Phonon spectra with asymmetrical sonic horizons

Stefano Finazzi[1] and Renaud Parentani[2]

[1]SISSA, Trieste

[2]LPT, Paris-Sud Orsay

September 6, 2010, ERE2010, Granada

SF and RP, work in progress

Motivations

- BH flux emitted by a relativistic massless field is exactly Planckian
- The temperature is given by Carter's $\kappa_C = \partial_x v|_{\rm horizon}$
- Dispersive fields ($\Omega^2 = c^2k^2 + c^4k^4/\Lambda^2$): Planckian spectrum recovered when $\Lambda \gg \kappa_C$
- Is the radiation characterized by the "local" ω-dependent surface gravity evaluated at the turning point?
- How does the dispersive spectrum follow the relativistic κ_C by replacing the symmetric flows by detuned ones?

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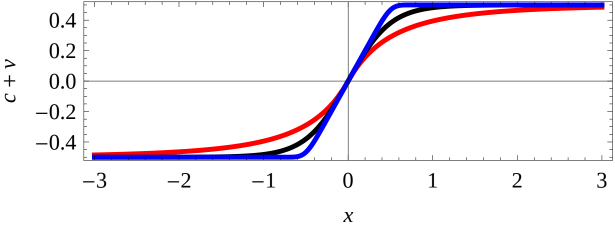
The metric of an acoustic black hole

$$ds^2 = \left[-c^2 \mathrm{d}t^2 + (\mathrm{d}x - v(x)\mathrm{d}t)^2\right]$$

- Acoustic horizon when $|v| = c$.
- $v < 0$ and $v + c = D_2 + D\,\mathrm{sign}(x)\left[\tanh\left(\frac{\kappa|x|}{D}\right)^n\right]^{1/n}$

Symmetric flows: $D_2 = 0$

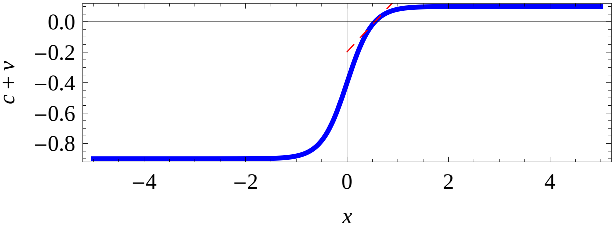


$$\kappa_C = \kappa$$

Asymmetric flows $D_2 \neq 0$

- $-D < D_2 < 0$

$$\kappa_C = \kappa \left|\frac{D_2}{D}\right|^{1-n} \left(1 - \left|\frac{D_2}{D}\right|^{2n}\right) \operatorname{arctanh}\left(\left|\frac{D_2}{D}\right|^{n}\right)^{1-\frac{1}{n}} < \kappa$$



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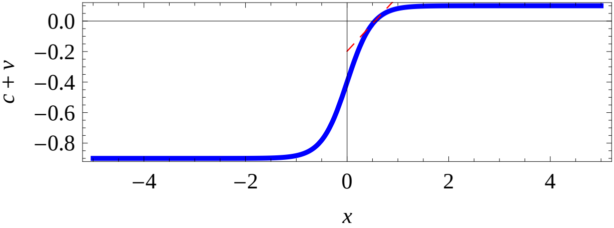

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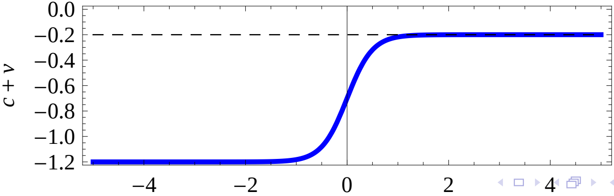


Phonons in BEC

Bogoliubov–de Gennes equation

$$i\hbar(\partial_t + v\partial_x)\hat{\phi} = \left[T_\rho + mc^2\right]\hat{\phi} + mc^2\hat{\phi}^\dagger$$

