w-singularities in cosmological models

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Spanish Relativity Meeting 2010

Sketch

Motivation

- Astronomical observations of luminosity distances derived from Type Ia supernovae, CMB spectrum and global matter distribution provide evidence of cosmic speed up of the Universe.
- One approach to deal with this riddle is to consider modifications of general relativity.
- Alternatively, cosmic acceleration might be due to an exotic fluid filling the Universe, known as dark energy.
- These have given rise to a collection of new cosmological evolutions, future singularites being the most perplexing ones (“big rip”, “sudden singularities”. . .).

Classification of singularities

- Big Bang / Crunch: zero a, divergent H, density and pressure. Strong.
- Type I: “Big Rip”: divergent a. Strong.
- Type II: “Sudden”: finite a, H, density, divergent $\dot{H}$ and pressure. Weak
- Type III: “Big Freeze”: finite a, divergent H, density and pressure. Weak/Strong
- Type IV: “Big Brake”: finite a, H, $\dot{H}$, density and pressure but divergent higher derivatives. Weak.

w-singularities

Dabrowski & Denkiewicz introduced a cosmological model,

$$
\begin{aligned}
a(t) \;=\; & \frac{a_s}{1-\frac{3\gamma}{2}\left(\frac{n-1}{n-\frac{2}{3\gamma}}\right)^{n-1}}+\frac{1-\frac{2}{3\gamma}}{n-\frac{2}{3\gamma}}\frac{n a_s}{1-\frac{2}{3\gamma}\left(\frac{n-\frac{2}{3\gamma}}{n-1}\right)^{n-1}}\left(\frac{t}{t_s}\right)^{\frac{2}{3\gamma}} \\
+ \; & \frac{a_s}{\frac{3\gamma}{2}\left(\frac{n-1}{n-\frac{2}{3\gamma}}\right)^{n-1}-1}\left(1-\frac{1-\frac{2}{3\gamma}}{n-\frac{2}{3\gamma}}\frac{t}{t_s}\right)^{n}, \quad \gamma>0, \quad n\neq 1.
\end{aligned}
$$

- The scale factor $a(t_s)=a_s$ is finite.
- Density and pressure vanish at t_s, $\rho(t_s)=0=p(t_s)$.
- If n is natural, the derivatives of a and the Hubble parameter are regular.
- Just the barotropic index w ($p=w\rho$) is singular at $t=t_s$.

Barotropic index expansions

The barotropic index w may be written in terms of the scale factor,

$$w = -\frac{1}{3} - \frac{2}{3}\frac{a\ddot{a}}{\dot{a}^2}.$$

Assuming that the scale factor admits a generalized power expansion with real exponents,

$$a(t) = c_0(t_s - t)^{\eta_0} + c_1(t_s - t)^{\eta_1} + \cdots, \quad \eta_0 < \eta_1 < \cdots, \quad c_0 > 0,$$

η_0	η_1	η_2	w_s
$\neq 0$	(η_0, ∞)	(η_1, ∞)	Finite
0	$(0, 1)$	(η_1, ∞)	Infinite
	1	$(1, 2)$	Infinite
	1	$[2, \infty)$	Finite
	$(1, \infty)$	$[\eta_1, \infty)$	Infinite

Density and pressure

Besides a diverging barotropic index, we need vanishing density and pressure.
For $\eta_0 = 0$, $\eta_1 \neq 1$,

$$\begin{aligned} \rho &= \frac{3c_1^2\eta_1^2}{c_0^2}(t_s - t)^{2(\eta_1 - 1)} + \cdots\,, \\ p &= -\frac{2c_1\eta_1(\eta_1 - 1)}{c_0}(t_s - t)^{\eta_1 - 2} + \cdots\,, \end{aligned}$$

$\eta_1 > 2$ is required.

Results

- A FLRW cosmological model has a singular barotropic index w with vanishing pressure and density at t_s if an only if the generalized power expansion of $a(t)$ is of the form

$$a(t) = c_0 + c_1(t_s - t)^{\eta_1} + \cdots ,$$

with $\eta_1 > 2$.

Results

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- A FLRW cosmological model has a w-singularity at a finite time t_s if an only if the scale factor $a(t)$ admits a Taylor series at t_s with vanishing linear and quadratic terms,

$$a(t) = c_0 + O((t - t_s)^3).$$

Scale factor vs. singularities

η_0	η_1	η_2	**Tipler**	**Królak**	**N.O.T.**
$(-s, 0)$	(η_0, s)	(η_1, s)	Strong	Strong	I
0	$(0, 1)$	(η_1, s)	Weak	Strong	III
	$(1, 2)$	(η_1, s)	Weak	Weak	II
	1	$(1, 2)$	Weak	Weak	II
		$[2, s)$	Weak	Weak	IV
	$[2, s)$	(η_1, s)	**Weak**	**Weak**	**IV**
$(0, s)$	(η_0, s)	(η_1, s)	Strong	Strong	Crunch

Hence w-singularities are weak singularities.

Further reading

- L. Fernández-Jambrina,
  On *w*-cosmological singularities
  (preprint)
- L. Fernández-Jambrina, R. Lazkoz,
  Singular fate of the universe in modified theories of gravity
  Phys. Lett. B **670** 254 (2009) [arXiv:0805.2284]
- L. Fernández-Jambrina, R. Lazkoz,
  Classification of cosmological milestones
  Phys. Rev. D **74** 064030 (2006) [arXiv:gr-qc/0607073]
- L. Fernández-Jambrina, R. Lazkoz,
  Geodesic behaviour of sudden future singularities
  Phys. Rev. D **70** 121503 (2004) [arXiv:gr-qc/0410124]

Thank you all!