$\mathrm{U}(N)$ INVARIANT DYNAMICS FOR A SIMPLIFIED LOOP QUANTUM GRAVITY MODEL

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Outline

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Introduction

The $\mathrm{U}(N)$ framework for LQG intertwiners

The 2-vertex graph and the $\mathrm{U}(N)$ symmetry

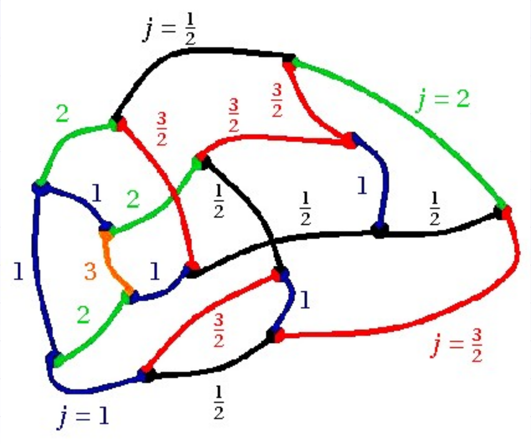
Dynamics on the 2-vertex graph

- The algebra of $\mathrm{U}(N)$ invariant operators
- An ansatz for dynamics
- Solving the model

Comparing with LQC

Introduction

- Hilbert space of LQG is constructed with spin-networks (functions over oriented graphs).



Introduction

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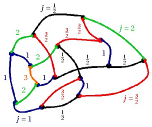

- Our Goal: Dynamics for the simplest class of graphs in LQG.
  - We consider 2 vertices linked with an arbitrary number of edges.
  - Generalization of the Rovelli-Vidotto model: Physical framework very similar to loop quantum cosmology.
- Use the $\mathrm{U}(N)$ framework recently introduced [F. Girelli, E. R. Livine, L. Freidel].
- Results:
  - Link between the $\mathrm{U}(N)$ operators and the holonomy ops. of LQG.
  - Global $\mathrm{U}(N)$ symmetry to select the reduced space of homogeneous/isotropic states.
  - $\mathrm{U}(N)$-invariant Hamiltonian operator encoding the dynamics of our 2-vertex model.

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E_{ij} and F_{ij} operators

- Space of intertwiners with N legs and fixed total area $J = \sum_i j_i$:

$$\mathcal{H}_N^{(J)} \equiv \bigoplus_{\sum_i j_i = J} \mathcal{H}_{j_1,..,j_N}. \equiv \bigoplus_{\sum_i j_i = J} \mathrm{Inv}[V^{j_1} \otimes .. \otimes V^{j_N}]$$

- Area conserving operators: $E_{ij} = a_i^\dagger a_j + b_i^\dagger b_j, \qquad E_{ij}^\dagger = E_{ji}.$

$$E_{ij} \,:\, \mathcal{H}_N^{(J)} \longrightarrow \mathcal{H}_N^{(J)}$$

- Annihilation and creation ops. to move between the spaces $\mathcal{H}_N^{(J)}$:

$$F_{ij} = (a_i b_j - a_j b_i) \qquad ; \qquad F_{ji} = -F_{ij}.$$

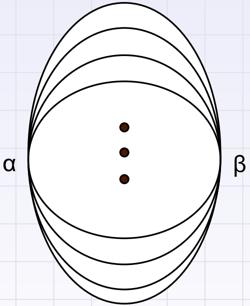
$$F_{ij} \,:\, \mathcal{H}_N^{(J)} \longrightarrow \mathcal{H}_N^{(J-1)} \qquad ; \qquad F_{ij}^\dagger \,:\, \mathcal{H}_N^{(J)} \longrightarrow \mathcal{H}_N^{(J+1)}$$

- Invariant under global $\mathrm{SU}(2)$ transformations, but they do not commute anymore with the total area operator $E = \sum_i E_{ii}$.
- F_{ij} with E_{ij} form a closed algebra.

Outline

The 2-vertex graph

- A slight generalization of the model introduced by C. Rovelli and F. Vidotto (related to models of quantum cosmology).
- The simplest non-trivial graph for spin network states in LQG: a graph with two vertices linked by N edges.



