FIRST GRAVITATIONAL WAVES IN FCF IN A DYNAMICAL SPACETIME WITH MATTER CONTENT

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in collaboration with

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Outline

· 3+1 formalism:

CFC approximation and FCF formulation.

· Numerical code and gravitational radiation:
First dynamical simulations in FCF with general background and matter content.

Teukolsky waves.

Stationary neutron star.

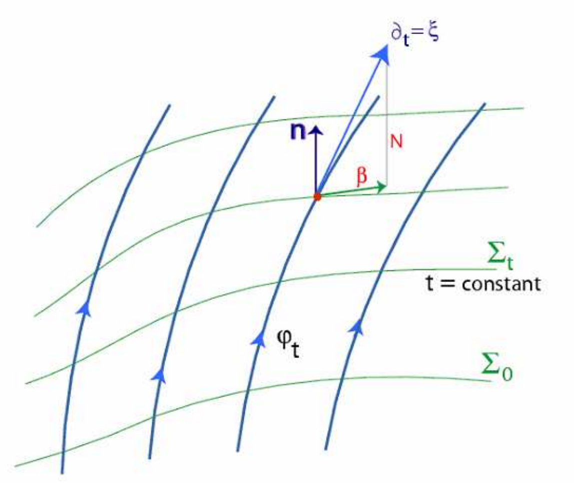
Perturbed neutron star.

· Conclusions.

3+1 formalism:

- Foliation of spacetime by spatial hypersurfaces Σ_t defined by constant t , normal vector $\boldsymbol{n}$.

- Decomposition: $\xi = N\boldsymbol{n} + \boldsymbol{\beta}.$



- Spatial metric onto the hypersurfaces, $\boldsymbol{\gamma} = \gamma_{ij} dx^i dx^j$, and metric of the spacetime:

$$g_{\alpha\beta} dx^\alpha dx^\beta = -N^2 dt^2 + \gamma_{ij} \left(dx^i + \beta^i dt\right) \left(dx^j + \beta^j dt\right)$$

- Extrinsic curvature: $\boldsymbol{K} := -\frac{1}{2}\mathcal{L}_n \boldsymbol{\gamma}$

⇒ Einstein equations: constraint equations + evolution equations: Spatial metric and extrinsic curvature dynamical variables. Constraints are fulfilled during the evolution.

- Non-vacuum case: coupling of the hydrodynamic equations.

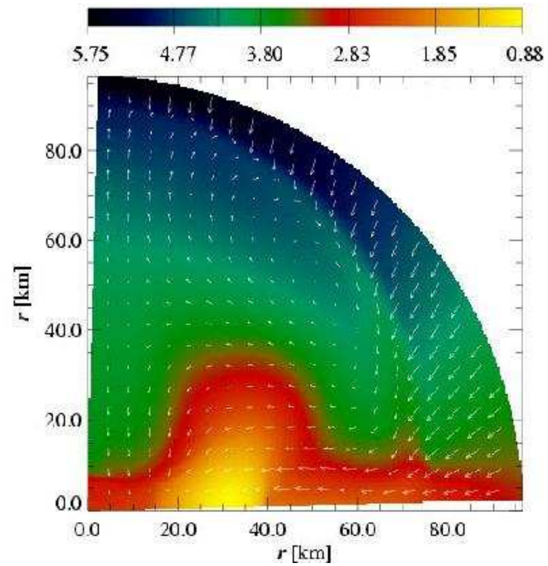
- CFC (Conformally Flat Condition) (Isenberg, 1979/2008; Wilson and Mathews, 1989).

