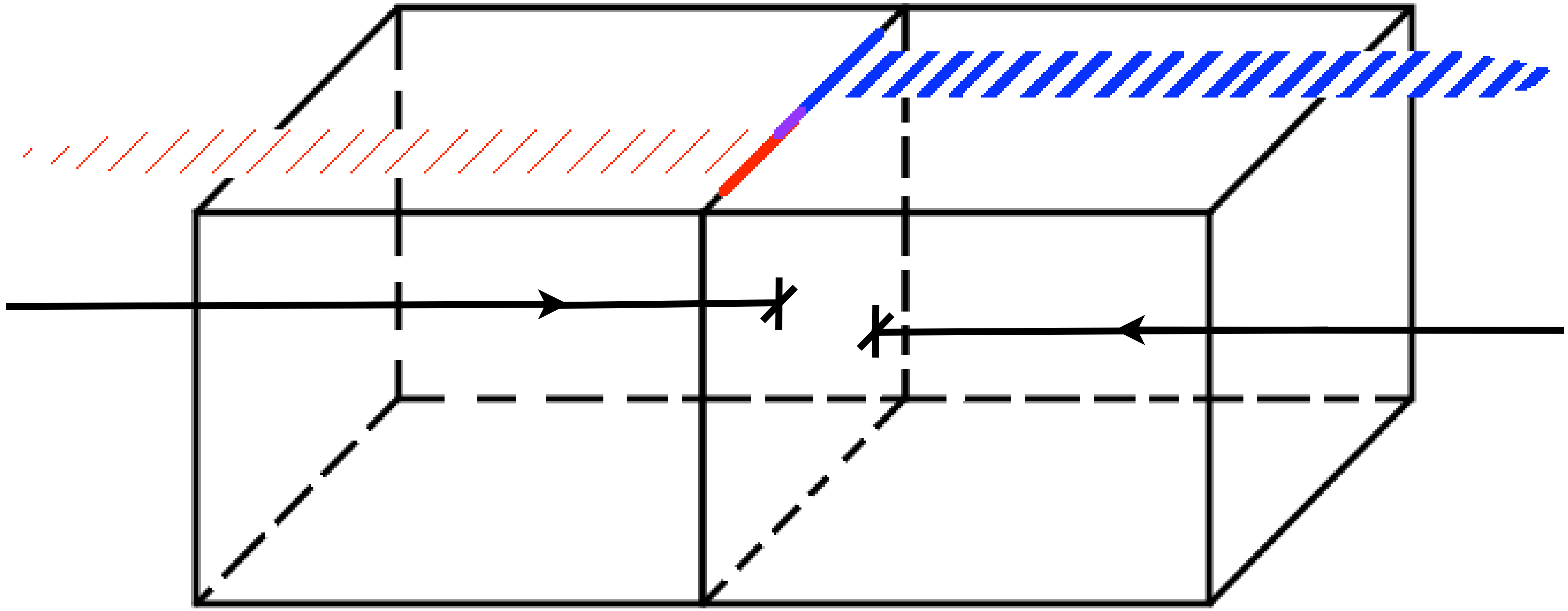
COLLISION OF SHOCK-WAVES IN AdS AND HOLOGRAPHY



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Based on: A. D-V. and M.A.Vazquez-Mozo, **arXiv:1004.2609 , JHEP 1007:021,2010**.

1.MOTIVATION

- Goal: To study thermalization after collision of energy-lumps in strongly coupled theories (QGP production in LHC and RHIC)

- We can not apply QFT $\longrightarrow$ Gauge/Gravity duality approach.

  * Einstein gravity with negative Λ $\longleftrightarrow$ fluid dynamics of strongly coupled SYM field theory.

  AdS black-hole $\longleftrightarrow$ SYM thermalized plasma

- What is the dual for collisions between energy-lumps in SYM??

  * AdS shock-waves are described holographically by plane-contracted energy distributions.

  * A good description for ultrarelativistic energy-lumps in SYM.