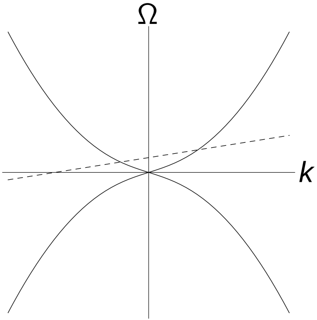
- Scalar massless field in curved background $\Rightarrow$ Acoustic metric
- UV supersonic dispersion relation

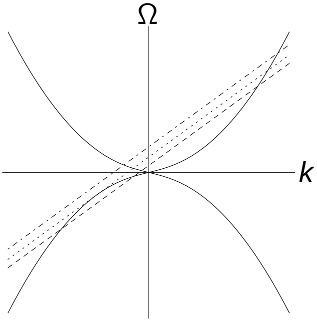
$$(\omega - kv)^2 = \Omega^2 = c^2k^2 + \hbar^2k^4/4m^2 \equiv c^2k^2 + c^4k^4/\Lambda^2$$

The phonon dispersion relation

Subsonic flow $|v| < c$

Supersonic flow $|v| > c$





- 2 solutions

- 4 solutions if $\omega < \omega_{\mathrm{max}}$
- 2 solutions if $\omega > \omega_{\mathrm{max}}$

- - - - - $\omega - vk$

——— $\pm\sqrt{c^2k^2 + \hbar^2k^4/4m^2}$

$D_2 = 0$

For one horizon, for $\omega < \omega_{\max}$: 3 asymptotically bound modes (ABM)

- positive norm rightgoing mode
- positive norm leftgoing mode
- negative norm mode: propagating in the supersonic region, decaying in the subsonic region

Results

- When $(c+v)(x \to +\infty) = -(c+v)(x \to -\infty)$, Hawking radiation at

$$T_H = \frac{\kappa_C}{2\pi}\left[1 + O\left(\frac{\kappa_C}{\omega_{\max}}\right)^4\right]$$

- $f_\omega \to 0$ for $\omega \to \omega_{\max}$.
- The radiation is robust even for strong dispersion.

J Macher and RP, Phys. Rev. A **80** 043601 (2009)

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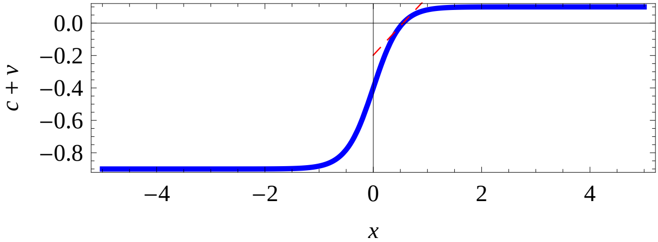
- Effective temperature depends on frequency

$$n_\omega = \frac{1}{\exp(\hbar\omega/\kappa_B T_\omega) - 1}$$

- T_ω is constant at low frequency
- Hawking temperature $T_H = \kappa_C/2\pi$



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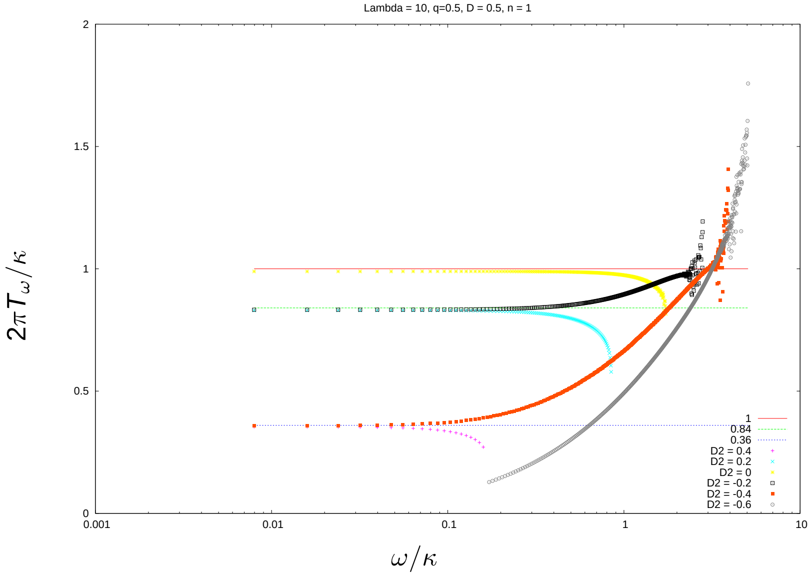
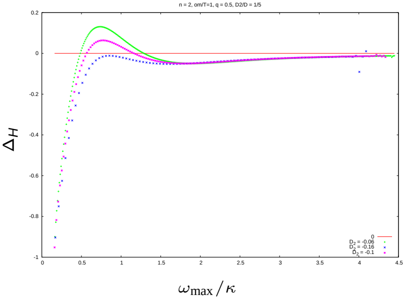