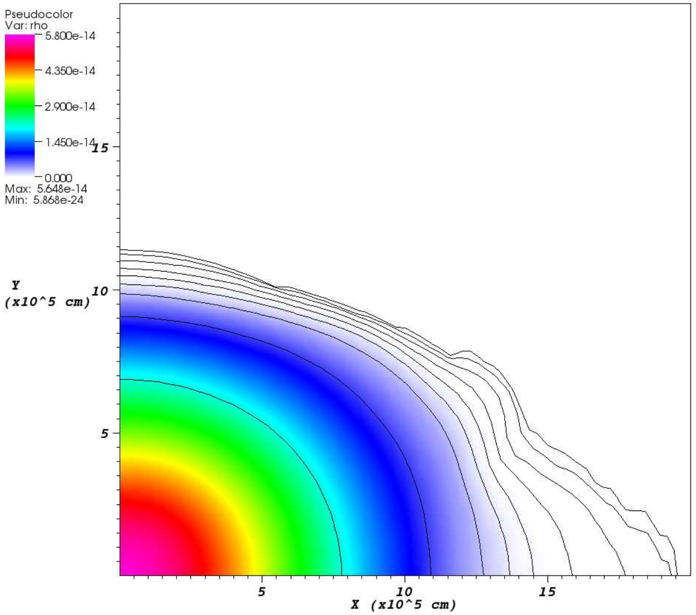
Many applications of evolving matter:

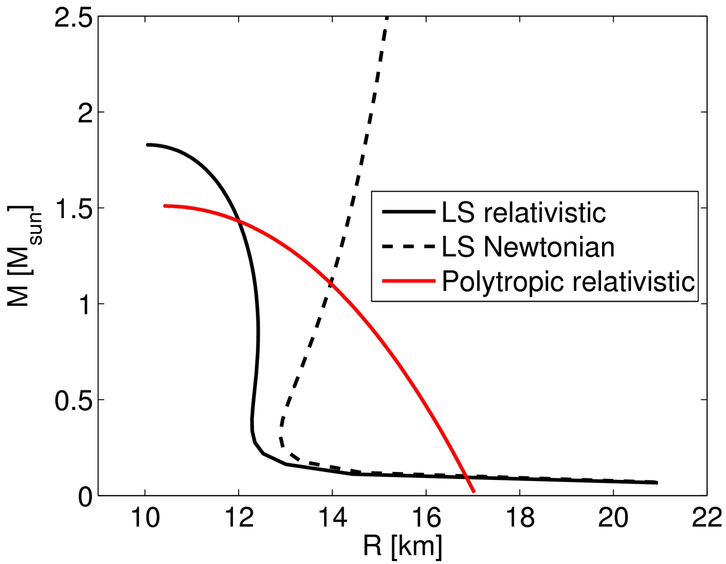
- H. Dimmelmeier & CoCoNuT code: Collapse of rotating cores of massive stars and gravitational waves catalog.

- P. Cerdá-Durán: Equilibrium model of rotating neutron stars and binary neutron star merger.

- A. Bauswein: Evolution of binary compact objects, NS-NS/BH. Necessity of recent new approach of CFC.







- FCF (Fully Constrained Formalism) (Bonazzola et al., 2004) :

· Flat metric f^{ij}, conformal factor $\psi := \left(\frac{\gamma}{f}\right)^{1/12}$, and conformal metric $\tilde{\gamma}^{ij} := \psi^4 \gamma^{ij}$.

· Decomposition of the conformal metric: $\tilde{\gamma}^{ij} =: f^{ij} + h^{ij}$

· Dirac gauge, $H^i := \mathcal{D}_j \tilde{\gamma}^{ij} = 0$, and maximal slicing , $\mathrm{tr}\mathbf{K} = 0$.

· CFC = FCF + ($h^{ij} = 0$) : Einstein equations can be cast into an elliptic system for $f = (\psi,\ N,\ \beta^i)$.