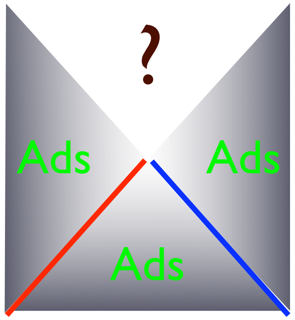
- Gravitational dual: Formation of an event horizont after collision of shock-waves in AdS.

Trapped surface production $\longleftrightarrow$ SYM plasma thermalization

2. GRAVITATIONAL SETUP

- Two Aichelburg-Sexl like colliding shock-waves (traveling along x^{D-2} coordinate):

$$(u = t - x^{D-2}, v = t + x^{D-2}, \vec{x} = (x^1, x^2, \dots, x^{D-3}))$$



$$\Rightarrow \quad ds^2 = \underline{ds^2_{AdS_D}} + \underline{\frac{L}{z}\phi_+(z,\vec{x})\delta(u)du^2} + \underline{\frac{L}{z}\phi_-(z,\vec{x})\delta(v)dv^2}$$

* Einstein equations:

$$T_{uu} = E\delta(u)\delta(z - z_+)\delta^{(D-3)}(\vec{x} - \vec{x}_+)$$
$$T_{vv} = E\delta(v)\delta(z - z_-)\delta^{(D-3)}(\vec{x} - \vec{x}_-)$$

$$\Rightarrow \left[\Box_{\mathbb{H}_{D-2}} - \frac{D-2}{L^2}\right]\phi_\pm(z,\vec{x}) = -16\pi G_E\left(\frac{z}{L}\right)^{D-1}\delta(z - z_\pm)\delta^{(D-3)}(\vec{x} - \vec{x}_\pm)$$

$$\phi_\pm(z,\vec{x}) = \frac{8\pi}{2^{D-3}(D-1)\Omega_{D-3}}\frac{G_D E}{L^{D-3}}\frac{z_\pm}{q_\pm}\,{}_2F_1(D-2, D/2; D; -1/q_\pm)$$

Hyperbolic space $\quad ds^2 = \frac{L^2}{z^2}\left(dz^2 + d\vec{x}^2\right)$

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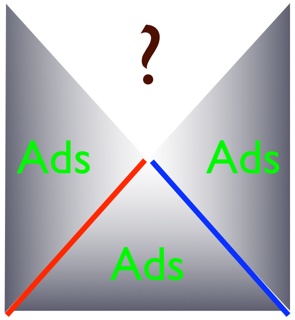

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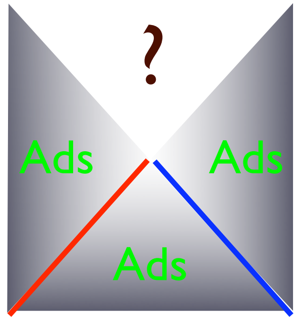

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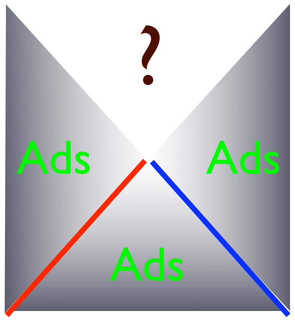
$$q_\pm = \frac{1}{4zz_\pm}\left[(z - z_\pm)^2 + (\vec{x} - \vec{x}_\pm)^2\right]$$

3. FINDING TRAPPED SURFACE

- Trapped surface in the union of the two shock-waves: Penrose approach



* Find trapped surfaces (in particular, MOTSs) over the suport of the shock-waves.

