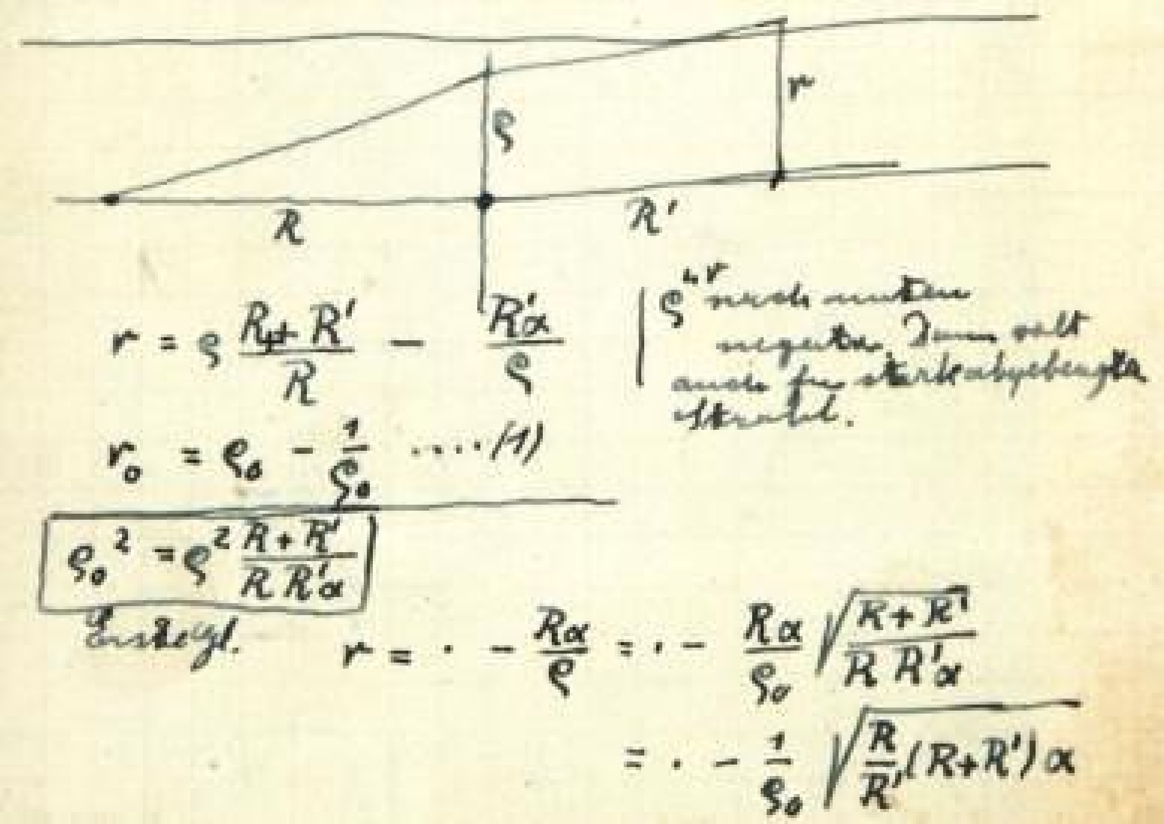
Statistical Bootstrap and Gravitational Microlensing



$$r = \varrho \frac{R+R'}{R} - \frac{R'\alpha}{\varrho}$$

$$r_0 = \varrho_0 - \frac{1}{\varrho_0} \quad \ldots\ldots(1)$$

$$\boxed{\varrho_0^2 = \varrho^2 \frac{R+R'}{RR'\alpha}}$$

[illegible]

$$r = \cdot - \frac{R\alpha}{\varrho} = \cdot - \frac{R\alpha}{\varrho_0}\sqrt{\frac{R+R'}{RR'\alpha}}$$

$$= \cdot - \frac{1}{\varrho_0}\sqrt{\frac{R}{R'}(R+R')\alpha}$$

ϱ nach unten negativ. Dann gilt auch für starkabgebogenen Strahl.

Guerras, E. - Mediavilla, E. - Muñoz, J.
Instituto de Astrofísica de Canarias

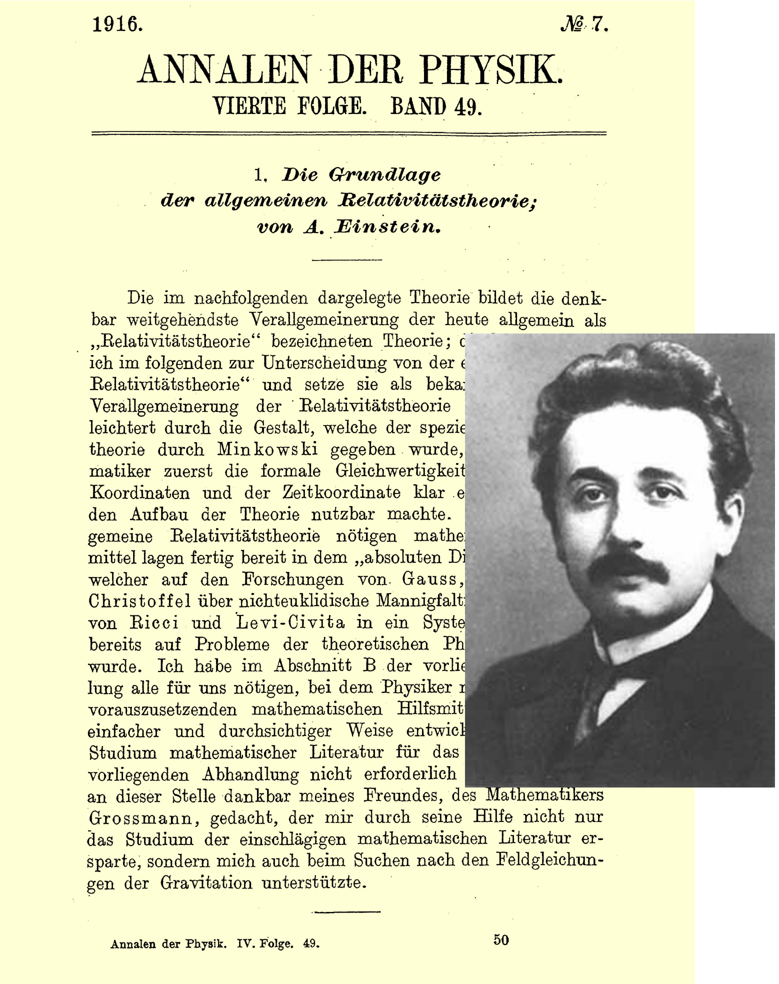
1. Microlensing

1. Microlensing

Light deflection by gravitating mass

$$a = \frac{4GM}{c^2 r}$$

1916AnP...354..769E



1916. № 7.