- Hilbert space of the two intertwiners:

$$\mathcal{H}_{\otimes 2} = \mathcal{H}_N \otimes \mathcal{H}_N = \bigoplus_{J_\alpha, J_\beta} \mathcal{H}_N^{(J_\alpha)} \otimes \mathcal{H}_N^{(J_\beta)} = \bigoplus_{\{j_i^\alpha, j_i^\beta\}} \mathcal{H}_{j_1^\alpha, \dots, j_N^\alpha} \otimes \mathcal{H}_{j_1^\beta, \dots, j_N^\beta}.$$

Matching conditions

- Each edge must carry a unique $\mathrm{SU}(2)$ representation, thus the spin j_i seen from α or β must be the same.

$$\mathcal{E}_i \equiv E_i^{(\alpha)} - E_i^{(\beta)} = 0.$$

- The Hilbert space of spin network states for this 2-vertex graph is:

$$^2\mathcal{H} \equiv \bigoplus_{\{j_i\}} \mathcal{H}^{(\alpha)}_{j_1,\ldots,j_N} \otimes \mathcal{H}^{(\beta)}_{j_1,\ldots,j_N}.$$

- Operators acting on $^2\mathcal{H}$, should be invariant under global $\mathrm{SU}(2)$ transformations and they should commute with the matching conditions $\mathcal{E}_i$.

- Operators deforming consistently the boundary between α and β.

$$e_{ij} \equiv E_{ij}^{(\alpha)} E_{ij}^{(\beta)}, \qquad f_{ij} \equiv F_{ij}^{(\alpha)} F_{ij}^{(\beta)}, \qquad f_{ij}^{\dagger} \equiv F_{ij}^{(\alpha)\dagger} F_{ij}^{(\beta)\dagger}.$$

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Global $\mathfrak{u}(N)$ algebra

- We can introduce the operators: $\mathcal{E}_{ij} \equiv E_{ij}^{(\alpha)} - E_{ji}^{(\beta)}$
- They form a $\mathfrak{u}(N)$ algebra: $[\mathcal{E}_{ij}, \mathcal{E}_{kl}] = \delta_{jk}\mathcal{E}_{il} - \delta_{il}\mathcal{E}_{kj}$.
- $\mathcal{E}_k$ are part of this larger $\mathfrak{u}(N)$ algebra.
- Look for vectors in ${}^2\mathcal{H}$ which are invariant under this $\mathrm{U}(N)$ action.

Looking for a $\mathrm{U}(N)$ invariant subspace

The subspace of spin network states invariant under the $\mathrm{U}(N)$-action:

$$
{}^2\mathcal{H}_{inv} \equiv Inv_{\mathrm{U}(N)}\left[{}^2\mathcal{H}\right] = Inv_{\mathrm{U}(N)}\left[\mathcal{H}_{\otimes 2}\right] = Inv_{\mathrm{U}(N)}\left[\bigoplus_{J_\alpha, J_\beta} \mathcal{H}_N^{(J_\alpha)} \otimes \mathcal{H}_N^{(J_\beta)}\right]
$$

- $\mathcal{H}_N^{(J)}$ are irreducible $\mathrm{U}(N)$-representations [L.Freidel, E.Livine].
- $\mathrm{U}(N)$-invariance $\Rightarrow J_\alpha = J_\beta$.
- There exists a unique invariant vector $|J\rangle \in \mathcal{H}_N^{(J)} \otimes \mathcal{H}_N^{(J)}$.

$$
{}^2\mathcal{H}_{inv} = \bigoplus_{J \in \mathbb{N}} \mathbb{C}\,|J\rangle
$$

Holonomy operator

- Link between our operators e_{ij} and f_{ij} with the usual holonomy operators of loop quantum gravity.