$$S = S_+ \cup \ S_-, \quad C = S_+ \cap \ S_-$$

$$S_+ \left| \begin{array}{ccc} u & \leq & 0 \\ v & = & -\psi_+(z,\vec{x}) \end{array} \right. \qquad S_- \left| \begin{array}{ccc} u & = & -\psi_-(z,\vec{x}) \\ v & \leq & 0 \end{array} \right.$$

$$\boxed{\begin{array}{l} \left[\nabla^2_{\mathbb{H}_{D-2}} - \dfrac{D-2}{L^2}\right](\psi_\pm - \phi_\pm) = 0 \quad \text{inside } C \\ \psi_+(z,\vec{x}) \mid_C = \psi_-(z,\vec{x}) \mid_C = 0 \\ g^{ab}\partial_a\psi_+\partial_b\psi_- \mid_C = 4, \end{array}}$$

$$ds^2_{\mathbb{H}_{D-2}} = g_{ab}dx^a dx^b, \quad a,b \in \{z,\vec{x}\}$$

4. COLLISIONS WITH IMPACT PARAMETER:

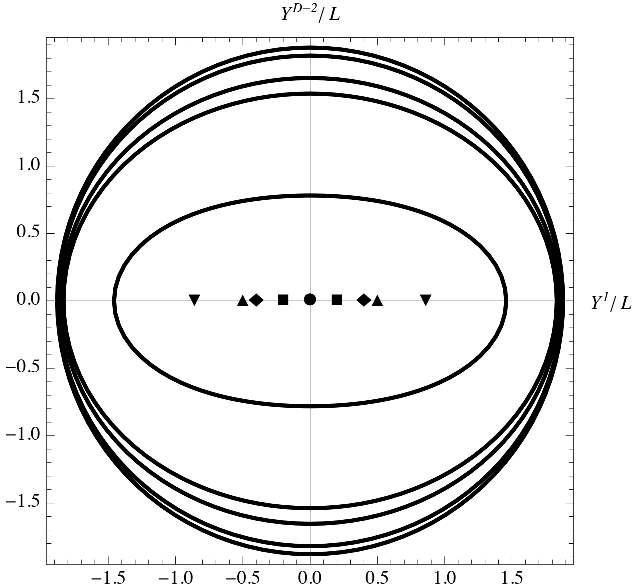
- Two colliding shock-waves with $\Delta x = x_+ - x_-,\ z_+ = z_- = L$

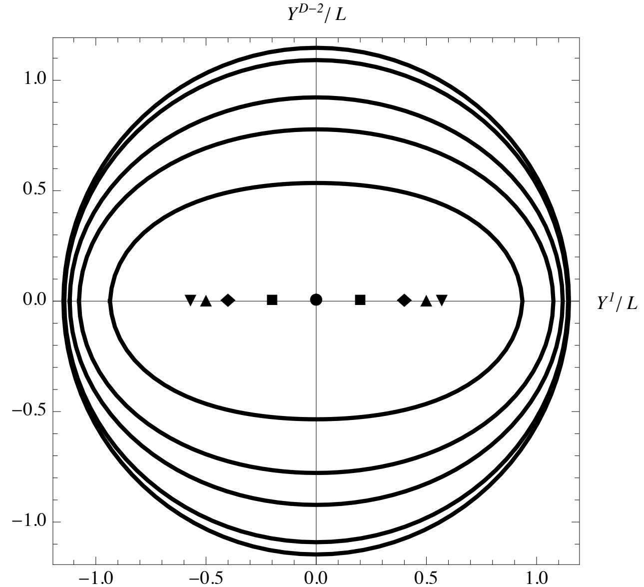
  * Δx impact parameter in field coordinates $\Longrightarrow$ impact parameter in the boundary theory.

