Hydromagnetic instabilities and magnetic field amplification in core collapse supernovae

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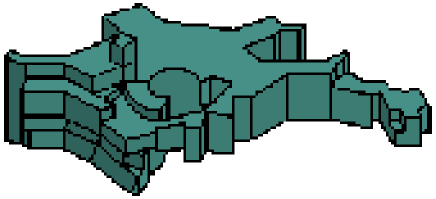

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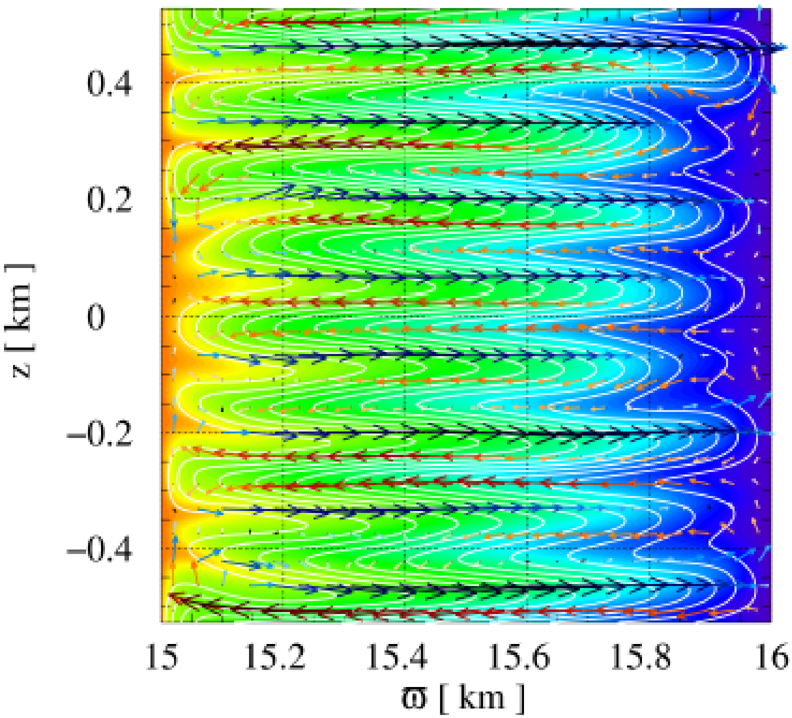
Magnetic field amplification in supernovae

- **Motivation (Core collapse supernovae):**
  - Magnetars : 10^{14}-10^{15} G
  - Long GRBs
    - Collapsar model (Whoosley et al 1993)
    - SNIc-BL – Long GRB association

- **Magnetic field amplification**
  - Compression (not effective)
  - Differential rotation
    - Wind-up (linear)
    - Magneto-rotational instabilities (MRI)
  - Dynamos

Magnetorotational instability (MRI)



- **Ingredients (simplified)**
  - Differential rotation

$$\partial_{\varpi} \Omega^2 < 0$$ (Balbus & Hawley 1991)

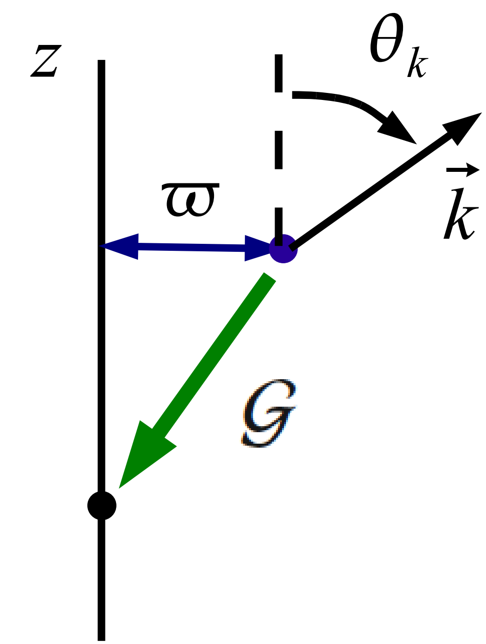
  - Seed magnetic field
- **Generic ingredients in CC**
- **Exponential growth**

$$\tau_{fgm} \approx \frac{4\pi}{q\bar{\Omega}} \approx 3\left(\frac{T}{1\,ms}\right)\left(\frac{1.5}{q}\right) ms$$

$$\lambda_{fgm} \approx 2\pi \frac{c_A}{\Omega} \approx 500\left(\frac{B_{polo}}{5\times10^{11}\,G}\right)\left(\frac{T}{1\,ms}\right) m$$

MRI theory in detail

- Based on Balbus 1995



$$\mathcal{G} \equiv \frac{\nabla P}{\rho}$$ **Gravity**

$$\mathcal{R} \equiv \varpi \nabla \Omega^2$$ **Rotation shear**

$$\mathcal{B} \equiv -\frac{1}{\Gamma_1} \left.\frac{\partial \ln P}{\partial s}\right|_{\rho} \nabla s$$ **Buoyancy**