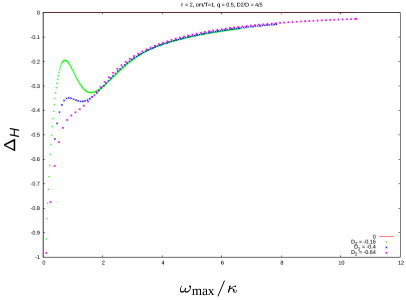
Relevant parameters: $\omega_{\max}$ and D_2/D

$$\Delta_H = \frac{f(\omega = T_H) - p_{T_H}(\omega = T_H)}{p_{T_H}(\omega = T_H)}$$

$D2/D = -1/5$



$D2/D = -4/5$

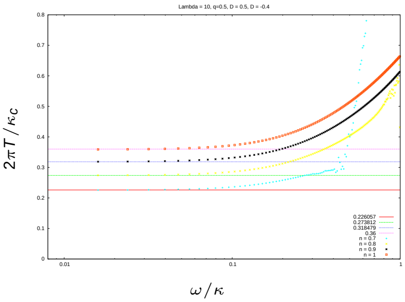


- For $\Lambda \to \infty$, $\omega_{\max} \to \infty$: the spectrum is Planckian.
- At fixed n, the relevant parameters are $\omega_{\max}/\kappa$ and D_2/D.

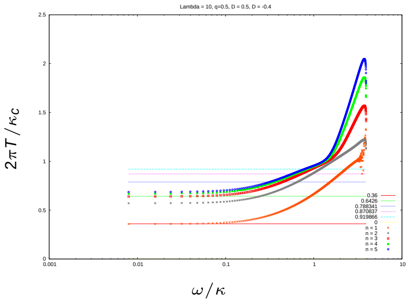
Relevant parameters: the slope of the transition n

- D_2=0 n plays no role
- $D_2 \neq 0$
  - n governs the surface gravity k_c
  - for large values of n the transition is steeper, higher derivatives $d^m(v+c)/dx^m$ are larger at the sonic horizon and affect significantly the flux and the effective temperature

$n \leq 1$



$n \geq 1$



$D_2 < -D$. Preliminary results.

- Radiation: the flux and the effective temperatures change continuously with D_2
- T_ω is no longer constant even at low frequency
  - The spectrum is not Planckian
  - No Hawking temperature

There exist a value $\omega_{\min}$ such that

	#PM($x < 0$)	#PM($x > 0$)	#ABM
$\omega < \omega_{\min}$	4	4	4
$\omega_{\min} < \omega < \omega_{\max}$	4	2	3
$\omega > \omega_{\max}$	2	2	2

PM=Propagating Mode (real k)
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Conclusions

- acoustic spacetime in BEC:
  - standard black holes
  - transitions between almost-supersonic/supersonic regions or supersonic/supersonic regions
- Hawking radiation is very robust
- asymmetric subsonic/supersonic transition: the spectrum is Planckian at low frequency
- relevant parameters: $\omega_{\rm max}/\kappa$, D_2/D, n
- supersonic/supersonic transition: no Planck spectrum

New projects

- regime $\omega < \omega_{\text{min}}$
  - 4 ABMs instead of 3
  - new interaction channels (4×4 scattering matrix)
- opposite of a black hole laser: supersonic – subsonic – supersonic (4 ABM)
- this is the analogue of a warp drive with modified dispersion relations