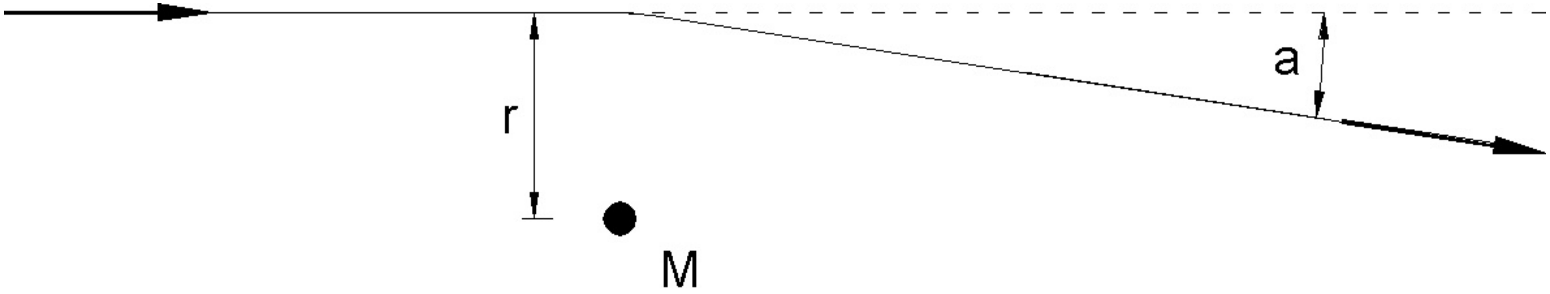
ANNALEN DER PHYSIK.

VIERTE FOLGE. BAND 49.

1. *Die Grundlage*
der allgemeinen Relativitätstheorie;
von A. Einstein.

Die im nachfolgenden dargelegte Theorie bildet die denkbar weitgehendste Verallgemeinerung der heute allgemein als
„Relativitätstheorie“ bezeichneten Theorie; d
ich im folgenden zur Unterscheidung von der e
Relativitätstheorie“ und setze sie als beka
Verallgemeinerung der Relativitätstheorie
leichtert durch die Gestalt, welche der spezie
theorie durch Minkowski gegeben wurde,
matiker zuerst die formale Gleichwertigkeit
Koordinaten und der Zeitkoordinate klar e
den Aufbau der Theorie nutzbar machte.
gemeine Relativitätstheorie nötigen mathe
mittel lagen fertig bereit in dem „absoluten Di
welcher auf den Forschungen von Gauss,
Christoffel über nichteuklidische Mannigfalt
von Ricci und Levi-Civita in ein Syste
bereits auf Probleme der theoretischen Ph
wurde. Ich habe im Abschnitt B der vorlie
lung alle für uns nötigen, bei dem Physiker n
vorauszusetzenden mathematischen Hilfsmit
einfacher und durchsichtiger Weise entwick
Studium mathematischer Literatur für das
vorliegenden Abhandlung nicht erforderlich
an dieser Stelle dankbar meines Freundes, des Mathematikers Grossmann, gedacht, der mir durch seine Hilfe nicht nur das Studium der einschlägigen mathematischen Literatur ersparte, sondern mich auch beim Suchen nach den Feldgleichungen der Gravitation unterstützte.

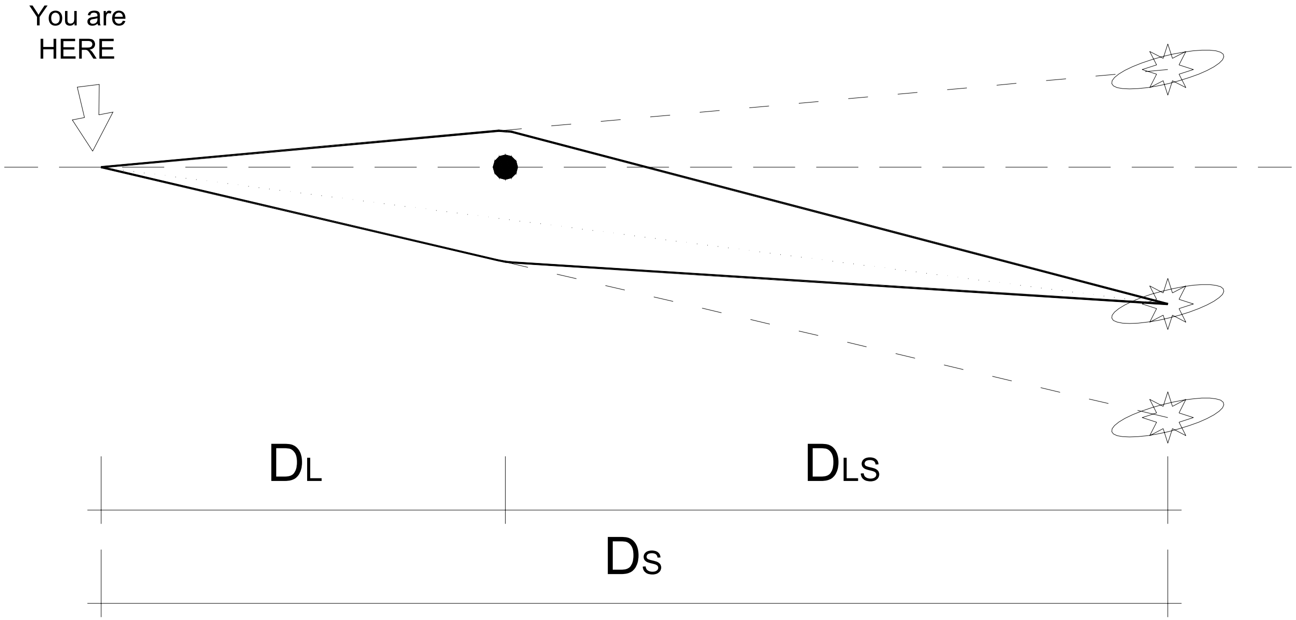
Annalen der Physik. IV. Folge. 49. 50



1. Microlensing

Орест Хвольсон (1852-1934) raised in 1924 the possibility that an alignment could result in a ficticious double star or a ring image

(1924AN....221..329C)