Holonomy operator:

$$\chi^{(ij)} = \frac{1}{\sqrt{E_i+1}\sqrt{E_j+1}} \left(f_{ij}^\dagger + e_{ij} + e_{ji} + f_{ij}\right) \frac{1}{\sqrt{E_i+1}\sqrt{E_j+1}}.$$

"Dictionary" between holonomy and $\mathrm{U}(N)$ operators:

$$\frac{1}{4}\Big([E_i,[E_j,\cdot]] + [E_i,\cdot] + [E_j,\cdot] + 1\Big)\chi^{(ij)} = \frac{1}{\sqrt{E_i+1}\sqrt{E_j+1}} f_{ij}^\dagger \frac{1}{\sqrt{E_i+1}\sqrt{E_j+1}}$$

$$\frac{1}{4}\Big([E_i,[E_j,\cdot]] - [E_i,\cdot] - [E_j,\cdot] + 1\Big)\chi^{(ij)} = \frac{1}{\sqrt{E_i+1}\sqrt{E_j+1}} f_{ij} \frac{1}{\sqrt{E_i+1}\sqrt{E_j+1}}$$

$$\frac{1}{4}\Big(-[E_i,[E_j,\cdot]] + [E_i,\cdot] - [E_j,\cdot] + 1\Big)\chi^{(ij)} = \frac{1}{\sqrt{E_i+1}\sqrt{E_j+1}} e_{ij} \frac{1}{\sqrt{E_i+1}\sqrt{E_j+1}}$$

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Structure of $\mathrm{U}(N)$-invariant operators on ${}^2\mathcal{H}_{inv}$

- $E = E^{(\alpha)} = E^{(\beta)}$ is invariant and $E\,|J\rangle = 2J\,|J\rangle$.
- We define the following operators

$$e \equiv \sum_{ij} e_{ij} = \sum_{ij} E_{ij}^{(\alpha)} E_{ij}^{(\beta)}, \qquad f \equiv \sum_{ij} f_{ij} = \sum_{ij} F_{ij}^{(\alpha)} F_{ij}^{(\beta)}.$$

- They obviously commute with the matching conditions.
- They form a surprisingly simple algebra:

$$\begin{aligned}
[e, f] &= -2(E + N - 1)f\,, \\
\left[e, f^\dagger\right] &= 2f^\dagger(E + N - 1), \\
\left[f, f^\dagger\right] &= 4(E + N)(e + 2(E + N - 1))\,.
\end{aligned}$$

- Introduce a shifted operator $\widetilde{e} \equiv e + 2(E + N - 1)$.

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- They obviously commute with the matching conditions.
- Introduce a shifted operator $\widetilde{e} \equiv e + 2(E + N - 1)$.
- Then the algebra reads:

$$\begin{aligned} \left[\widetilde{e}, f\right] &= -2(E+N+1)f\,, \\ \left[\widetilde{e}, f^\dagger\right] &= 2f^\dagger(E+N+1), \\ \left[f, f^\dagger\right] &= 4(E+N)\widetilde{e}\,. \end{aligned}$$

Structure of $\mathrm{U}(N)$-invariant operators on ${}^2\mathcal{H}_{inv}$

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- They obviously commute with the matching conditions.
- Introduce a shifted operator $\widetilde{e} \equiv e + 2(E + N - 1)$.
- Our invariant Hilbert space ${}^2\mathcal{H}_{inv}$ is spanned by the states

$$|J\rangle_{un} \equiv f^{\dagger J}|0\rangle = \left(\sum_{ij} F_{ij}^{(\alpha)\dagger} F_{ij}^{(\beta)\dagger}\right)^J |0\rangle\,; \qquad E\,|J\rangle_{un} = 2J\,|J\rangle_{un}$$

- The states $|J\rangle_{un}$ diagonalize $\widetilde{e}$, while $f^\dagger$ and f act respectively as creation and annihilation operators.