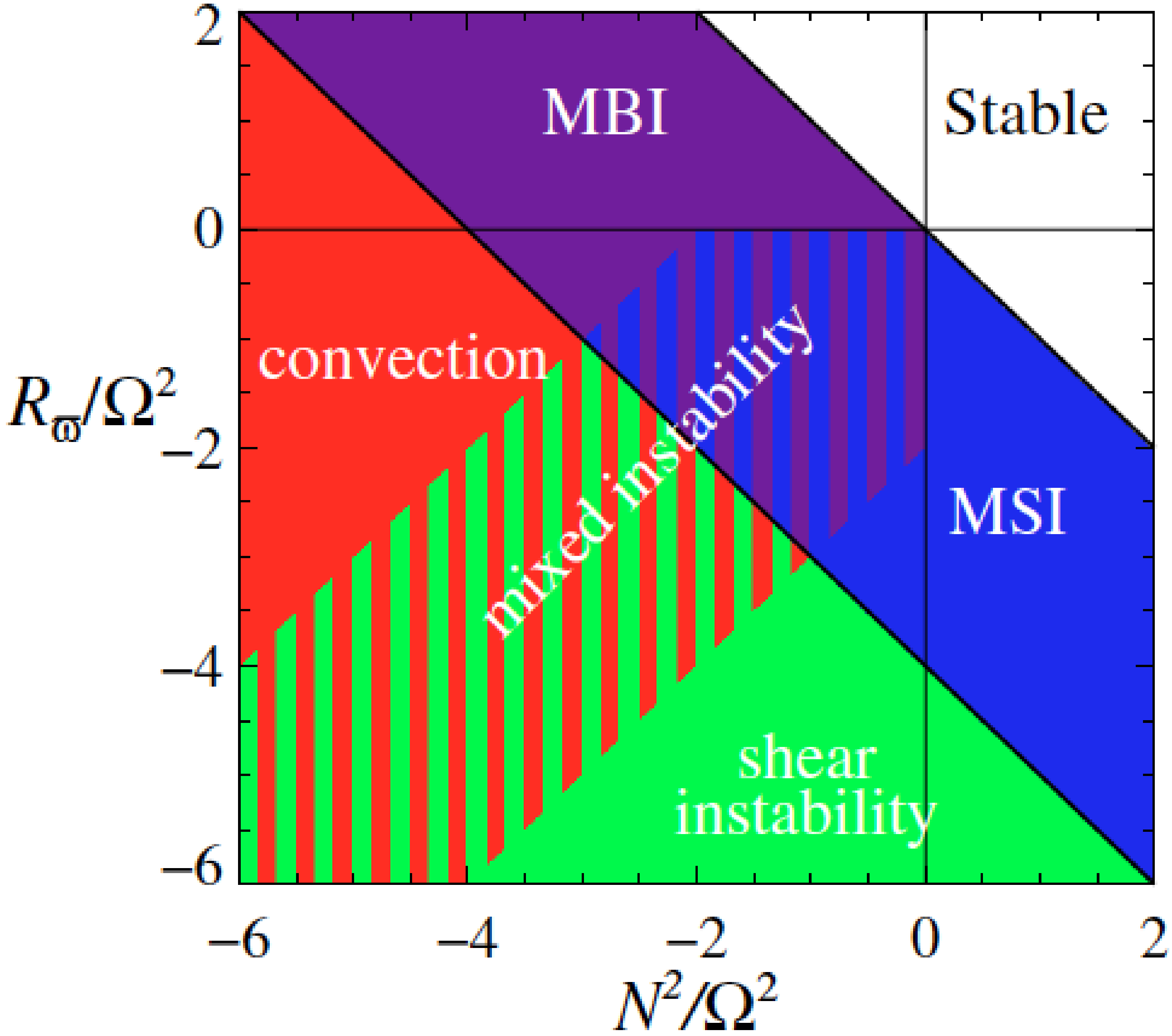
$$C = (\mathcal{G}_z \mathcal{B}_z \tan^2 \theta_k - 2\mathcal{B}_\varpi \mathcal{G}_z \tan\theta_k + \mathcal{G}_\varpi \mathcal{B}_\varpi + \mathcal{R}_\varpi)/\Omega^2$$

