The regular cosmic string in Born-Infeld gravity

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Contents

WHY A THEORY OF MODIFIED GRAVITY?

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- To search for alternative geometrical descriptions of the present universe, without resorting to dark matter and dark energy
- To smooth singularities (BH's, Big-Bang)

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SOME WAYS TO MODIFIED GRAVITY:

- Lovelock's Lagrangians: polynomials in Riemann curvature that lead to second order equations for the metric. However, they differ from General Relativity only if the dimension is bigger than 4.

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- $f(R)$ theories: for instance, the Lagrangian

$$L[g] \propto R + \alpha R^2$$

departs from General Relativity if $\alpha R \gtrsim 1$. This could work to modify the dynamics at the big-bang scale. However, it would be unable to modify a $(R = 0)$ vacuum solution such as a BH. Worst yet, the dynamical equations will result in 4th order equations.

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- "$f(T)$" theories or modified teleparallelism: they lead to second order equations. They work whatever the dimension is. They could modify even vacuum solutions.

Contents

Elements of teleparallelism

Dynamical variables

The field of frames (tetrads or vierbeins): $\{\mathbf{e}_a(\mathbf{x})\}, \quad a = 0, 1, 2, 3$

and their respective co-frames: $\{\mathbf{e}^a(\mathbf{x})\}, \quad e_a^\mu \, e_\mu^b \doteq \delta_a^b$

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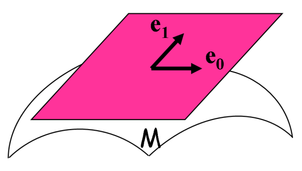
Relation with the metric

The frame is orthonormal:

$$\eta_{ab} \;=\; g_{\mu\nu}\, e^\mu_a \, e^\nu_b$$

Then,

$$g_{\mu\nu} = \eta_{ab}\; e^a_\mu \, e^b_\nu \quad \Rightarrow \quad \sqrt{-g} = \det[e^a_\mu] \;\doteq\; e$$



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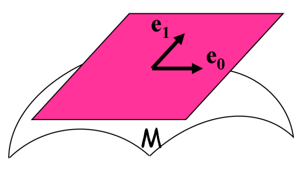

Dynamics for the tetrad induces dynamics for the metric.

Elements of Teleparallelism

The gravitational field is encoded in the *torsion* instead of the *curvature*.

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Weitzenböck connection

$$\overset{\mathbf{w}}{\Gamma}{}^{\mu}_{\ \rho\nu} \doteq e^{\mu}_{a}\, \partial_{\nu} e^{a}_{\rho} \quad \Rightarrow \quad \overset{\mathbf{w}}{Riemann} \equiv 0$$

The torsion is $$T^{\mu}{}_{\nu\rho} \doteq \overset{\mathbf{w}}{\Gamma}{}^{\mu}_{\ \rho\nu} - \overset{\mathbf{w}}{\Gamma}{}^{\mu}_{\ \nu\rho} = e^{\mu}_{a}\left(\partial_{\nu} e^{a}_{\rho} - \partial_{\rho} e^{a}_{\nu}\right)$$

Then, $e^{a}_{\mu}\, T^{\mu}{}_{\nu\rho}$ are the components of four exact 2-forms: $\mathbf{T}^{a} \doteq d\mathbf{e}^{a}$

▸ Appendix

Teleparallel equivalent of General Relativity

Lagrangian density

The Lagrangian density is quadratic in the torsion:

$$\mathcal{L}_{\mathbf{T}}[\mathbf{e}^a] = \frac{1}{16\pi G}\; e\; S_\rho{}^{\mu\nu}\, T^\rho{}_{\mu\nu} \doteq \frac{1}{16\pi G}\; e\; \mathbb{S}\cdot\mathbb{T}$$

where

$$S_\rho{}^{\mu\nu} \doteq -\frac{1}{4}\,(T^{\mu\nu}{}_\rho - T^{\nu\mu}{}_\rho - T_\rho{}^{\mu\nu}) + \frac{1}{2}\,(\delta^\mu_\rho\, T^{\theta\nu}{}_\theta - \delta^\nu_\rho\, T^{\theta\mu}{}_\theta)$$

See, for instance, Hayashi and Shirafuji, PRD 19 (1979), 3524.

