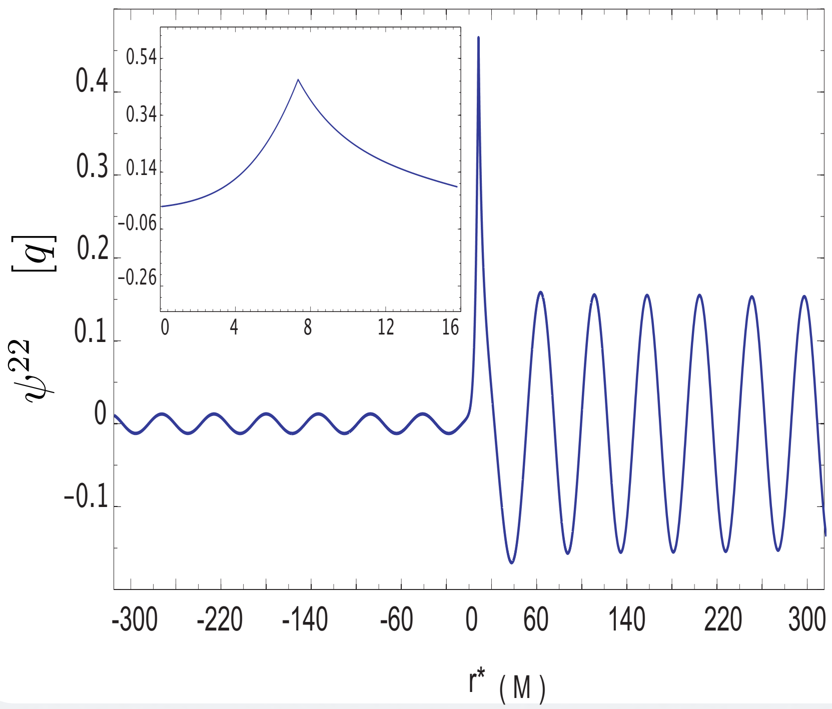


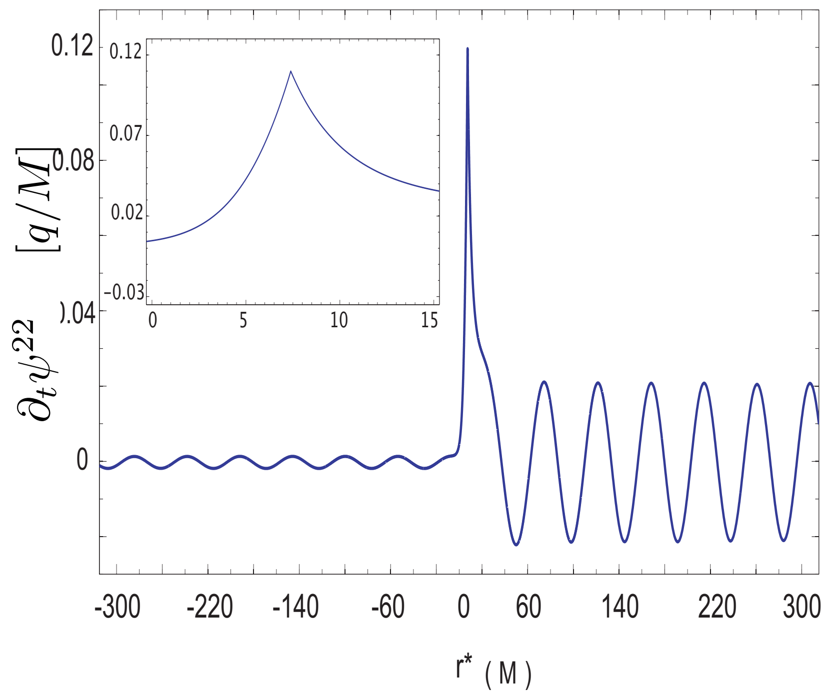
The dependence of the truncation error (~ $|a_N|$) with respect increasing numbers of collocation points, N, give us an estimation of the exponential convergence of the solutions: e^{-N}