At the equator: $$C_{90} = (N^2 + \mathcal{R}_\varpi)/\Omega^2$$

$$N^2 \equiv \mathcal{B} \cdot \mathcal{G}$$ **Brunt-Väisälä frequency**

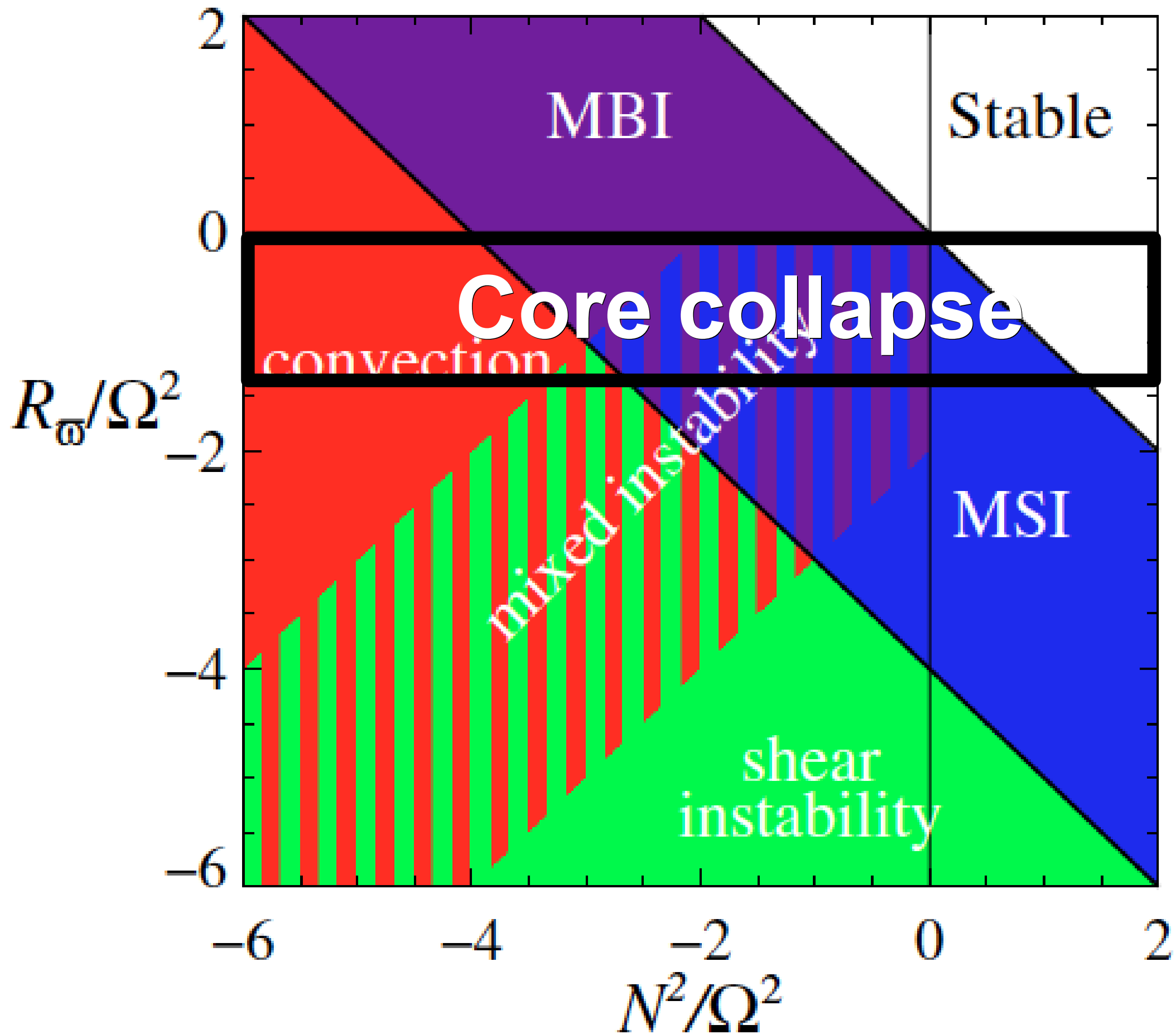
MRI theory in detail

- Equatorial plane : $\mathcal{C}_{90} = (N^2 + \mathcal{R}_{\varpi})/\Omega^2$



MRI theory in detail

- Equatorial plane : $$C_{90} = (N^2 + \mathcal{R}_{\varpi})/\Omega^2$$

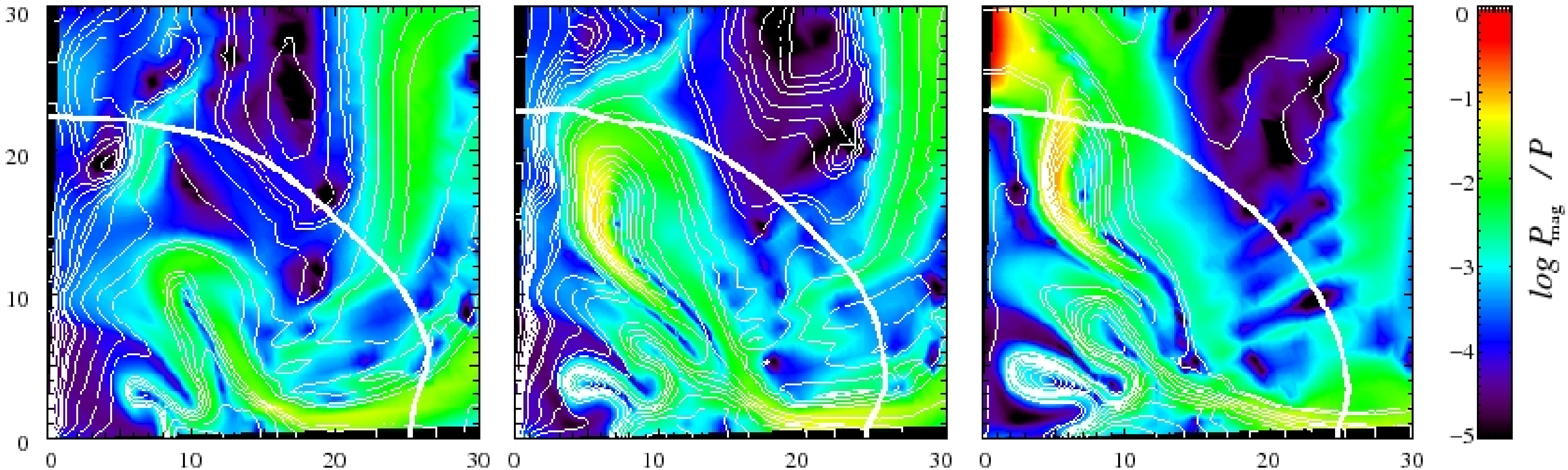


GR simulations - CoCoNuT

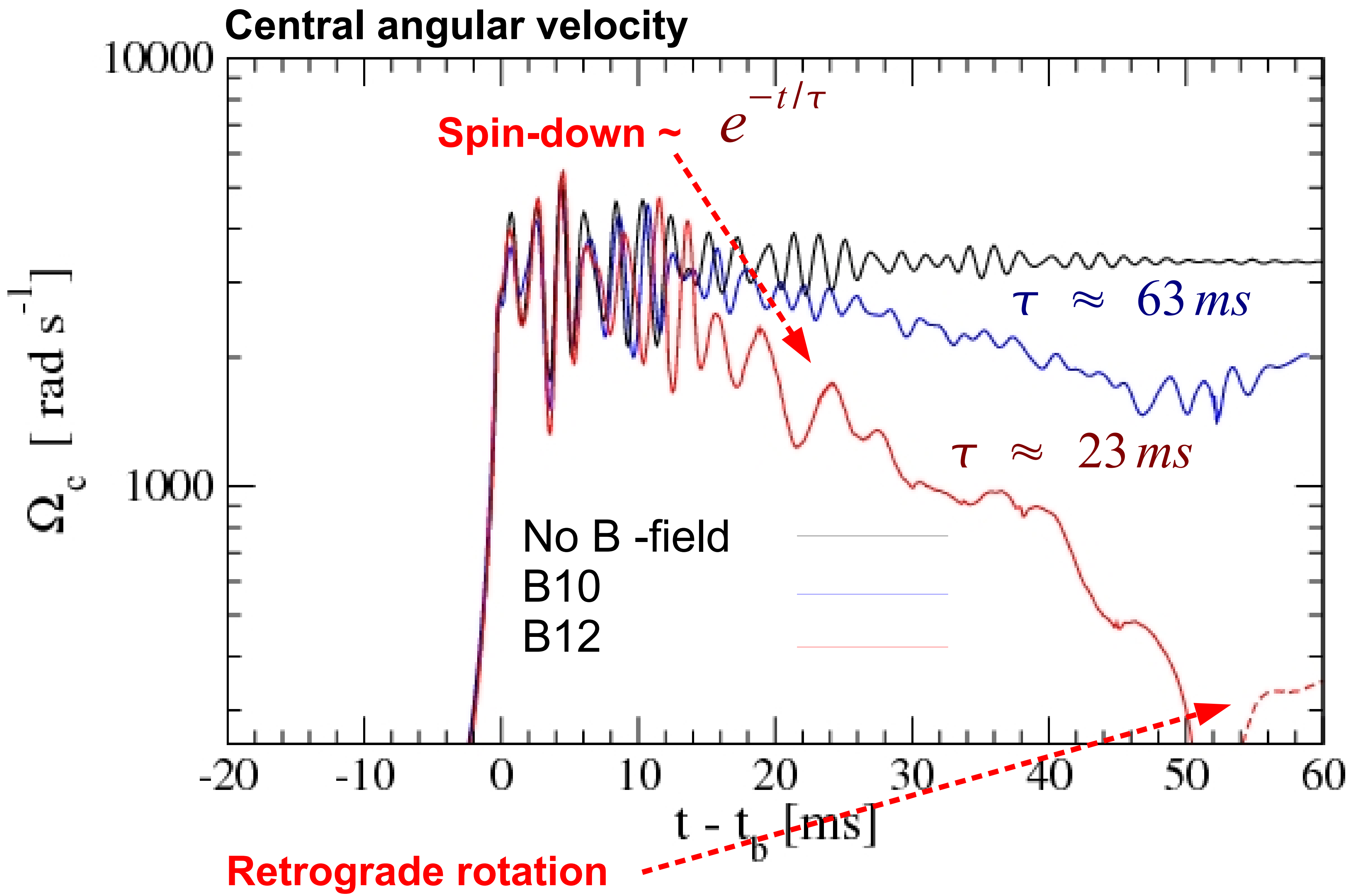
- **Magnetorotational core collapse:**

  20 $M_\odot$ star with B~10^{10} G and 10^{12} G.

- **Ideal MHD code in dynamical space-time (CFC)**
  (Dimmelmeier et al 2001, Cerdá-Durán et al. 2008)