· FCF: Einstein equations can be cast into a similar elliptic system + hyperbolic system for gravitational radiation:

$$\Delta f = S_{\mathrm{CFC}}(f) + S_f(f, h)$$

+

$$\frac{\partial^2 h^{ij}}{\partial t^2} - \frac{N^2}{\psi^4}\tilde{\gamma}^{kl}\mathcal{D}_k\mathcal{D}_l h^{ij} - 2\mathcal{L}_\beta \frac{\partial h^{ij}}{\partial t} + \mathcal{L}_\beta \mathcal{L}_\beta h^{ij} = S_h^{ij}$$

Numerical code: Basic structure and tests.

$$\frac{\partial^2 h^{ij}}{\partial t^2} - \frac{N^2}{\psi^4}\tilde{\gamma}^{kl}\mathcal{D}_k\mathcal{D}_l h^{ij} - 2\mathcal{L}_\beta \frac{\partial h^{ij}}{\partial t} + \mathcal{L}_\beta\mathcal{L}_\beta h^{ij} = S_h^{ij}$$

Numerical evolution of the system:

$$\hat{A}^{ij} = \psi^{10}\left(K^{ij} - \tfrac{1}{3}K\gamma^{ij}\right) \longrightarrow \begin{cases} \frac{\partial h^{ij}}{\partial t} = \dots \\ \frac{\partial \hat{A}^{ij}}{\partial t} = \dots \\ \frac{\partial w_k^{ij}}{\partial t} = \dots \end{cases}$$

(Cordero-Carrión et al., *2009*)

· Lapse, shift, conformal factor, energy-momentum tensor as sources.

· FD for the spatial derivatives and RK methods for the evolution.

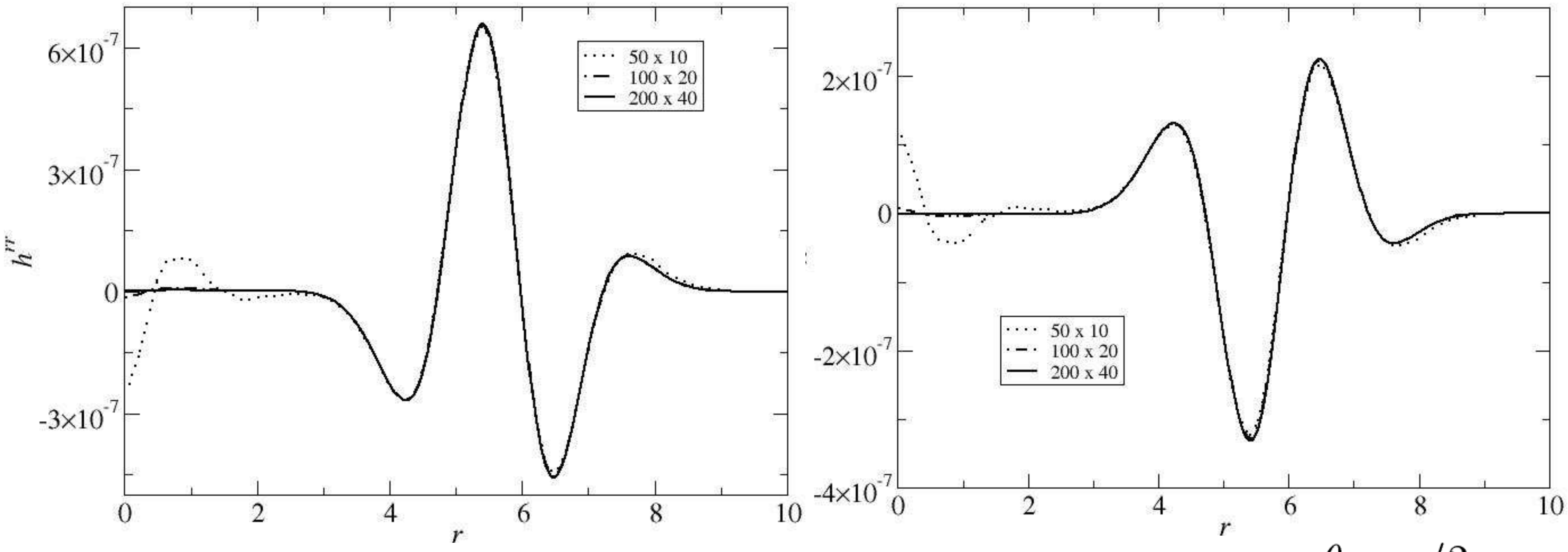
· Spherical orthonormal coordinates. Axisymmetry and symmetry with respect to the equatorial plane.