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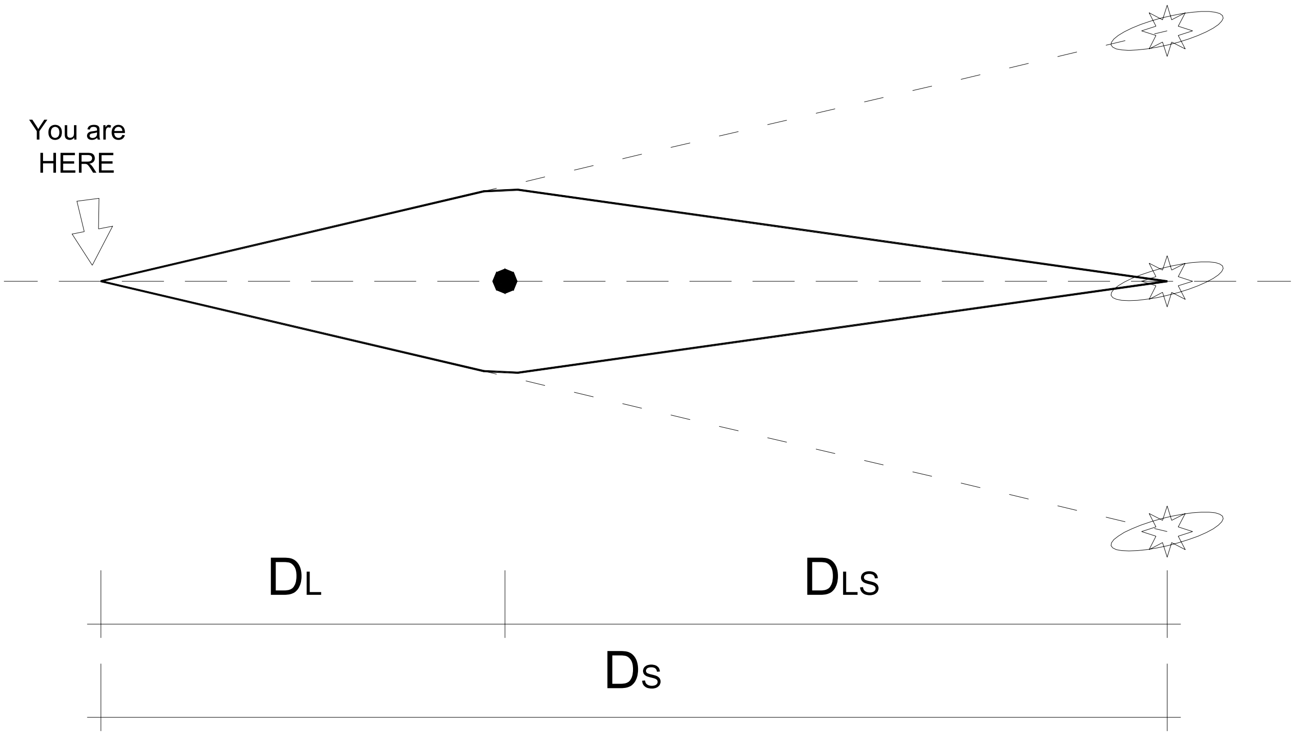


$$R_0 = \sqrt{\frac{4GMD_{LS}}{c^2 D_S D_L}} \approx 1\times10^{-3} \text{ arcsec}$$

below telescope resolution!

(D_L ,D_{LS} = 5 kpc, M = M_{Sun})

1. Microlensing

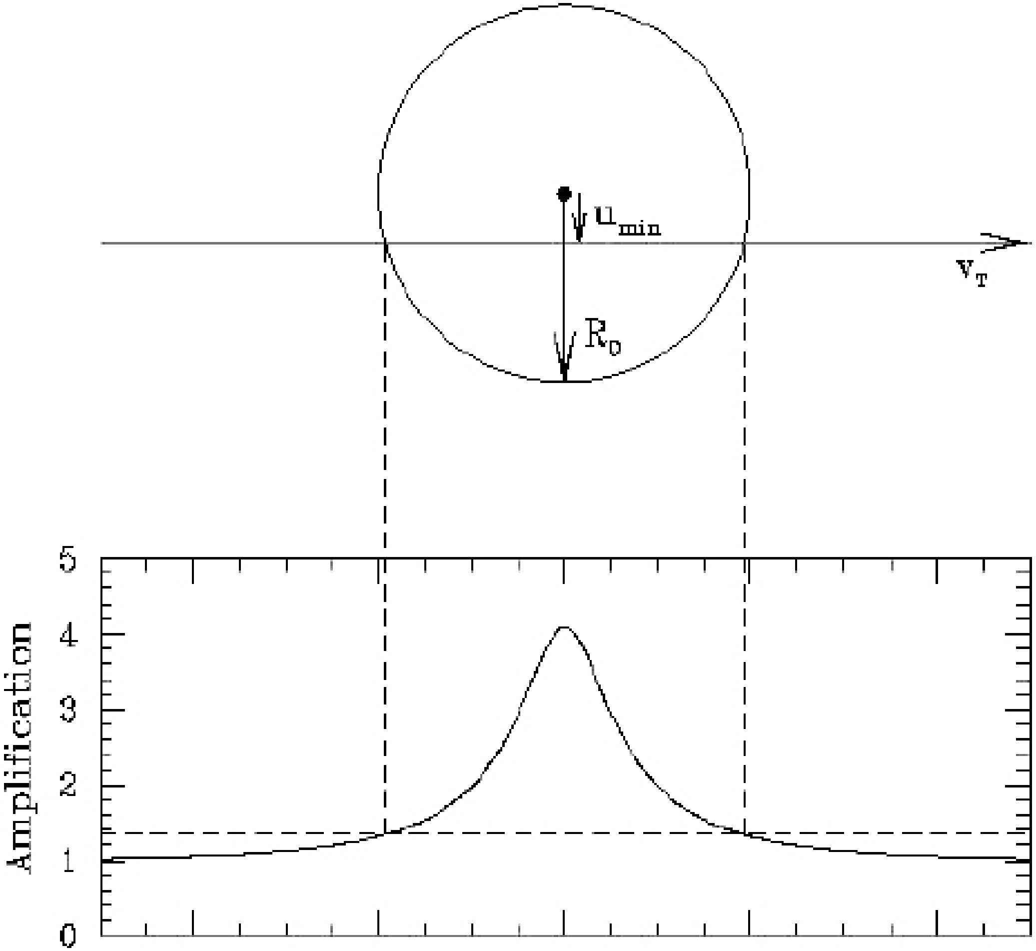
Magnification is position dependent

$$A=\frac{u^2+2}{u\sqrt{u^2+4}}$$

Time span of the event

$$t_E=\frac{R_0}{v_T}$$

(Point-source-point lens approximation; transversal speed is angular apparent speed)