- **Spherical polar coordinates in axisymmetry (2D)**
- **Microphysical EOS (SHEN) and deleptonization scheme**
  (Liebendörfer 2005)
  - Time-scale: ~ 10 ms (PNS) , Length-scale: ~ 1-5 km (PNS) for 10^{12} G
    ~ 10-50 m (PNS) for 10^{10} G



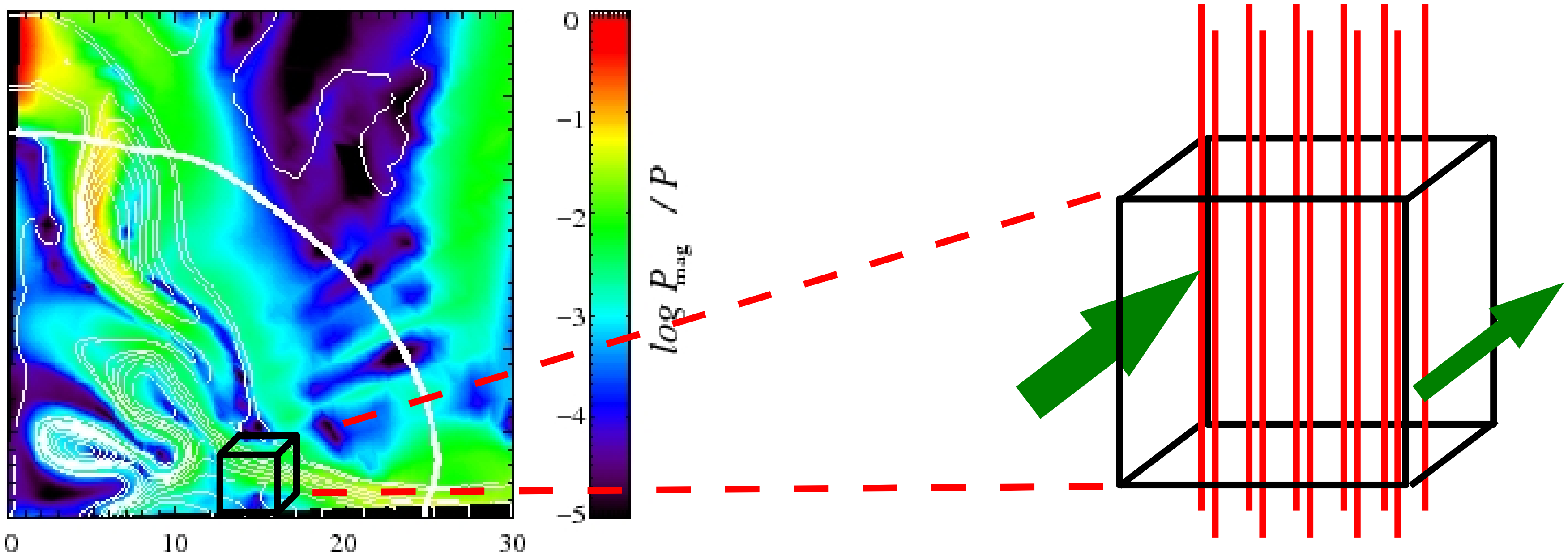
GR simulations - CoCoNuT



(also Müller & Hillebrant 1979 and Obergaulinger et al 2006)

Shearing-box simulations

- Box size: 0.5 – 4 km at ~ 15 km (resolution: 2.5 – 20 m)
- Initial field: 10^{10} – 10^{11} G, Ideal gas
- Ideal MHD (Newtonian)
- Numerical code – **AENUS** (Obergaulinger et al. *in prep.*)
  - Approximate Riemann solvers + CT scheme
  - High order schemes: MP5/7/9 and WENO4