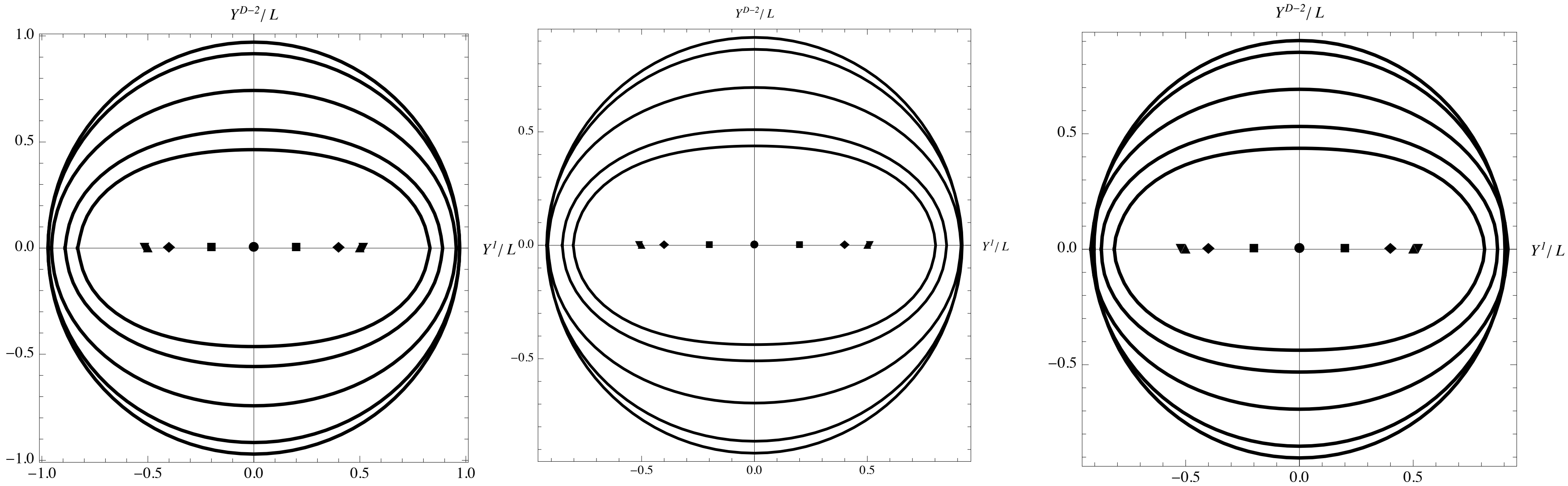
  * It is expected that for large enough impact parameter a trapped surface/ thermalization is not going to appear.

- Solution requires numerical methods in a computer: We use a finite difference method (50 x 100 grid) combined with a trial-and-correction loop.

- Solutions for the shape of C in D=4,5,6,7 and 8:



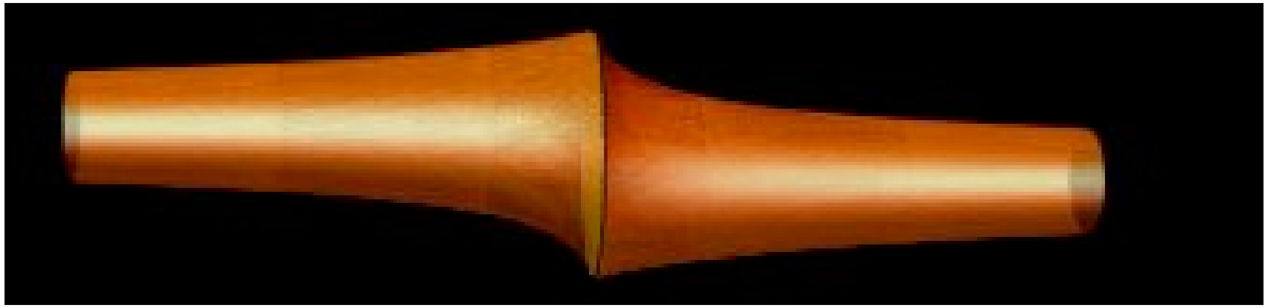




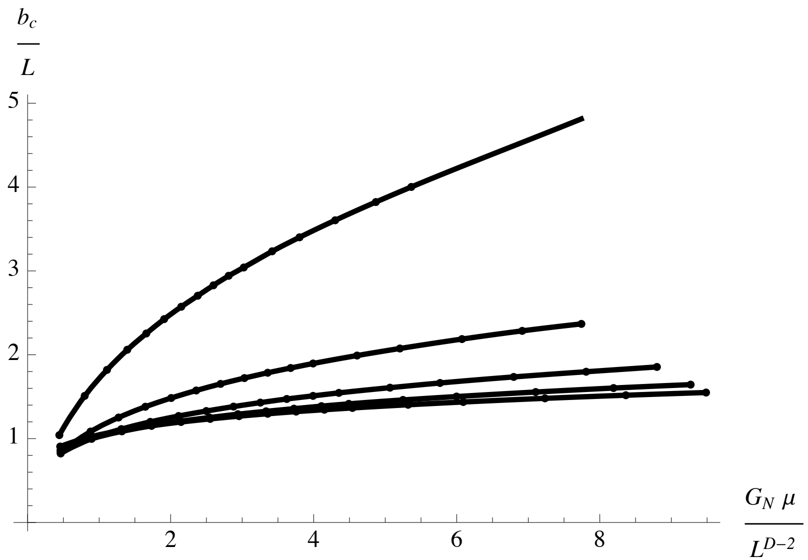
* Critical behaviour observed in all dimensions: For each energy, there exists a critical impact parameter such that not trapped surface is formed above such impact parameter $\Longrightarrow$ NO THERMALIZATION FOR LARGE ENOUGH IMPACT PARAMETER.

* All the critical trapped surfaces have a minimun size not equal to zero.

- Artistic representation of trapped surface in D=5 from numerical data



- Scaling of the critical impact parameter with the energy of the shock-waves:



$$\Rightarrow \qquad \frac{\Delta x_c}{L} \sim \left(\frac{G_D E}{L^{D-3}}\right)^{\frac{1}{D-2}}$$

* It suggests independence of critical impact parameter from energy at very high dimensions.

5. COLLISIONS BETWEEN NON-EQUALLY SIZED ENERGY-LUMPS:

- Holographic energy-momentum of the lump: Computed from asymptotic behaviour of the metric.

$$T_{uu} = \frac{2Ez_0^4}{\pi(\vec{x}^2 + z_0^2)}$$

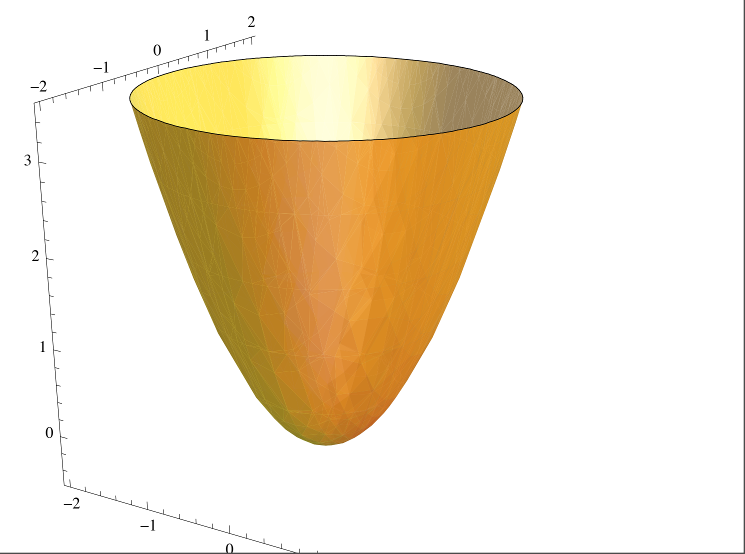
    * Energy-weighted RMS size: $\frac{\int d^{D-2}x < T_{vv} > \vec{x}^2}{\int d^{D-2}x < T_{vv} >} = z_0^2$

$\Rightarrow$ depth of shock-waves in AdS gives rise to the size of the energy-lumps in the boundary.

- Using colliding shock-waves with $\Delta z \neq 0, \ \Delta x = 0$ we have an holographically description to head-on collisions between non-equally sized energy-lumps.