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Equivalence between $\mathcal{L}_{\mathbf{T}}$ and $\mathcal{L}_{\mathbf{GR}}$

$$\mathcal{L}_{\mathbf{GR}}[\mathbf{e}^a] = \mathcal{L}_{\mathbf{T}}[\mathbf{e}^a] + \text{ divergence}$$

Example: flat FRW minisuperspace, $\quad e^a_\mu = \mathrm{diag}[N(t), a(t), a(t), a(t)],$

Then

$$\mathcal{L}_{\mathbf{T}}[N, a] \propto -N^{-1} \, a \, \dot{a}^2, \qquad \mathcal{L}_{\mathbf{GR}}[N, a] \propto -N^{-1} \, a \, \dot{a}^2 + \tfrac{d}{dt}(N^{-1} \, a^2 \, \dot{a})$$

Contents

$f(T)$ theories

A $f(T)$ theory is a deformation of the teleparallel equivalent of General Relativity:

$f(T)$ theories

$$\mathcal{L}_{\mathbf{T}} = \frac{e}{16\pi G}\,\mathbb{S}\cdot\mathbb{T} \longrightarrow \mathcal{L}_{\mathbf{T}} = \frac{e}{16\pi G}\,f(\mathbb{S}\cdot\mathbb{T})$$

- R.Ferraro and F. Fiorini, PRD 75 (2007) 084031.
- R.Ferraro and F. Fiorini, PRD 78 (2008) 124019.
- G.R. Bengochea and R.Ferraro, PRD 79 (2009) 124019.
- E.V. Linder, PRD 81 (2010) 127301.

R.F. and F.F., PRD 75 (2007) 084031, *Inflation without inflaton*

Deformation à la Born-Infeld

$$f(\mathbb{S}\cdot\mathbb{T}) \;=\; \lambda\left[\sqrt{1+\frac{2\,\mathbb{S}\cdot\mathbb{T}}{\lambda}}-1\right]$$

The scale factor $a(t)$ of the flat modified FRW cosmology is governed by the equation

$$\left(1-\frac{12\,H^2}{\lambda}\right)^{-\frac{1}{2}}-1 \;=\; \frac{16\pi G}{\lambda}\rho$$

where $H \equiv \dot{a}/a$ is the Hubble parameter.

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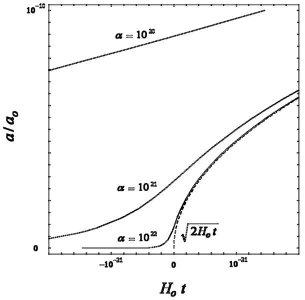
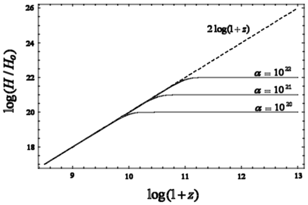
where $H \equiv \dot{a}/a$ is the Hubble parameter.

- The initial singularity is smoothed.
- $H_{max} \doteq \lim_{t\to-\infty} H(t) = \sqrt{\lambda/12}$ for state equations $p = w\,\rho$ with $w > -1$.
- The particle horizon diverges; so the whole universe is causally connected.





$\alpha \equiv H_{max}/H_o$, $w = 1/3$ (radiation)

Contents

R.F. and F.F., PLB 692 (2010) 206, *The taming of the conical singularity in 2+1 D*

Cosmic string in GR

$$ds^2 = d(t + 4J\,\theta)^2 - d\rho^2 - (1 - 4\mu)^2\,\rho^2\,d\theta^2 - dz^2$$

R.F. and F.F., PLB 692 (2010) 206, *The taming of the conical singularity in 2+1 D*

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$$ds^2 = d(t + 4J\,\theta)^2 - d\rho^2 - (1-4\mu)^2\,\rho^2\,d\theta^2 - dz^2$$

- In (2+1) dimensions (z is absent), this metric solves the Einstein equations for

  $T^{00} = \mu\,\delta(x,y)$ and $T^{0i} = (J/2)\,\epsilon^{ij}\,\partial_j\delta(x,y).$

  So the solution is a particle of mass μ and spin J (a "cosmon").

  Deser, Jackiw and 't Hooft, Ann. Phys. 152 (1984), 220.

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- No gravitational field surrounds the cosmon since the metric is manifestly flat.