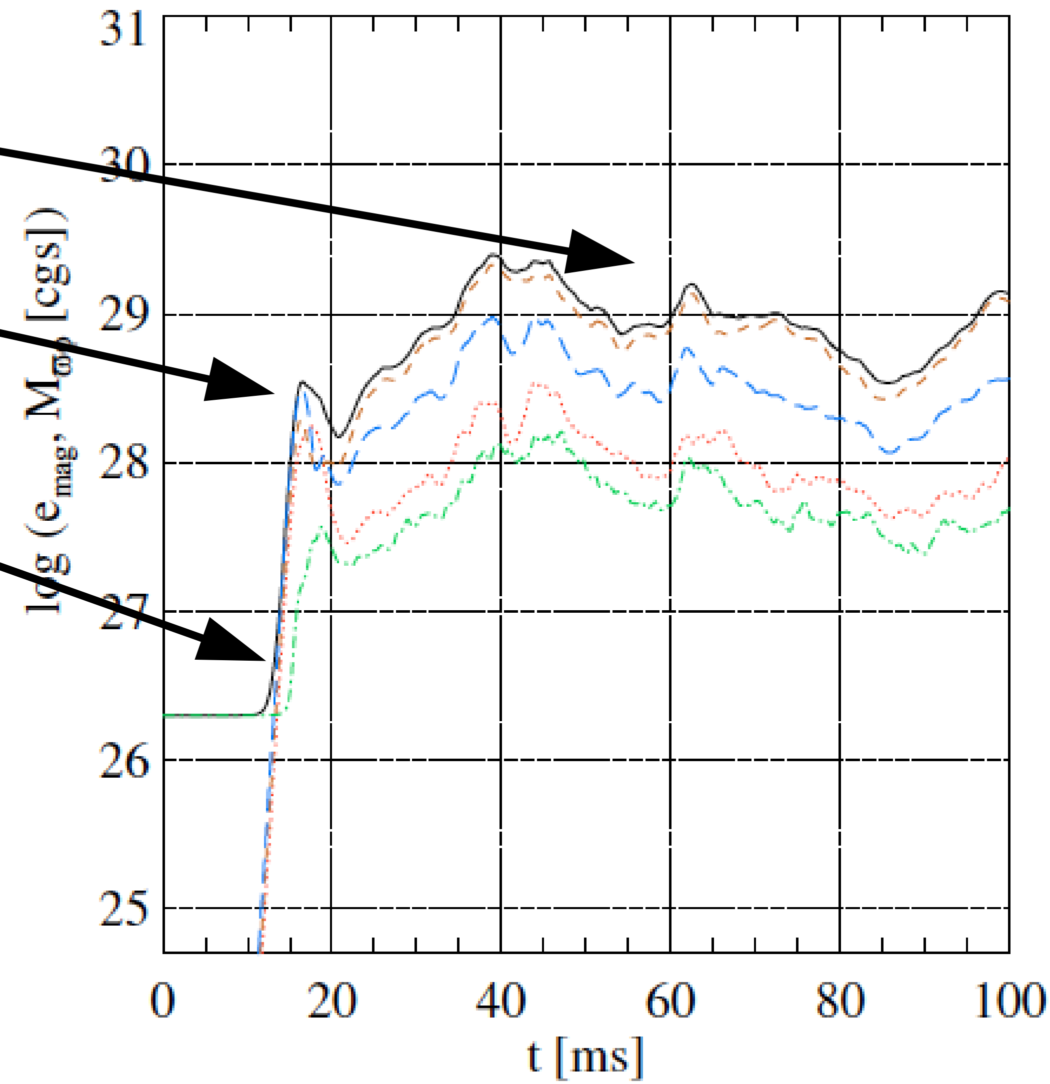
Shearing-box simulations

- **Typical simulation**

saturation

termination

exponential growth