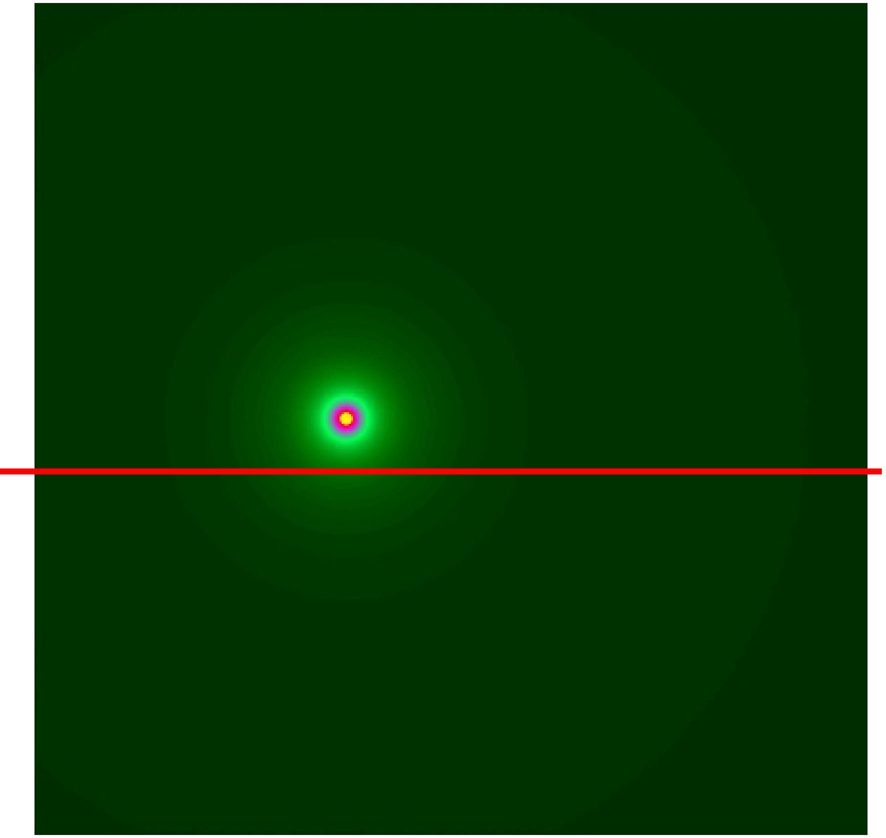


Mao, 2008arXiv0811.0441M

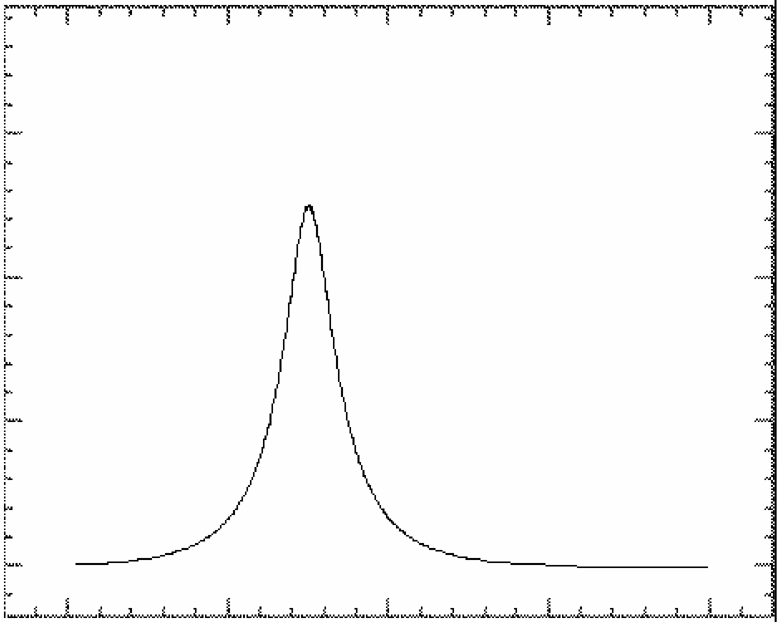
1. Microlensing

Main Tool in microlensig numerical calculations: **Magnification Map**

- Divides source plane in *cells*, (so every pixel represents a square area)
- Assigns value of magnification for hypothetical source *within* every cell
- Does *not* gives information about deflections



Single point-deflector



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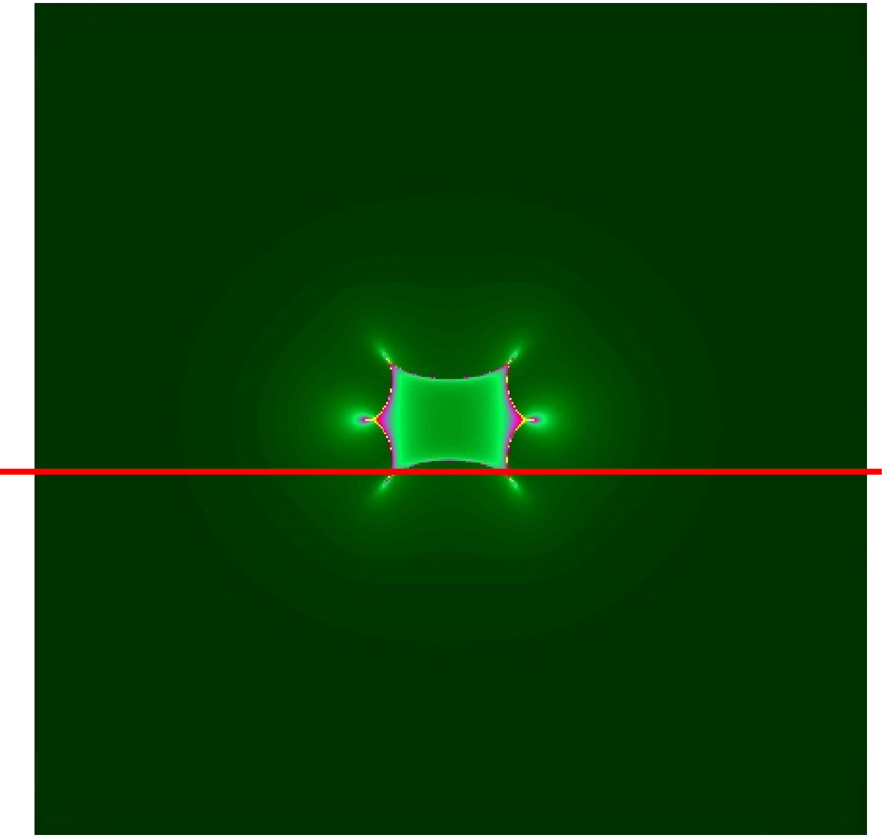
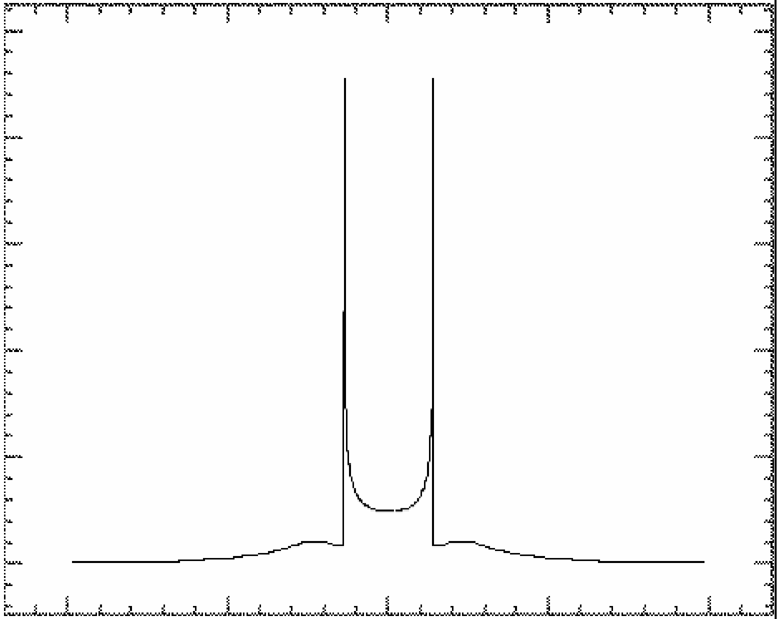
Binary point-deflector