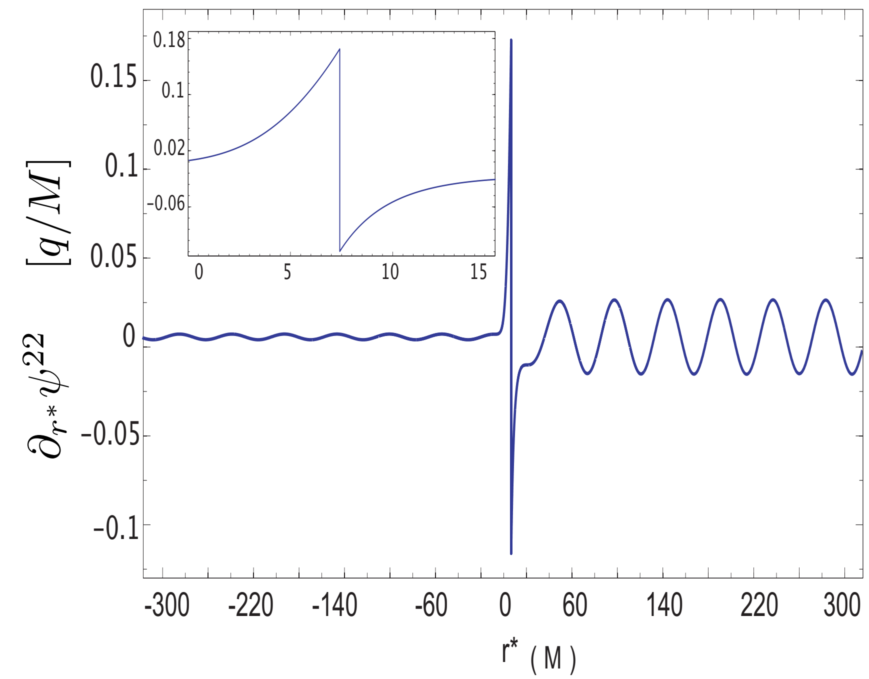
Eccentric Orbit



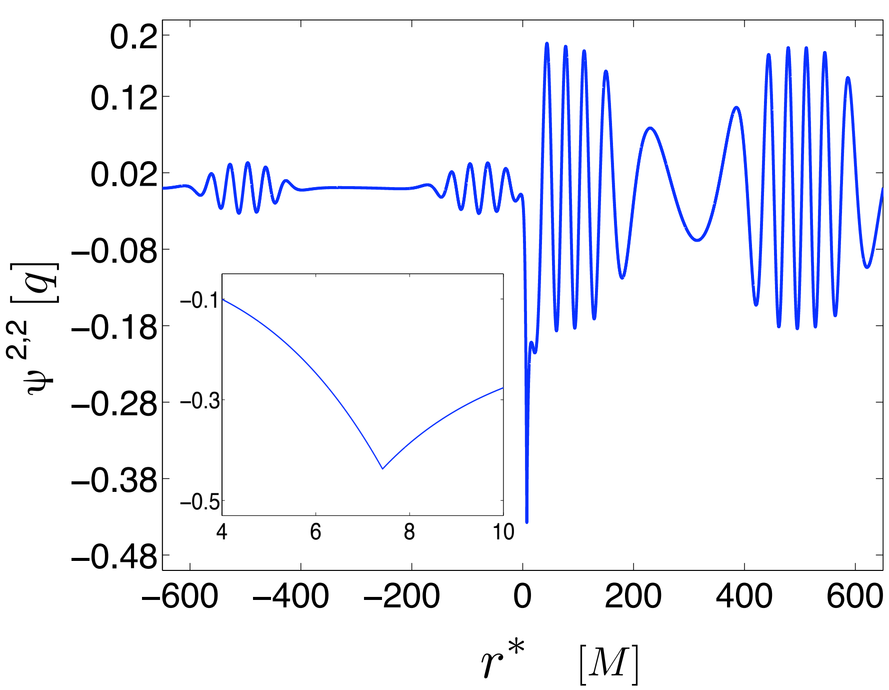
Snapshots from the Circular case (D=12, N=50). Mode (2,2)