Other operators

$\mathfrak{sl}(2,\mathbb{R})$ operators

$$
\begin{aligned}
Z &\equiv \frac{1}{\sqrt{E+2(N-1)}}\,\widetilde{e}\,\frac{1}{\sqrt{E+2(N-1)}}\\
X_- &\equiv \frac{1}{\sqrt{E+2(N-1)}}\,f\,\frac{1}{\sqrt{E+2(N-1)}}\\
X_+ &\equiv \frac{1}{\sqrt{E+2(N-1)}}\,f^\dagger\,\frac{1}{\sqrt{E+2(N-1)}}
\end{aligned}
$$

They satisfy a $\mathfrak{sl}(2,\mathbb{R})$ Lie algebra:

$$
[Z, X_\pm] = \pm X_\pm\,, \qquad [X_+, X_-] = -2Z
$$

Action

$$
\begin{aligned}
Z\,|J\rangle &= (J+1)\,|J\rangle\\
X_-\,|J\rangle &= \sqrt{J(J+1)}\,|J-1\rangle\\
X_+\,|J\rangle &= \sqrt{(J+1)(J+2)}\,|J+1\rangle
\end{aligned}
$$

Renormalized Operators

$$
\frac{1}{\sqrt{\widetilde{e}}}\,\widetilde{e}\,\frac{1}{\sqrt{\widetilde{e}}} = \mathbb{I}
\qquad
\frac{1}{\sqrt{\widetilde{e}}}\,f\,\frac{1}{\sqrt{\widetilde{e}}}
\qquad
\frac{1}{\sqrt{\widetilde{e}}}\,f^\dagger\,\frac{1}{\sqrt{\widetilde{e}}}
$$

Action

$$
\begin{aligned}
\mathbb{I}|J\rangle &= |J\rangle\\
\frac{1}{\sqrt{\widetilde{e}}}\,f\,\frac{1}{\sqrt{\widetilde{e}}}|J\rangle &= |J-1\rangle\,, \quad \forall J\geq 1\\
\frac{1}{\sqrt{\widetilde{e}}}\,f^\dagger\,\frac{1}{\sqrt{\widetilde{e}}}|J\rangle &= |J+1\rangle
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Outline

Hamiltonian operator

- Simplest $\mathrm{U}(N)$-invariant ansatz:

$$H \equiv \lambda \tilde{e} + (\sigma f + \bar{\sigma} f^\dagger)$$

- It corresponds to the *evolution operator* $\hat{\Theta}$ of LQC.
- Looking for eigenstates: three regimes ($\lambda > 0$, $\cos\omega = -\lambda/2\sigma$):
  1. The *oscillatory regime*: $|\sigma| > \lambda/2$
  2. The *discrete regime*: $|\sigma| < \lambda/2$
  3. The *critical regime*: $\sigma = \pm\lambda/2$

- H is unique up to a renormalization by a E-dependent factor.
- We can propose a $\mathfrak{sl}_2$ Hamiltonian:

$$\mathbf{h} \equiv \frac{1}{\sqrt{E + 2(N-1)}} H \frac{1}{\sqrt{E + 2(N-1)}} = \lambda Z + (\sigma X_- + \bar{\sigma} X_+) \quad \in \mathfrak{sl}_2$$

  1. It is an element in the Lie algebra $\mathfrak{sl}_2$.
  2. It has the same three regimes as H.
  3. It corresponds to the gravitational part of the LQC Hamiltonian constraint $\hat{C}_{\mathrm{grav}}$

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H: The spectrum

- The action of the Hamiltonian is:

$$
\begin{aligned}
H|J\rangle &= \lambda\varphi(J)\,|J\rangle + \sigma\psi(J)\,|J-1\rangle + \bar{\sigma}\psi(J+1)\,|J+1\rangle \\
\varphi(J) &= (J+1)(N+J-1) \\
\psi(J) &= \sqrt{J(J+1)(N+J-1)(N+J-2)}
\end{aligned}
$$

- Looking for the eigenstates:

$$
H_c|\psi\rangle = \sum_J \alpha_J\, H_c\,|J\rangle \;=\; \beta \sum_J \alpha_J\,|J\rangle
$$

$$
\lambda\varphi(J)\alpha_J + \bar{\sigma}\psi(J)\alpha_{J-1} + \sigma\psi(J+1)\alpha_{J+1} \;=\; \beta\alpha_J
$$

1. H is positive if $2|\sigma| \leq \lambda$.
2. H is essentially self-adjoint as soon as $2|\sigma| \leq \lambda$.
3. H has a strictly positive discrete spectrum when $2|\sigma| < \lambda$.