· Outer boundary: Sommerfeld condition. Kreiss-Oliger dissipative term to avoid high frequency numerical noise.

· Non-vacumm case: Implementation of evolution of matter fields with HRSC methods with the CFC approximation.

Testing the code: Teukolsky waves

Combination of ingoing and outgoing even-parity axisymmetric waves: linear equation in vacuum with a flat background.



$\theta = 0$ $t = 6$ $\theta = \pi/2$

· Order of convergence: ~ 3.7

· The velocity and the amplitude of the wave is recovered.

Equilibrium configuration of a rotating neutron star:

We first simulate a relativistic axisymmetric uniformly rotating neutron star in equilibrium:

- CFC approximation for comparison in perturbed case

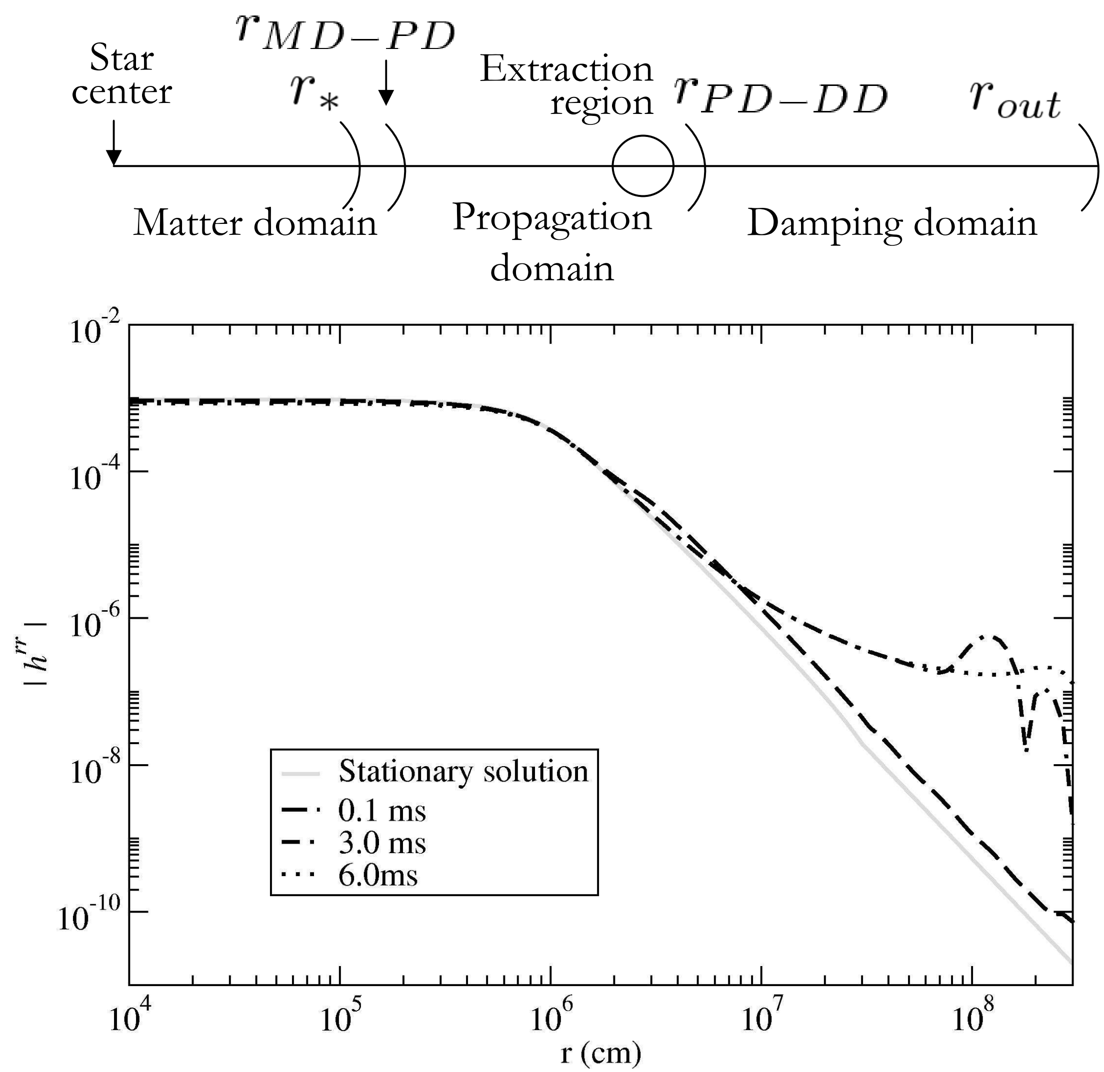
- $r_{out} \geq$ 300 stellar radii

- Rotation frequency: 550 Hz

- EOS: relativistic polytrope.