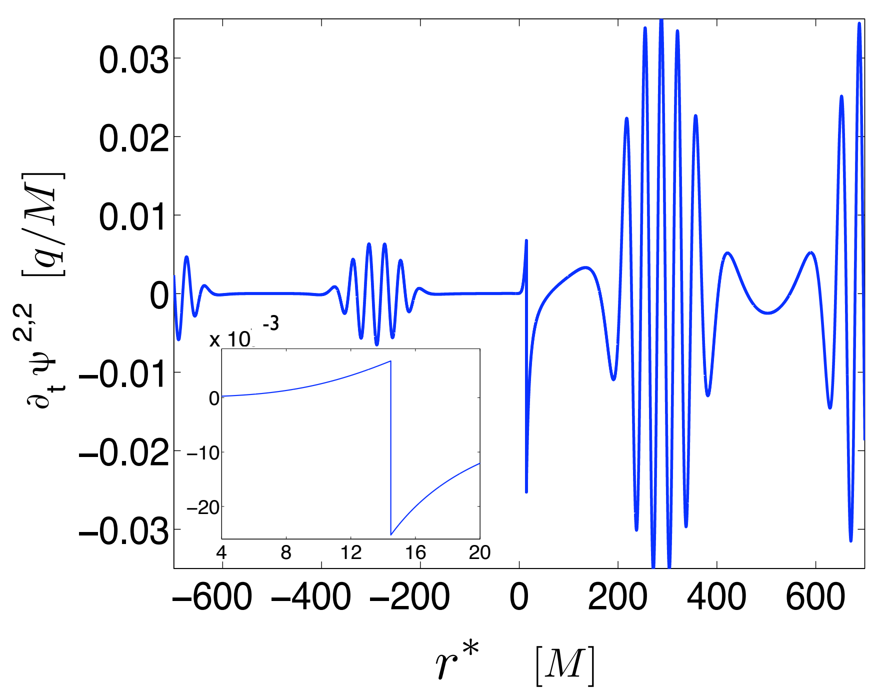


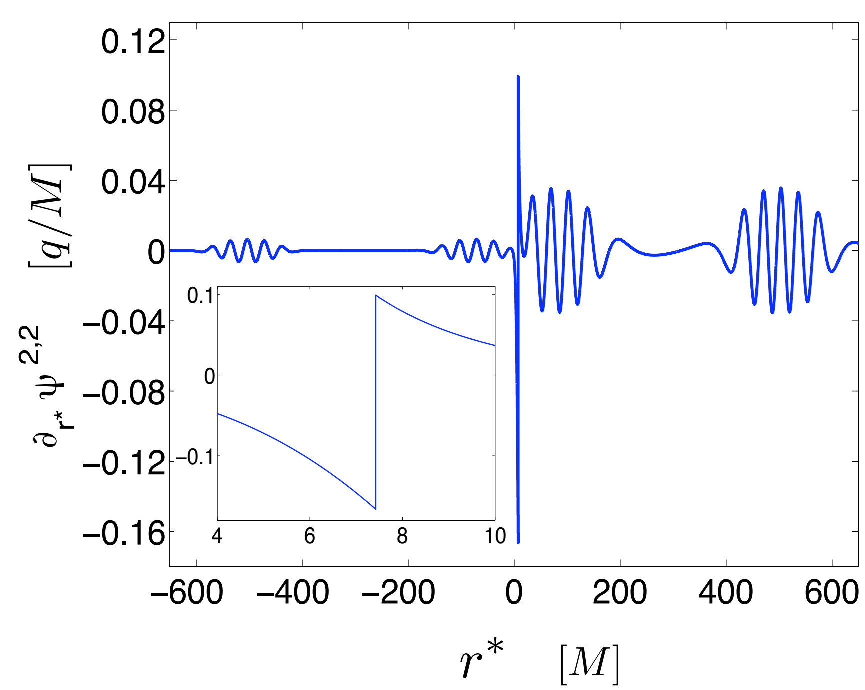




Snapshots from the Eccentric (e=0.5, p= 7.1) case (D=10, N= 100). Mode (2,2)







- Results for the retarded field derivatives (Self–force components) for D = 12, N = 50

Circular Orbit

$r(M)$	Component of Φ^{R}_{α}	Estimation using the PSC Method	Estimation from Frequency-domain (a,b)	Error relative to Frequency-domain (a,b)	Error relative to Time-domain (c)
6	$(\Phi^{R,-}_{t}, \Phi^{R,+}_{t})$	$(3.60777, 3.60778) \cdot 10^{-4}$	$3.609072 \cdot 10^{-4}$	$(0.03, 0.03)\%$	$(0.12, 0.12)\%$
	$(\Phi^{R,-}_{r}, \Phi^{R,+}_{r})$	$(1.67364, 1.67362) \cdot 10^{-4}$	$1.67728 \cdot 10^{-4}$	$(0.2, 0.2)\%$	$(0.18, 0.18)\ \%$
	$(\Phi^{R,-}_{\varphi}, \Phi^{R,+}_{\varphi})$	$(-5.3042, -5.3044) \cdot 10^{-3}$	$-5.304231 \cdot 10^{-3}$	$(4 \cdot 10^{-4}, 10^{-3})\ \%$	$(6 \cdot 10^{-4}, 10^{-3})\%$

(a) [Diaz-Rivera et al. PRD 70, 124018 (2004)] , (b) [Haas, Poisson. PRD 74, 044009 (2006)] (c) [Hass. PRD 75, 124011 (2007)]

P. Canizares & C. F. Sopuerta '09;

> We obtain **accurate Self-Force results** with small amount of computational resources.

Conclusions & Future work

Conclusions & Future work

- We have developed a new time-domain technique for the simulations of EMRIs:
  - It avoids the introduction of a small scale in our code, and provides precise determination of the field and its derivatives near an on the SCO.
  - It is an efficient method to make time-domain computations of the self-force because it preserves the properties of the PSC method.
- We would like to apply these techniques to the Schwarzschild and Kerr gravitational cases.

Thank you for your attention!

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