- The cosmon reveals itself through topological properties:

  i) the deficit angle $8\pi\mu$ (conical singularity),

  ii) the existence of closed timelike curves (CTC) of constant (t, ρ, z):

$$ds^2 = \left(\frac{16J^2}{M^2} - \rho^2\right)\,M^2\,d\theta^2\,, \qquad M \doteq 1 - 4\mu$$

  which means that curves with radio $\rho < \rho_o \doteq 4J/M$ are CTC.

R.F. and F.F., PLB 692 (2010) 206, *The taming of the conical singularity in 2+1 D*

The singular structure of cosmic strings can be prevented in modified gravity.

Born-Infeld gravity: determinantal Lagrangian density

$$\mathcal{L} \; \propto \; -\lambda \left[\sqrt{|g_{\mu\nu} - 2\lambda^{-1} F_{\mu\nu}|} - \sqrt{|g_{\mu\nu}|} \right] \quad \xrightarrow[\lambda\to\infty]{} \quad \sqrt{|g_{\mu\nu}|} \; Tr(F)$$

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In the context of teleparallelism we can use $F_{\mu\nu} = \alpha\, S_{\mu\lambda\rho}\, T_\nu{}^{\lambda\rho} + \beta\, S_{\lambda\mu\rho}\, T^\lambda{}_\nu{}^\rho,$

since $Tr(F) \; = \; (\alpha + \beta)\, \mathbb{S} \cdot \mathbb{T}.$

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since $Tr(F) = (\alpha+\beta)\, \mathbb{S}\cdot\mathbb{T}$.

Thus, choosing $\alpha+\beta=1$, it results

$$\mathcal{L} \propto e \left[\mathbb{S}\cdot\mathbb{T} - \frac{\lambda^{-1}}{2}(\mathbb{S}\cdot\mathbb{T})^2 + \lambda^{-1} F_\mu{}^\nu F_\nu{}^\mu\right] + \mathcal{O}(\lambda^{-2}).$$

which shows that this theory differs from a $f(T)$ theory. This feature is essential for our purpose because the cosmon has $\mathbb{S}\cdot\mathbb{T}=0$.

R.F. and F.F., PLB 692 (2010) 206, *The taming of the conical singularity in 2+1 D*

The modified cosmic string

We propose the cylindrically symmetric tetrad

$$\mathbf{e}^0 = d(t + 4J\theta), \quad \mathbf{e}^1 = Y(\rho)\, d\rho, \quad \mathbf{e}^2 = \rho\, M\, d\theta, \quad \mathbf{e}^3 = dz,$$

So, the respective metric is

$$ds^2 = d(t + 4J\,\theta)^2 - Y^2(\rho) d\rho^2 - \rho^2\, M^2\, d\theta^2 - dz^2.$$

The function $Y(\rho)$ is the sole difference between Born-Infeld determinantal gravity and GR.

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Dynamical equations ($\alpha = 1, \beta = 0$)

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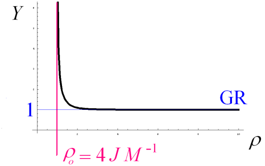


$Y(\rho)$ is defined for $\rho_o \leq \rho < \infty$.

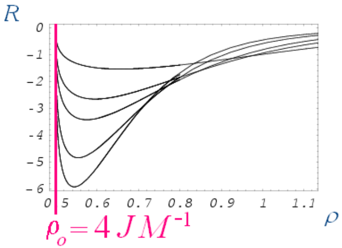
R.F. and F.F., PLB 692 (2010) 206, *The taming of the conical singularity in 2+1 D*

RESULTS

- The modified cosmic string geometry is curved instead of flat:

$$R(\rho) \;=\; \frac{2Y'(\rho)}{\rho\, Y(\rho)^3},$$

$$R^{\mu\nu} R_{\mu\nu} = \frac{1}{2} R^2, \;\; R^{\mu}{}_{\nu\eta\pi} R_{\mu}{}^{\nu\eta\pi} = R^2$$



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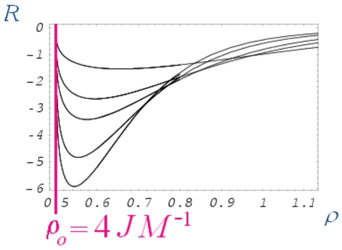

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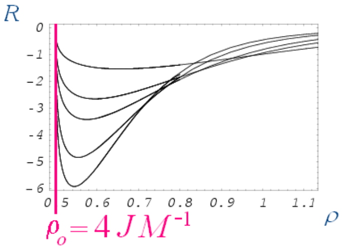

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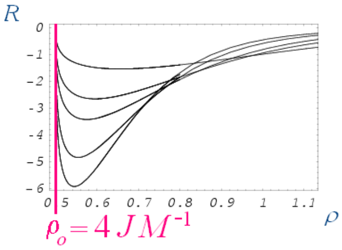

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- The proper time to reach the minimal circle $\rho = \rho_o$ is infinite: the conical singularity has disappeared.