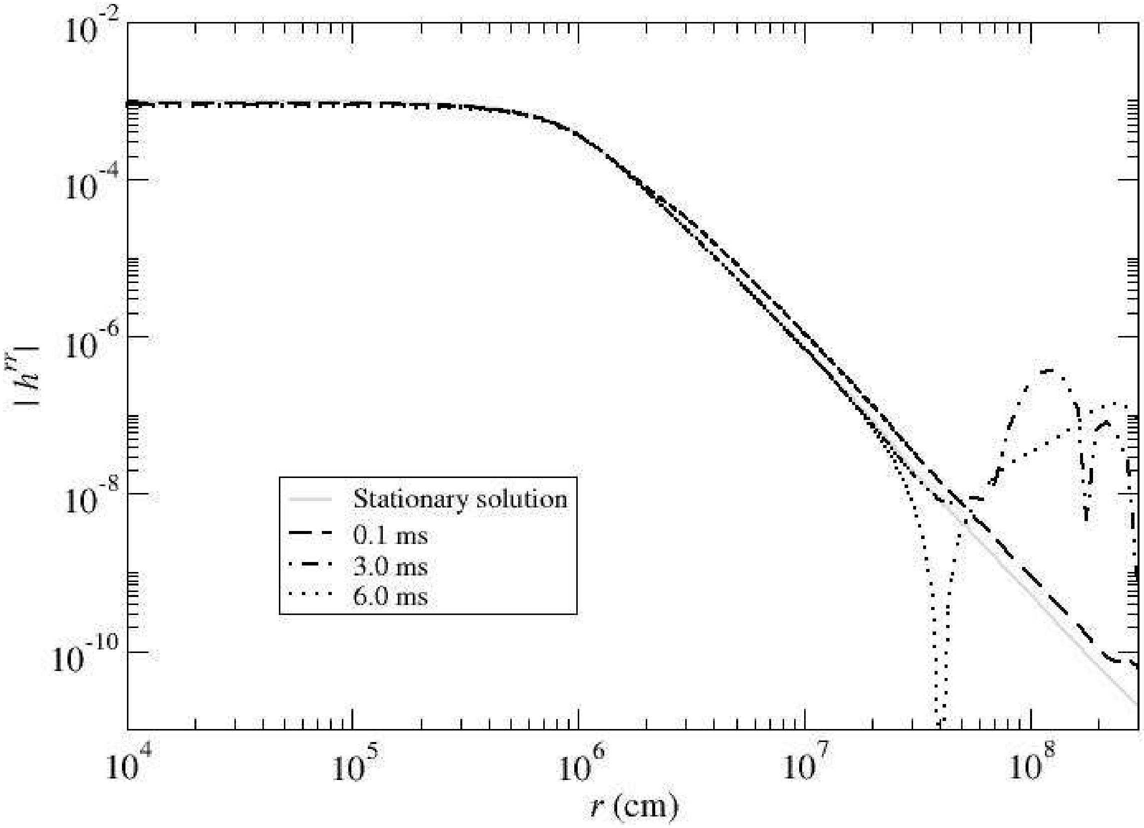
- Baryonic mass: 1.6 $M_{\odot}$

- Coordinate equatorial radius: 12.86 km



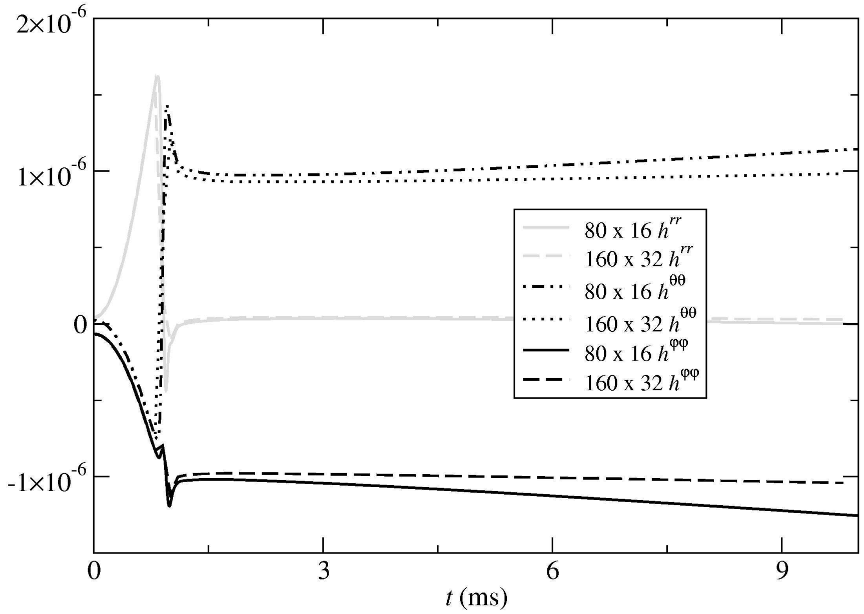
Simulation of a relativistic axisymmetric uniformly rotating neutron star in equilibrium:

- CFC approximation: IMPORTANCE OF ACCURACY OF SOLUTION OF CFC ELLIPTIC EQUATIONS.

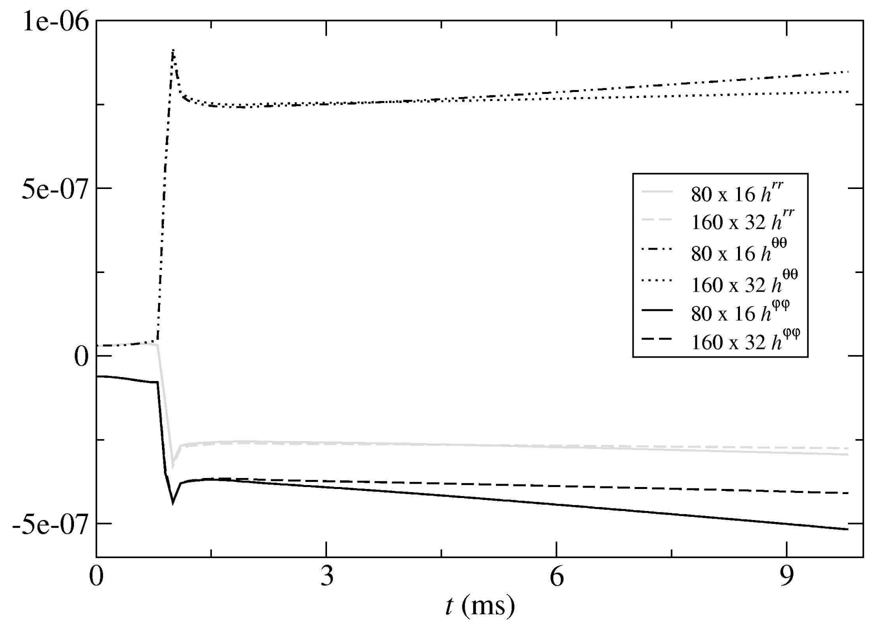


Bad accuracy in resolution of elliptic equation will produce an unphysical off-set.