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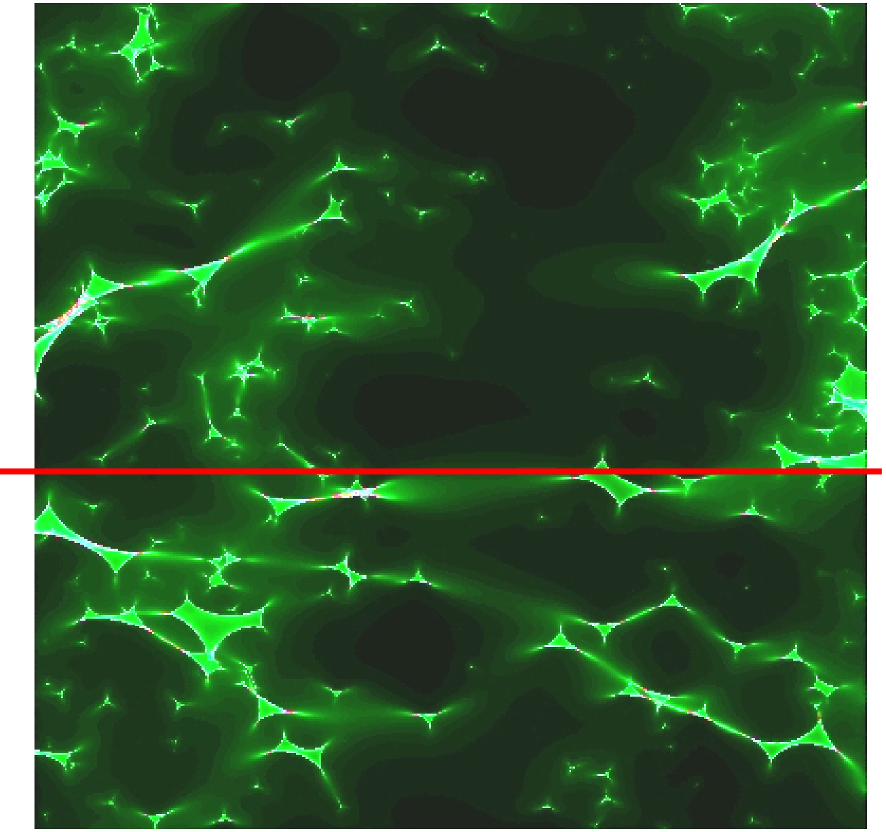
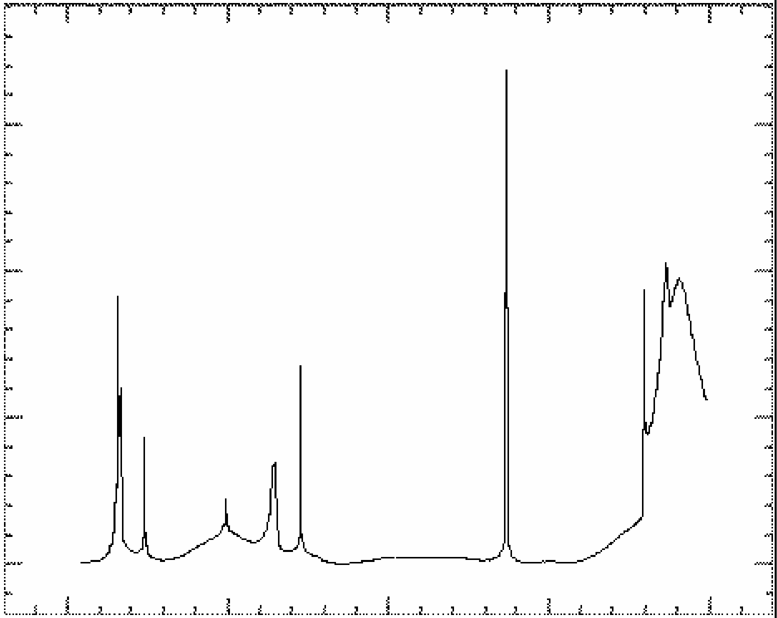