- The closed timelike curves (CTC) have disappeared: since $\rho > \rho_o$ then the curves of constant (t, ρ, z) are always spacelike:

$$ds^2 = \left(\frac{16J^2}{M^2} - \rho^2\right)\, M^2\, d\theta^2 \;<\; 0\;.$$

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Thank you!

Elements of Teleparallelism

Geometrical meaning of Weitzenböck connection

The covariant derivatives of a vector $\mathbf{V} = V^a \, \mathbf{e}_a = V^a \, e_a^\mu \, \partial_\mu = V^\mu \, \partial_\mu$ reduce to

$$\overset{\mathbf{w}}{\nabla}_\nu V^\mu \; = \; \partial_\nu V^\mu + \overset{\mathbf{w}}{\Gamma}{}^{\mu}_{\;\rho\nu} V^\rho = \; e_a^\mu \, \partial_\nu V^a$$

A vector $\mathbf{V}$ is parallel transported along a curve iff its components V^a are constant on the curve.

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Weitzenböck connection is metric compatible: $\overset{\mathbf{w}}{\nabla}_\nu e_a^\mu \equiv 0$.

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Relation with the Levi-Civita connection. Geodesics.

$$\overset{\mathbf{w}}{\Gamma}{}^{\lambda}_{\ \mu\nu} - \overset{\mathbf{L}}{\Gamma}{}^{\lambda}_{\ \mu\nu} \;=\; -\frac{1}{2}\,(T^\lambda{}_{\mu\nu} - T_\mu{}^\lambda{}_\nu - T_\nu{}^\lambda{}_\mu) \;\doteq\; K^\lambda{}_{\mu\nu}$$

Geodesics: $$\frac{d^2x^\lambda}{d\tau^2} + \overset{\mathbf{w}}{\Gamma}{}^{\lambda}_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} = K^\lambda{}_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}$$

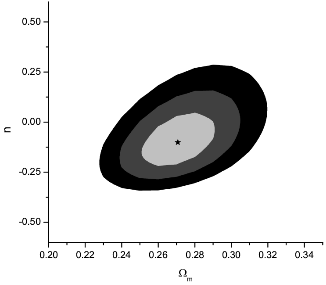
The *contorsion* tensor $K^\lambda{}_{\mu\nu}$ can be regarded as a gravitational field moving particles away from Weitzenböck autoparallel lines.

Return

G.R. Bengochea and R.F., PRD 79 (2009) 124019, *Dark torsion as the cosmic speed-up*

Deformation à la Carroll et al.

$$f(\mathbb{S}\cdot\mathbb{T}) = \mathbb{S}\cdot\mathbb{T} - \frac{\alpha}{(-\mathbb{S}\cdot\mathbb{T})^n}$$



The modified (flat) FRW equation is

$$H^2 - \frac{(2n+1)\,\alpha}{6^{n+1}H^{2n}} = \frac{8}{3}\pi G\rho$$

which can be rephrased as

$$\left(\frac{H}{H_o}\right)^{2n}\left[\left(\frac{H}{H_o}\right)^2 - \Omega_{m\,o}(1+z)^3 - \Omega_{r\,o}(1+z)^4\right] = 1-\Omega_{m\,o}-\Omega_{r\,o}, \qquad \Omega_{r\,o} = 5\times 10^{-5}$$

α is encoded in the difference $1-\Omega_{m\,o}-\Omega_{r\,o}$. In GR it is $\alpha = 0$ and $\Omega_{m\,o}+\Omega_{r\,o} = 1$.

The best fit for the data coming from SNIa, BAO and CMB is

$$\Omega_{m\,o} = 0{,}27 \qquad n = -0{,}10$$