Conserved Charges of Quadratic Curvature Gravity Theories in Arbitrary Backgrounds

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- Assume a generic quadratic gravity theory coupled to a matter source τ_{ab} with a coupling κ [6]

$$\Phi_{ab}(g, R, \nabla R, R^2, \cdots) = \kappa\tau_{ab}. \tag{1}$$

- Decompose the metric as $g_{ab} = \bar{g}_{ab} + h_{ab}$, where we assume $\bar{g}_{ab}$ solves the field equations. Then the field equations can be written as

$$\Phi_{ab} = \bar{\Phi}_{ab} + (\Phi_{ab})_L + \mathcal{O}(h^2) + \cdots, \tag{2}$$

- All higher order terms and τ_{ab} are included in total energy momentum tensor (matter+field) T_{ab} then (2) reads

$$(\Phi_{ab})_L = T_{ab}. \tag{3}$$

Reformulation of ADT charge

- Contracting with the background Killing vector $\bar{\xi}_b$ one can define a surface charge

$$\text{if} \quad \bar{\nabla}_b(T^{ab}\bar{\xi}_a) = 0 \quad \text{conserved.}$$

$$Q^b(\bar{\xi}) = \int_\Sigma \sqrt{-g}T^{ab}\bar{\xi}_a \; dx^{n-1} = \int_\Sigma \bar{\nabla}_a \mathscr{F}^{ab} \; dx^{n-1} = \int_{\partial\Sigma} \mathscr{F}^{ab} \; dS_a,$$

where $\mathscr{F}^{ab}$ is a 2-form.

Calculation details

- We will deal with the action

$$I = \int d^D x \sqrt{-g} \left\{ \frac{1}{\kappa}(R + 2\Lambda_0) + \alpha R^2 + \beta R_{ab} R^{ab} \right\}. \quad (4)$$

  Which leads to equation of motion

$$\begin{aligned} T_{ab} =& \frac{1}{\kappa}\left(R_{ab} - \frac{1}{2} g_{ab} R - \Lambda_0 g_{ab}\right) + 2\alpha R \left(R_{ab} - \frac{1}{4} g_{ab} R\right) \\ &+ (2\alpha + \beta)\left(g_{ab}\Box - \nabla_a \nabla_b\right) R \\ &+ \beta \Box \left(R_{ab} - \frac{1}{2} g_{ab} R\right) + 2\beta \left(R_{acbd} - \frac{1}{4} g_{ab} R_{cd}\right) R^{cd}. \end{aligned} \quad (5)$$

- Notice that when linearized, this equations will have fourth order derivatives hitting on h_{ab}.

Calculation details

- For the Einstein-Hilbert term one has charge+extra terms, as

$$\bar{\xi}_b(\mathscr{G}^{ab})_L = \bar{\nabla}_b\left(\sum_{i=1}^{5} Q_{\mathrm{i}}^{ab}\right) - \underbrace{\bar{\mathscr{G}}^{ab}h_{bc}\bar{\xi}^c + \frac{1}{2}\bar{\mathscr{G}}_{cd}\bar{\xi}^a h^{cd} - \frac{1}{2}h\bar{\xi}_b\bar{\mathscr{G}}^{ab}}_{Extra}. \tag{6}$$

- So whatever one have as an extra term other than charges, those must be in the form of *Extra*. For example

$$\bar{\xi}_b\alpha(A^{ab})_L = \alpha\bar{\nabla}_b\left(\sum_{i=1}^{n} F_{\mathrm{i}}^{ab}\right) - \alpha\bar{A}^{ab}h_{bc}\bar{\xi}^c + \alpha\frac{1}{2}\bar{A}_{cd}\bar{\xi}^a h^{cd} - \alpha\frac{1}{2}h\bar{\xi}_b\bar{A}^{ab} \tag{7}$$

where

$$\alpha\bar{A}_{ab} \equiv \alpha\left[2\bar{R}\bar{R}_{ab} - \frac{1}{2}\bar{g}_{ab}\bar{R}^2 + 2\bar{g}_{ab}\bar{\Box}\bar{R} - 2\bar{\nabla}_a\bar{\nabla}_b\bar{R}\right],$$