Multiple point-deflector



1. Microlensing

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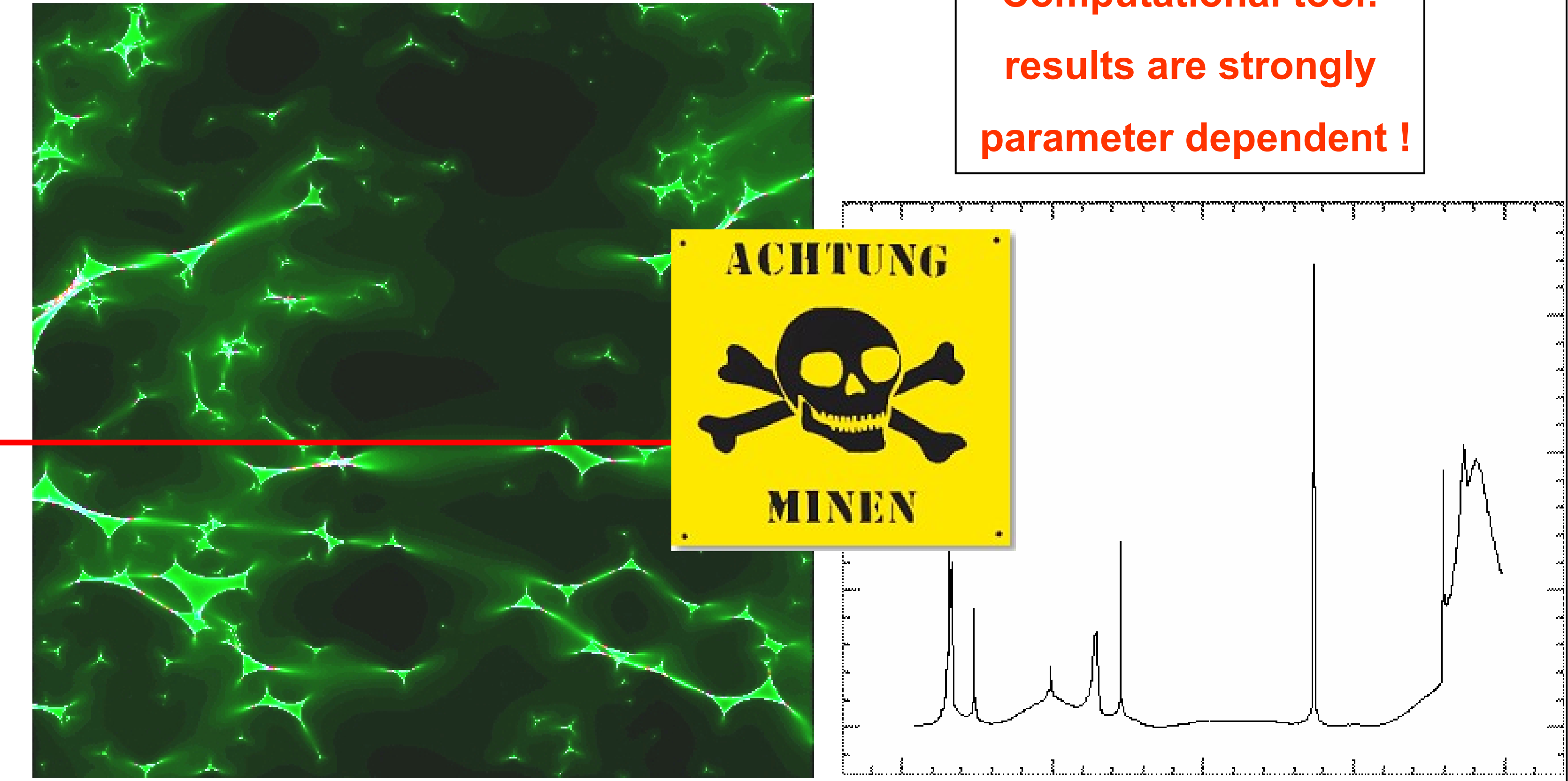
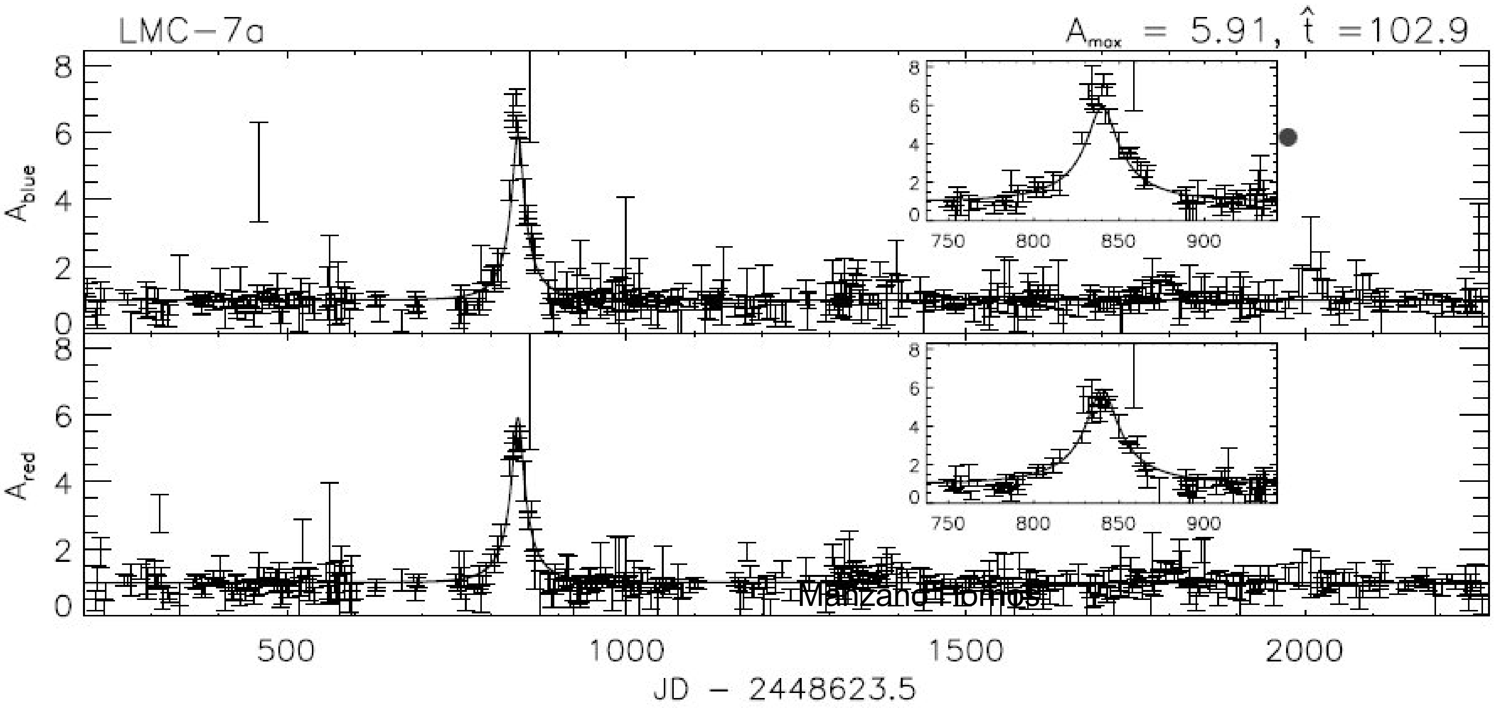
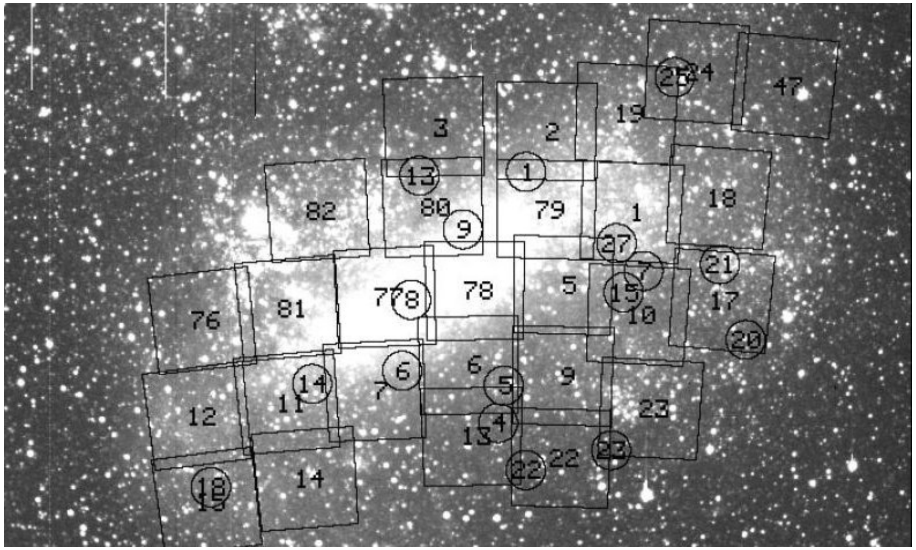
- Does *not* gives information about deflections

Computational tool:
results are strongly
parameter dependent !