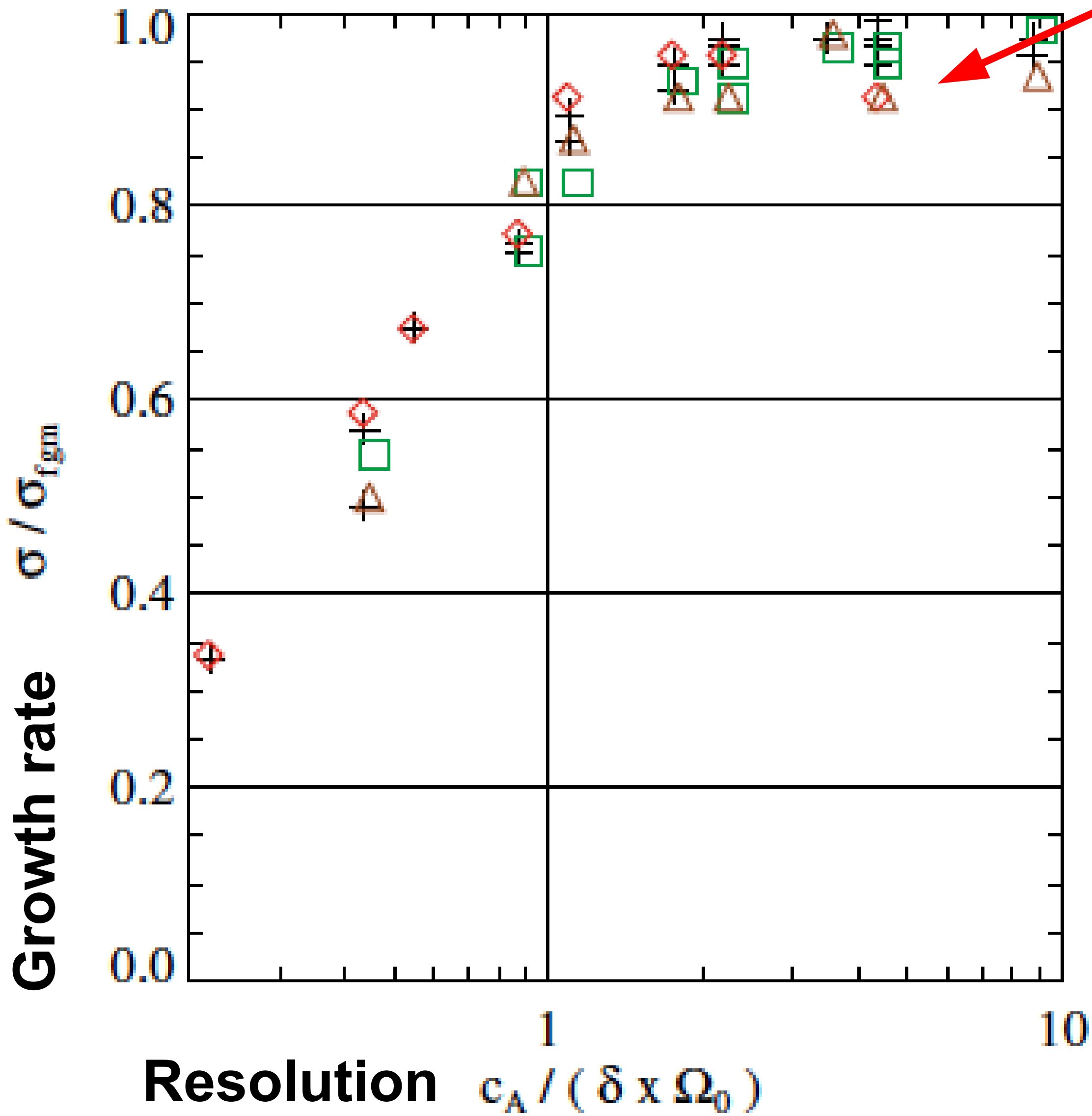
Maxwell stresses

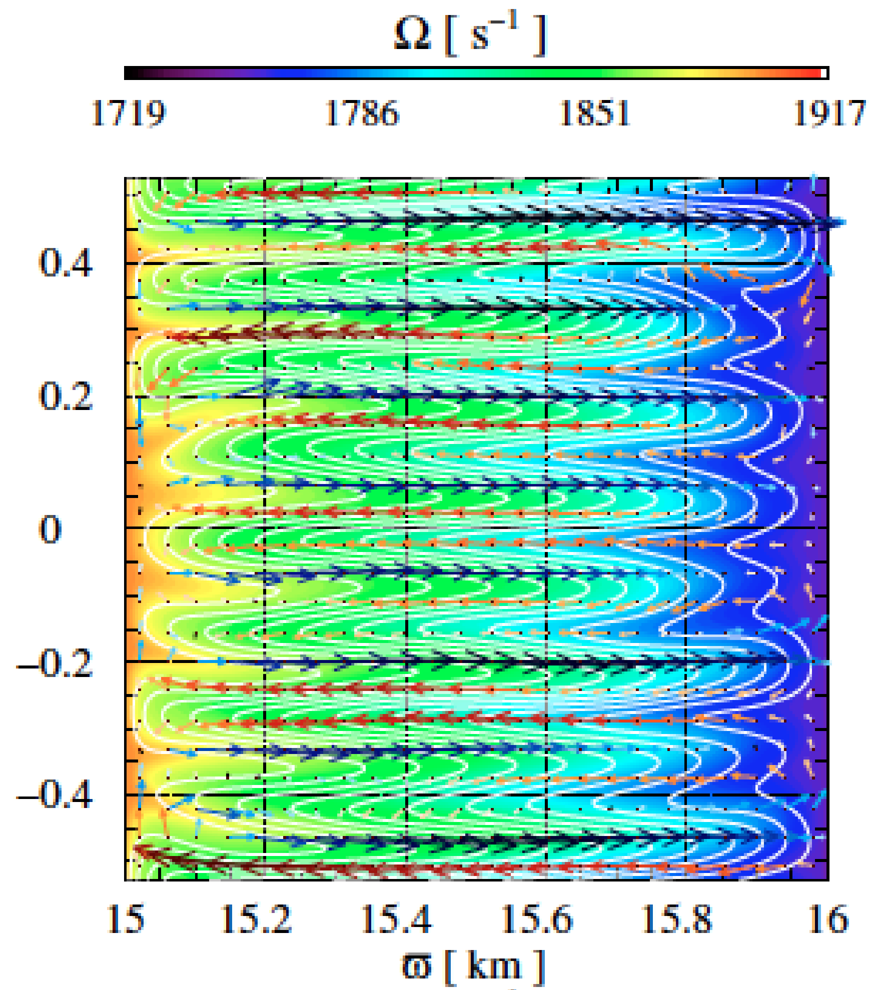
$$M_{ij} = \frac{1}{V} \int dV \, B^i B^j$$