Improvement of resolution in the CFC case



Improvement of resolution in the FCF case

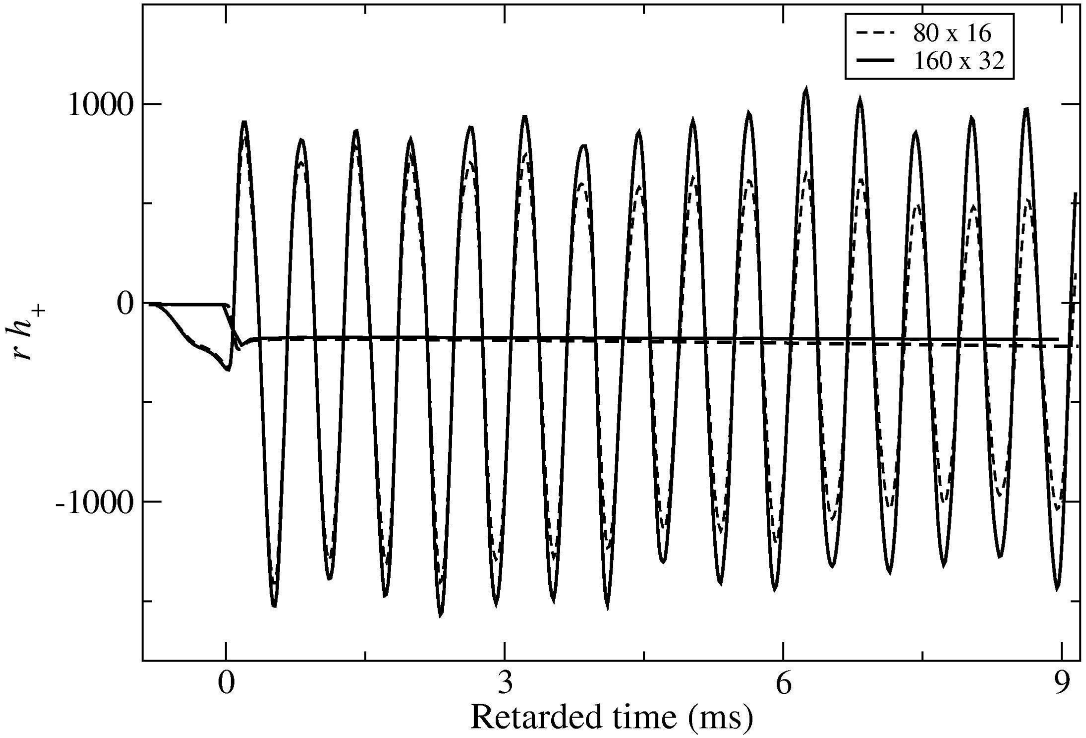


Bad accuracy in the resolution of elliptic equations: main reason of the unphysical off-set, no corrected with resolution.

FCF elliptic equations helps in the aim of reducing the off-set.

Perturbed equilibrium configuration of a rotating neutron star:

(l=2 velocity perturbation)