AdS limit

- After cumbersome calculations and finding charges, first thing to check is the already known AdS limit [6]

$$Q_{\alpha}^{ab} = 2\bar{R} Q_E^{ab} + 2(\bar{\nabla}_c \bar{R}) \bar{\xi}^{[b} h^{a]c} + 4\bar{\xi}^{[a} \bar{\nabla}^{b]} R_L + 2 R_L \bar{\nabla}^{[a} \bar{\xi}^{b]} \tag{8}$$

$$Q_{_{AdS}}^{ab} = constant \times Q_E^{ab} + 4\bar{\xi}^{[a} \bar{\nabla}^{b]} R_L + 2 R_L \bar{\nabla}^{[a} \bar{\xi}^{b]} \tag{9}$$

- As easily seen third term in general background vanishes in AdS and constant depends on dimension. β terms are too cumbersome.
- Since we didn't specified deviation h_{ab} explicitly, charge must be invariant under a transformation

$$h_{ab} \rightarrow h_{ab} + 2\bar{\nabla}_{(a}\zeta_{b)} \tag{10}$$

generated by a vector ζ_b.

Applications in NMG

- The “cosmological” New Massive Gravity [10] is given by

$$S = \frac{1}{16\pi G}\int d^3x\,\sqrt{-g}\left[R - 2\lambda - \frac{1}{m^2}\left(R_{ab}R^{ab} - \frac{3}{8}R^2\right)\right], \tag{11}$$

  by identifying constants with ours $(\kappa,\,\alpha,\,\beta)$ one can compute the charge easily.
- BTZ blackhole [9] can be written as

$$ds^2 \;=\; N^2 dr^2 - N^{-2} dt^2 + r^2\left(d\phi + N_\phi dt\right)^2 \tag{12}$$

where $\quad N(r) \;=\; \left(-8GM + \frac{r^2}{\ell^2} + \frac{16G^2J^2}{r^2}\right)^{\frac{1}{2}}\;, \quad N_\phi \;=\; -\frac{4GJ}{r^2}\;.$ (13)

Applications in NMG

- The limit $M, J \to 0$ gives us the background and the difference with the whole metric leads to the deviation. Inserting those in our charge definition leads to

$$M_{\rm BTZ} = M,\, J_{\rm BTZ} = J. \tag{14}$$

  Whereas in [5] they found vanishing mass and angular momentum using boundary stress tensor method.
- Another blackhole solution to NMG is given by Clement [4] as

$$ds^2 = -\frac{4\rho^2}{l^2 f(\rho)} d\bar{t}^2 + f(\rho)\left[q\bar{\varphi} - \frac{ql \ln|\rho/\rho_0|}{f(\rho)} d\bar{t}\right]^2 + \frac{l^2 d\rho^2}{4\rho^2} \tag{15}$$

$$(f(\rho) = 2\rho + ql^2 \ln|\rho/\rho_0|). \tag{16}$$

- This blackhole has AdS background therefore can be calculated by Deser-Tekin definition. Our results agree with the one in [4].

Applications in NMG

- Finally the Lifshitz blackhole in 3 dimensions [1]

$$ds^2 = -\frac{r^6}{\ell^6}\left(1 - \frac{M\ell^2}{r^2}\right)dt^2 + \frac{dr^2}{\left(\frac{r^2}{\ell^2} - M\right)} + \frac{r^2}{\ell^2}dx^2. \tag{17}$$

- This blackhole has Lifshitz background, which is not constant curvature. Therefore the charge definition given by Deser-Tekin will not work here.
- With our extension charge of Lifshitz reads $M_{Lifshitz} = \frac{7M^2}{8G}$.
- With boundary stress tensor method [5] it is found $M_{Lifshitz} \;=\; \frac{M^2}{4G}$.
- One needs the Gauss-Bonnet term to calculate charges of higher dimensional Lifshitz blackholes [2].

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