Shearing-box simulations

- **Exponential growth**

Converged growth rates



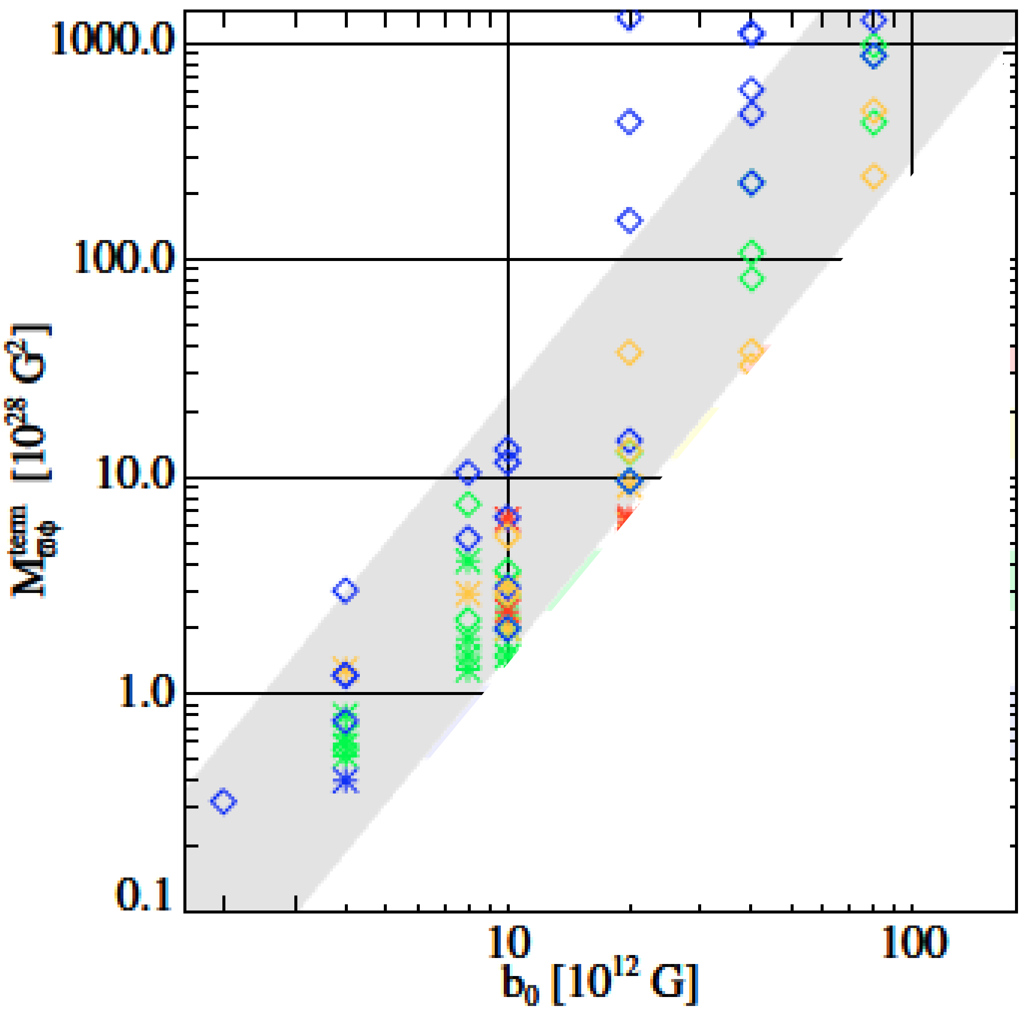


MRI : solution of the non-linear incompressible MHD equations (Goodman & Xu 1994)

Shearing-box simulations

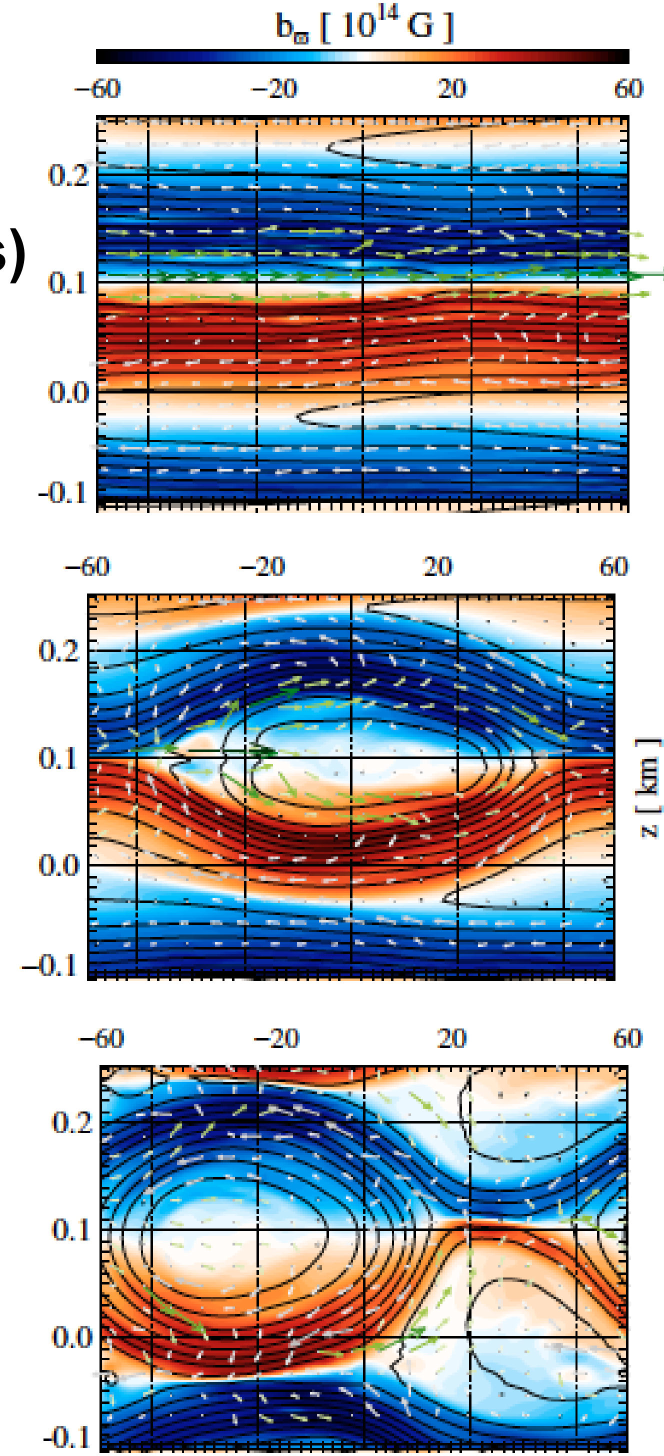
- **Termination: parasitic instabilities (tearing modes)**