- We have not to repeat all the numerical work thanks to symmetries of $\mathbb{H}_{D-2}$: SO(D-2)

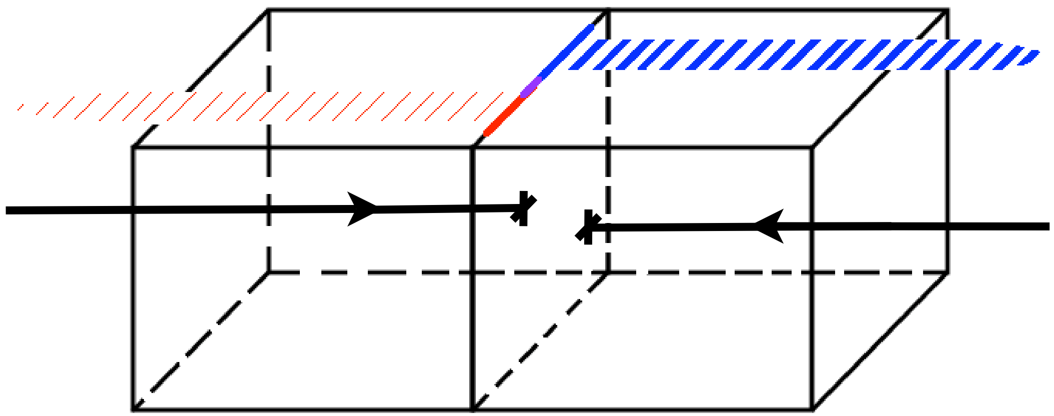
    * SO(2) rotation in the plane $Y^1 Y^{D-2}$ can pass from $\Delta x = b, \ \Delta z = 0$ to $\Delta x = 0, \ \Delta z \neq 0$



* Under such transformations the shock-waves energies change†:

$$\left[\nabla^2_{\mathbb{H}_{D-2}} - \frac{D-2}{L^2}\right]\phi_\pm = -16\pi G_D E_\pm \left(\frac{z}{L}\right)^{D-1} \delta(z - z_\pm)\delta^{(D-3)}(\vec{x} - \vec{x}_\pm)$$

$$E_\pm = E' \frac{\sqrt{1+\beta^2} \pm \beta\cos\theta}{\sqrt{1+\beta^2} \pm \beta}$$