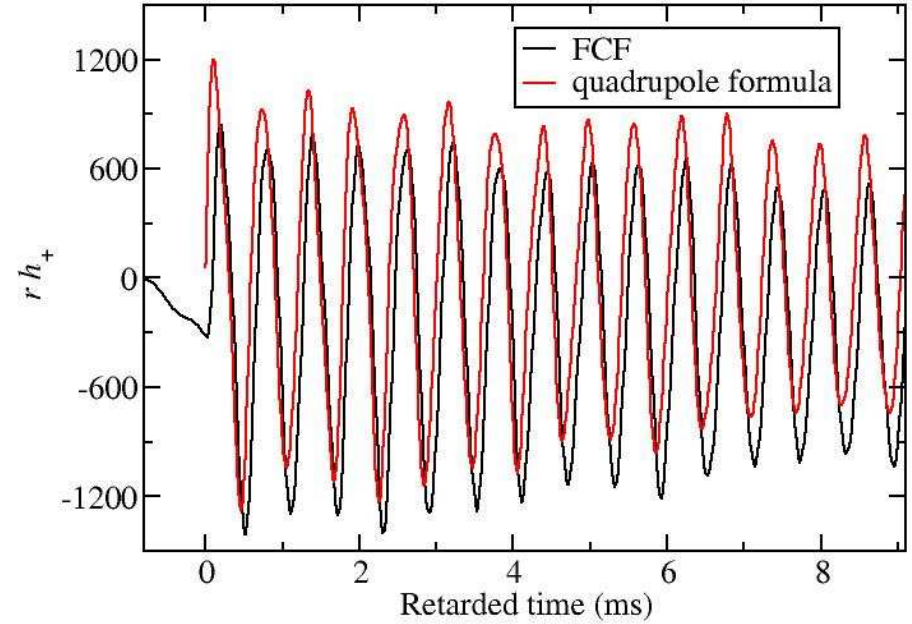
$$r_{MD-PD} = 3 \cdot 10^7 \text{ cm}$$

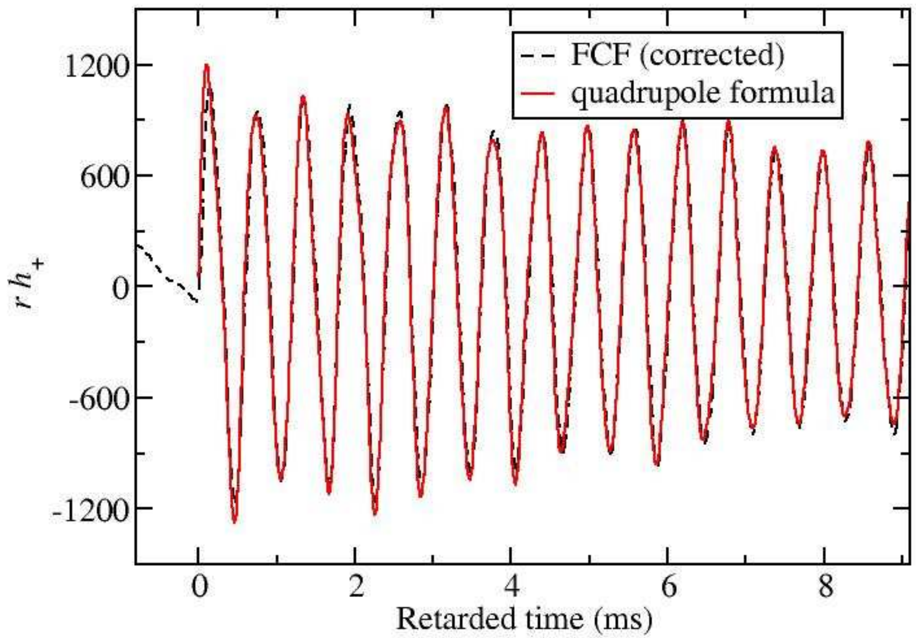
Wave extracted far from the source, but inside the propagation domain.

Speed (velocity of light) and decay (as 1/r) expected of the gravitational wave.

Evolution of the elliptic and hydro equations: important contribution to wave amplitude of each system.

Off-set coincides with the one in the stationary case: accuracy in the elliptic equations and CFC approximation.





Comparison: Weyl scalar and quadrupole formula.

Correction of the off-set with the stationary case.

Correction in phase due to non flat metric.

Conclusions

FCF: Relativistic formulation for Einstein equations.
Extension of CFC approximation: gravitational radiation.

Numerical implementation:

Usual techniques of numerical relativity.

Teukolsky waves: convergence in vacuum flat background.

Stationary neutron star: general background, study of unphysical off-set and coupling of equations, comparison CFC and FCF.

Perturbed neutron star: physical gravitational radiation, comparison with quadrupole formula.

Future work:

Full implementation of FCF in CoCoNuT code: feedback reaction in elliptic equations.

Gravitational radiation of astrophysical scenarios: collapse of rapidly rotating stellar cores, evolution of isolated NS…