Maxwell stress at termination



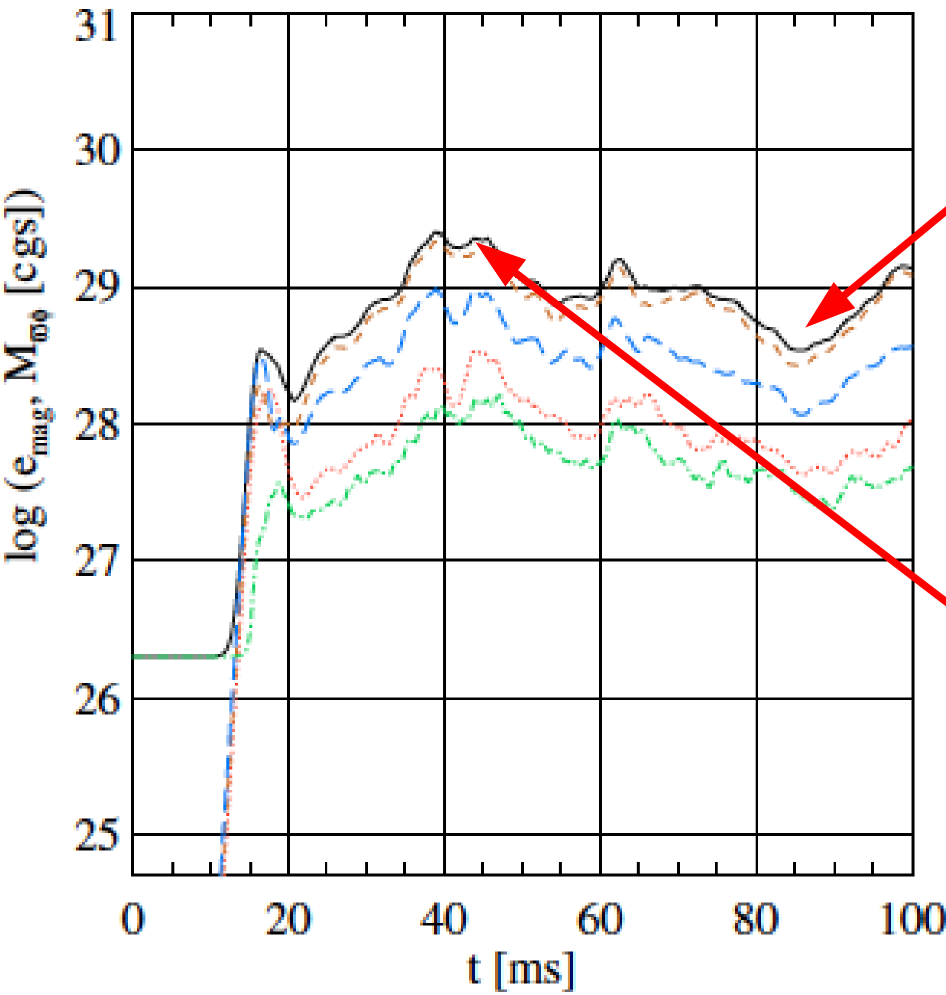
Magnetic field ⟶

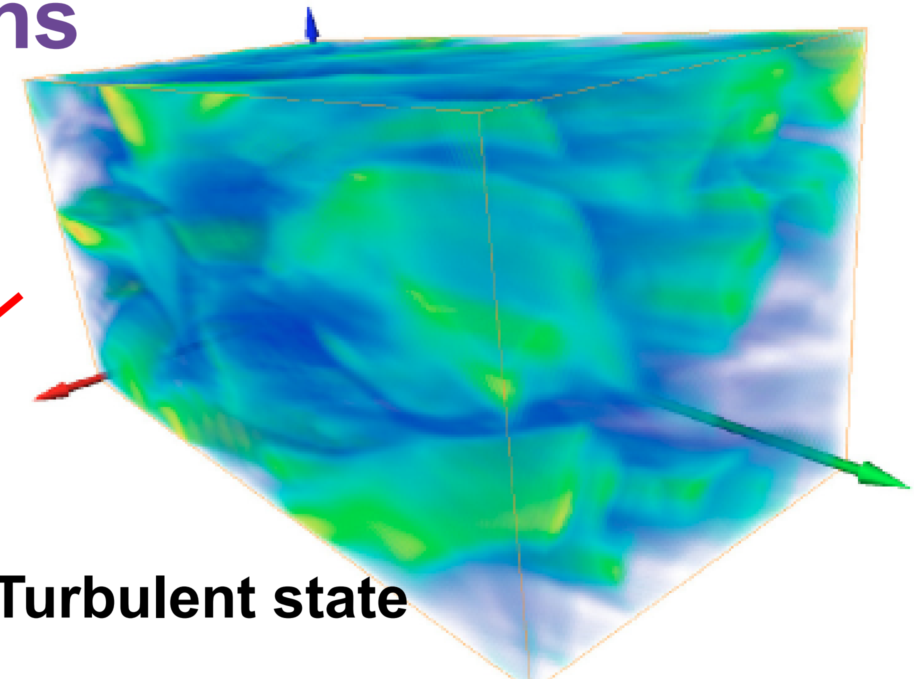
⟵ **Numerical resistivity**



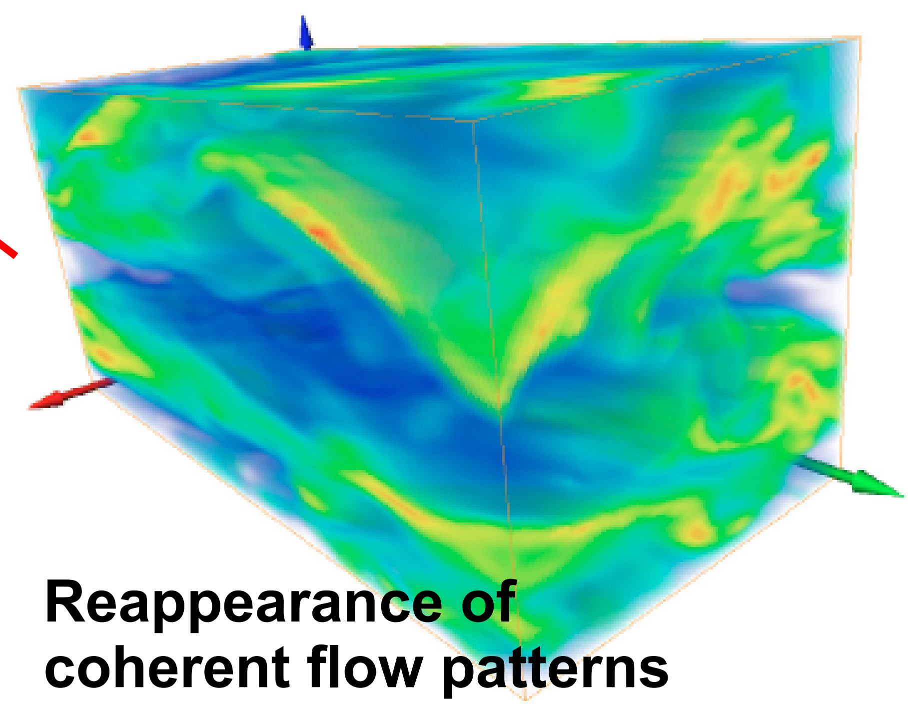
Shearing-box simulations

- **Saturation (3D)**





Turbulent state



Reappearance of coherent flow patterns

Conclusions

- **Global CC simulations**
  - MRI resolved only for unrealistic magnetic field
  - Strong spin down: slow rotating magnetized PNS
- **Semi-local simulations**
  - Exponential growth well resolved and understood
  - Termination: depends on numerical resistivity
  - Saturation: important differences between 2D and 3D
  - Resistive MHD simulations needed