* Results: Applying

1) The coordinate rescaling

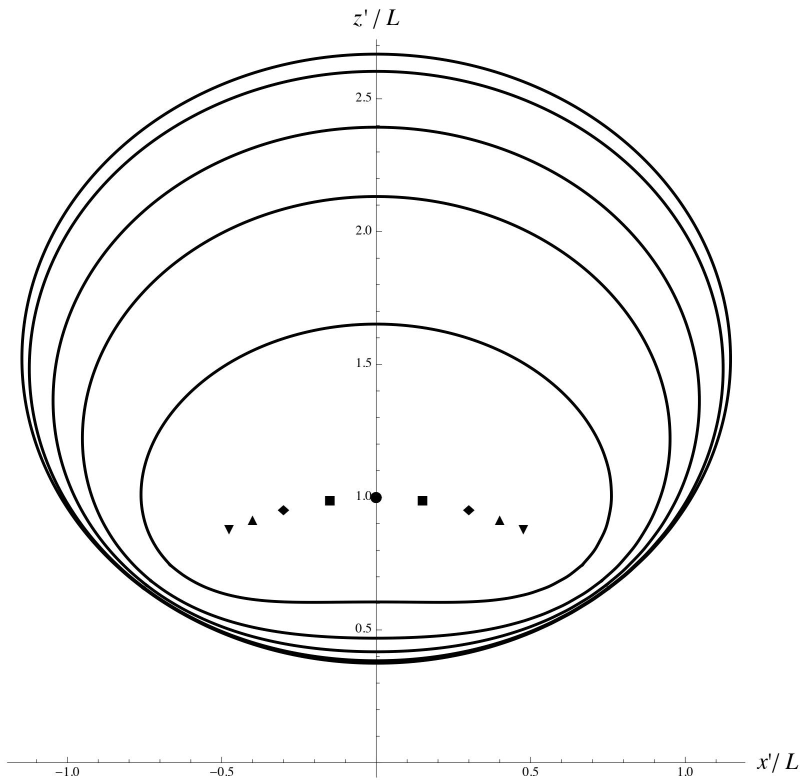
2) The rotation

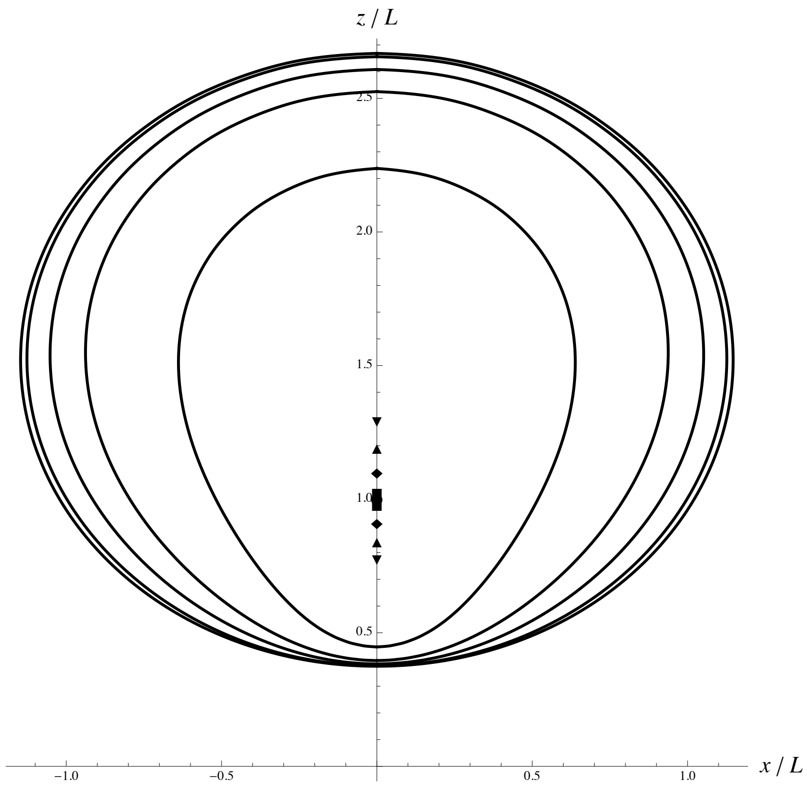
we can relate collisions between equally sized energy-lumps with non-zero impact parameter with head-on collisions between non-equally sized energy-lumps

$\Longrightarrow$ thermalization dependence in difference of sizes, measured by Δz

† $\Delta x = \dfrac{2L\beta}{\sqrt{1+\beta}}, \quad z_+ = z_- = \dfrac{L\beta}{\sqrt{1+\beta}} \Rightarrow x_\pm = \dfrac{2L\beta\cos a}{\sqrt{1+\beta} \pm \beta\sin a} \quad z_\pm = \dfrac{L}{\sqrt{1+\beta} \pm \beta\sin a}$

- Shape of C before and after the rotation and the resacling for D =5 and $\frac{G_D E}{L^{D-3}} = 1$





* As was expected, a critical behaviour also apears: If the difference of sized between the two energy-lumps is enough, there is no thermalization.

6.CONCLUDING REMARKS AND OUTLOOK:

- Gravity/gauge duality is a powerful tool to study the formation of strongly coupled SYM plasma in energy-lumps collisions.

- For collisions with non-zero impact parameter, there is a critical behaviour: For large enough impact parameter there is no thermalization. Each energy-lump does not see the other.

- Such behaviour happens in each dimension, and the critical impact parameter have a scaling $1/(D-2)$ with the energy. It suggests that for very high dimension the critical impact parameter is independent from the energy.

- For head-on collision between non equally-sized energy-lumps there is also a critical behaviour: If the difference between the size of the two energy-lumps is enough, there is no thermalization. Physically this could be interpreted as that the smallest energy-lump does not have enough degrees of freedom to thermalize with the biggest one.

- Thermalization time?