1. Microlensing



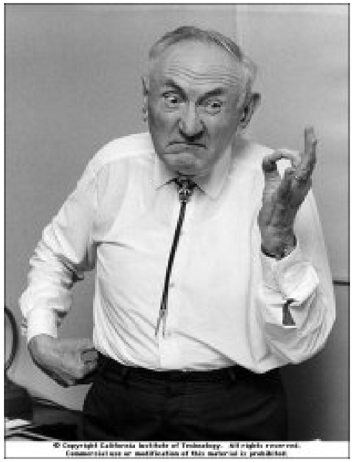


The MACHOs
project

Alcock et al, 2000ApJ…542..281A

2. Strong Lensing

2. Strong Lensing



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Fritz Zwicky (1898 - 1974) predicted in 1937 the detection of multiple images when extragalactic nebulae instead of stars were involved

(1937PhRv...51..290Z, 1937PhRv...51..679Z)

$$R_0 = \sqrt{\frac{4GMD_{LS}}{c^2 D_S D_L}} \approx 5\,\mathrm{arcsec}$$

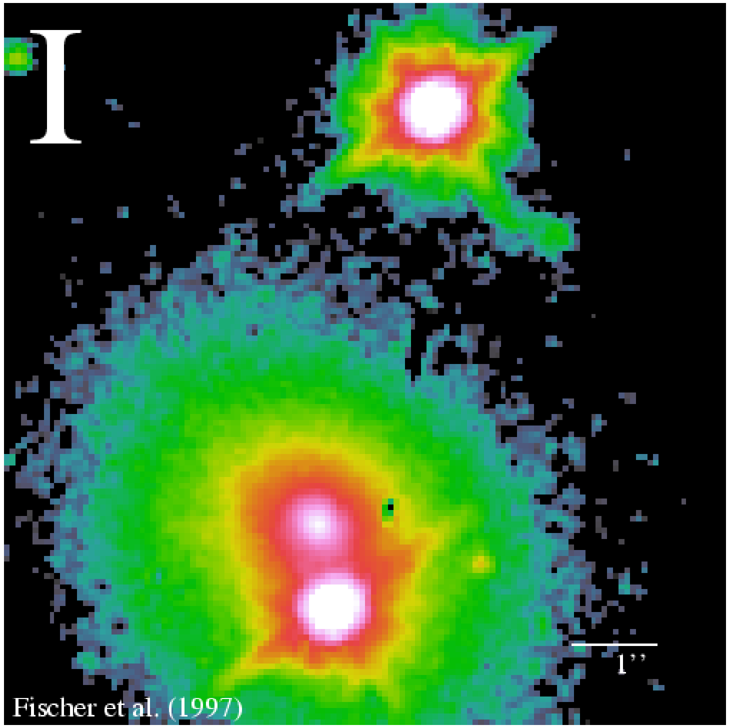
Zwicky's calculations and predictions include:

- **multiple images**
- **ring images**
- **amplification bias**
- **mass determinations**
- **GR test**
- **lens as telescopes**

First detection in 1979:

QSO 0957+561

1979Natur.279..381W



Fischer et al. (1997)

2. Strong Lensing

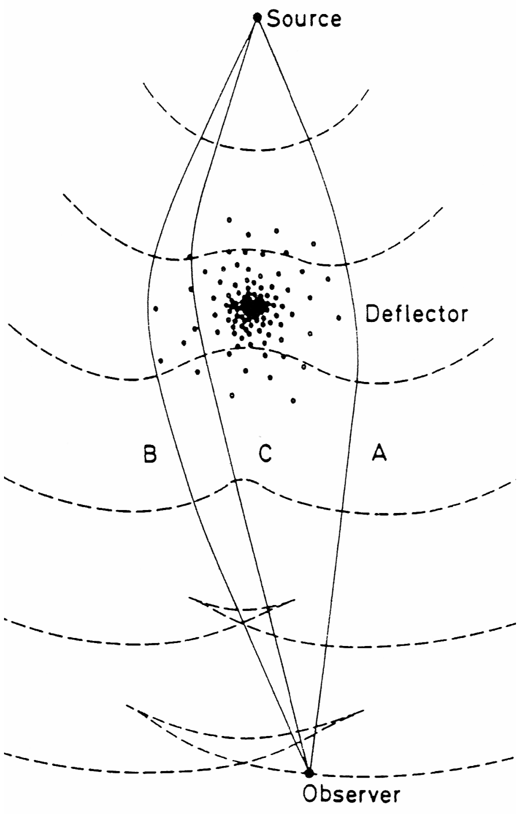
Good old Fermat's principle

δL=0

but in curved spacetime:

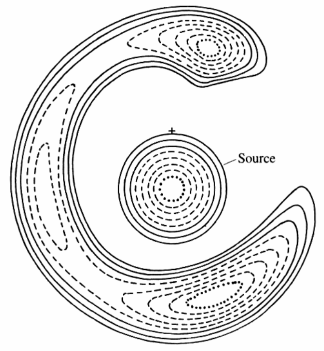
$$\mathcal{L}\left(x^{\alpha},\dot{x}^{\beta}\right)=\frac{1}{2}g_{\alpha\beta}(x^{\gamma})\dot{x}^{\alpha}\dot{x}^{\beta}$$

$$\delta\left\{\frac{1}{2}\int g_{\alpha\beta}\dot{x}^{\alpha}\dot{x}^{\beta}dv\right\}=0$$



A mass model for the lens is required, which leads to the assumption of a deflection potential dependent from several parameters, usually redshifts, angular separations, etc.