H_c: The critical regime

The critical regime: $2|\sigma| = \lambda$

$\sigma = \exp(-i\theta)$

- Looking for eigenstates:

$$2\varphi(J)\alpha_J + e^{+i\theta}\psi(J)\alpha_{J-1} + e^{-i\theta}\psi(J+1)\alpha_{J+1} \;=\; \beta\alpha_J$$

- LQC inspired ansatz:

$$\alpha_J \sim \frac{(-1)^J}{\sqrt{J}}\, e^{i\theta J}\, e^{ik\ln J}\,; \qquad k \in \mathbb{R}$$

- Eigenvalues (strictly positive):

$$\boxed{\beta = \frac{1}{4} + k^2}$$

H: Strong coupling

Strong coupling regime: $2|\sigma| > \lambda$

- The ansatz for the leading order of the eigenvectors:

$$\alpha_J = \frac{1}{J + c} e^{i\omega J} \qquad c \in \mathbb{C}$$

- The eigenvalue:

$$\beta = \left(\frac{N}{2} - c \right) \left(\bar{\sigma} e^{-i\omega} - \sigma e^{+i\omega} \right)$$

- The eigenvalues are complex !!

Outline

Comparing with loop quantum cosmology

- The 2-vertex graph is a perfect setting to derive a quantum cosmology sector from the full LQG. [C.Rovelli, F. Vidotto]
- Gravitational part of the Hamiltonian constraint in LQC:
$$\hat{C}_{gr}\,|v\rangle \propto 2v\,|v\rangle - v\,|v+4\rangle - v\,|v-4\rangle\,,$$
- Evolution operator in LQC ($\hat{\Theta} = \sqrt{v}\hat{C}_{gr}\sqrt{v}$):
$$\hat{\Theta}\,|v\rangle \propto 2v^2\,|v\rangle - v^2\,|v+4\rangle - v^2\,|v-4\rangle$$

Analogies

- $\hat{\Theta}$ corresponds to H (coefficients grow as J^2).
- $\hat{C}_{gr}$ corresponds to $\mathfrak{s}l_2$-Hamiltonian $\mathbf{h}$ (coefficients grow as J).
- Spectral properties will be very similar. Apply to our framework techniques developed in LQC.
- LQC operators for the flat case $\Lambda = 0$ correspond to our critical regime with $\sigma = -\lambda/2$.

Cosmological constant

- Gravitational part of the Hamiltonian constraint with Λ:

$$\hat{C}_{gr}\,|v\rangle = (A(v+2)+A(v-2))\,|v\rangle - A(v+2)|v+4\rangle - A(v-2)|v-4\rangle - \Lambda \hat{V}\,|v\rangle$$

$$\hat{V}\,|v\rangle = V_0\, v|v\rangle \qquad A(v) \sim 2A_0\, v.$$

- Substitution at mathematical level: $v \equiv 4J$

$$\hat{\Theta}|J\rangle \sim 16(4A_0 - \Lambda V_0)J^2|J\rangle - 32A_0(J+\frac{1}{2})\sqrt{J(J+1)}|J+1\rangle - 32A_0(J-\frac{1}{2})\sqrt{J(J-1)}|J-1\rangle$$

- Comparison with H:

$$\lambda \equiv 16(4A_0 - \Lambda V_0), \qquad \sigma = \bar{\sigma} \equiv -32A_0$$

Different regimes

- $\Lambda = 0 \;\Rightarrow\; \sigma = -\lambda/2$. *Critical regime*.
- $\Lambda > 0$, but close to 0 $\Rightarrow\; 0 < \lambda < 2|\sigma|$. *Strong coupling regime*.
- $\Lambda < 0 \;\Rightarrow\; \lambda > 2|\sigma|$. *Weak coupling regime*.

Conclusions

- The new $\mathrm{U}(N)$ framework represents a new way to study the dynamics in LQG.
- The model: 2 vertex (linked with N edges) glued by matching conditions.
- Global $\mathrm{U}(N)$ symmetry to select the isotropic/homogeneous states $|J\rangle$. Deriving LQC from LQG?
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thanks!!

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