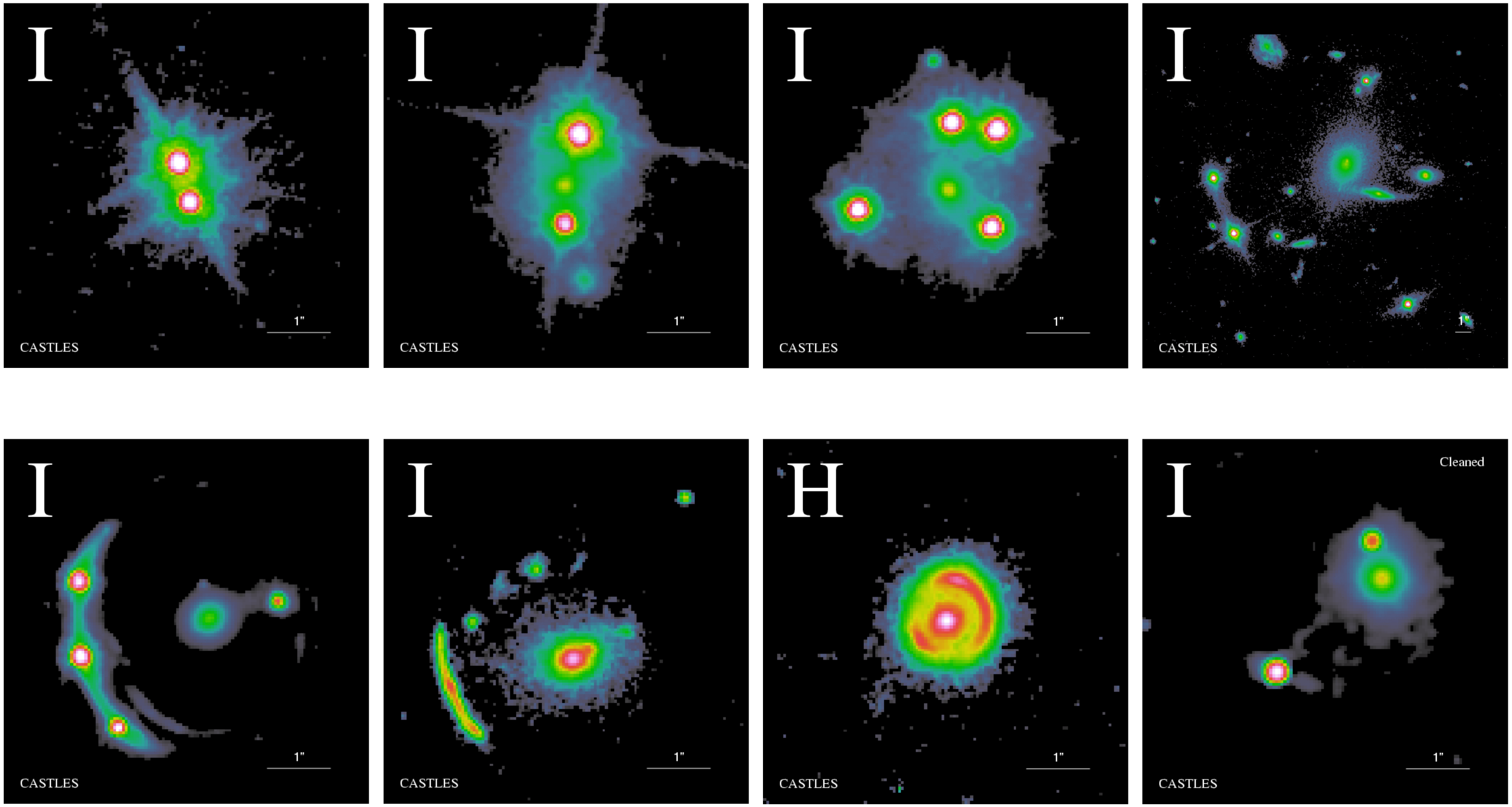
$$f:\mathrm{R}^2\rightarrow\mathrm{R}^2,\mathrm{x}\mapsto\mathrm{y}$$



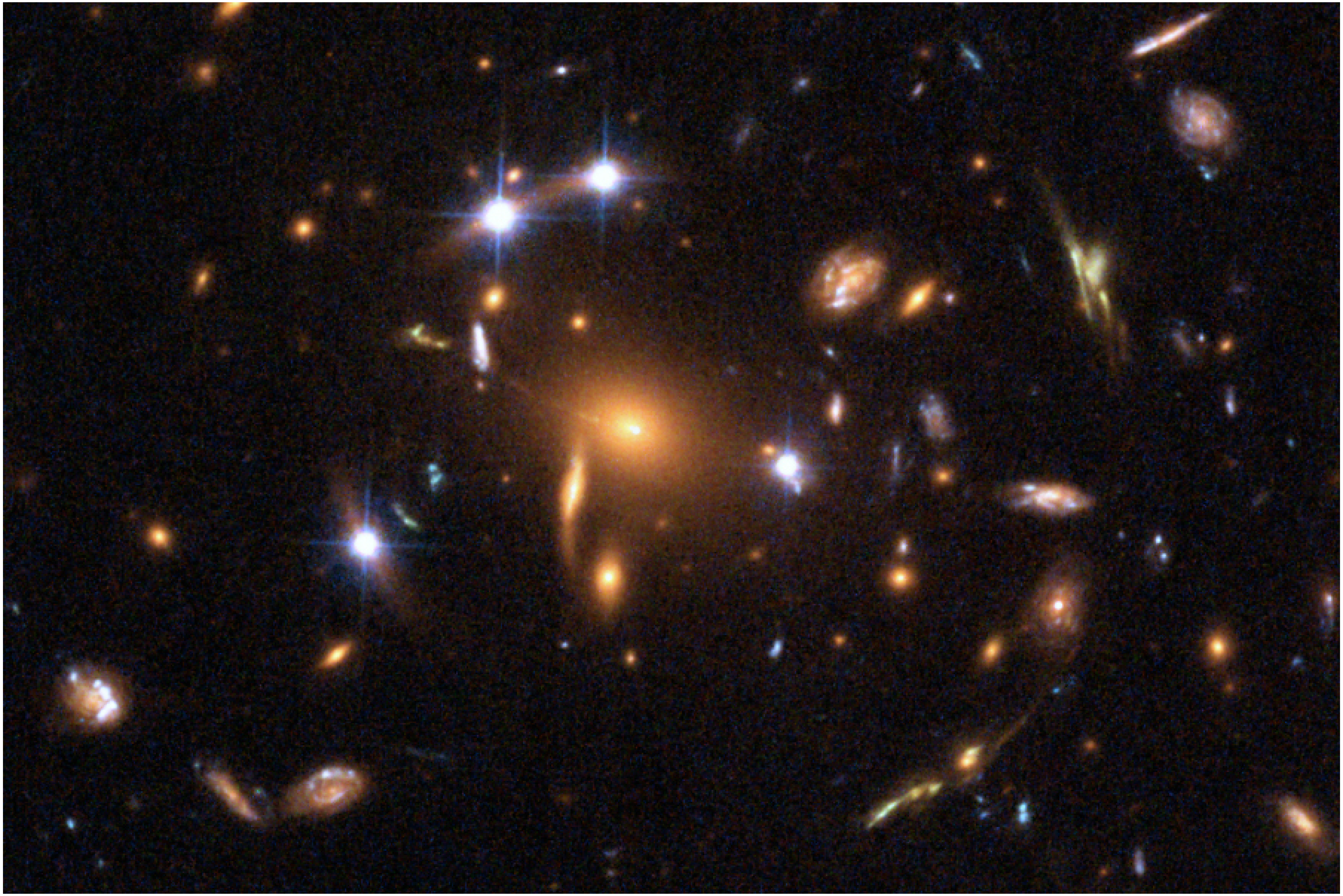
Imaging is modeled as a mapping from the *lens plane* to the *source plane*. which is only *locally* homeomorphic due to image plane domains to whom the jacobian of the transformation diverges.

2. Strong Lensing

Gravitational lens zoo

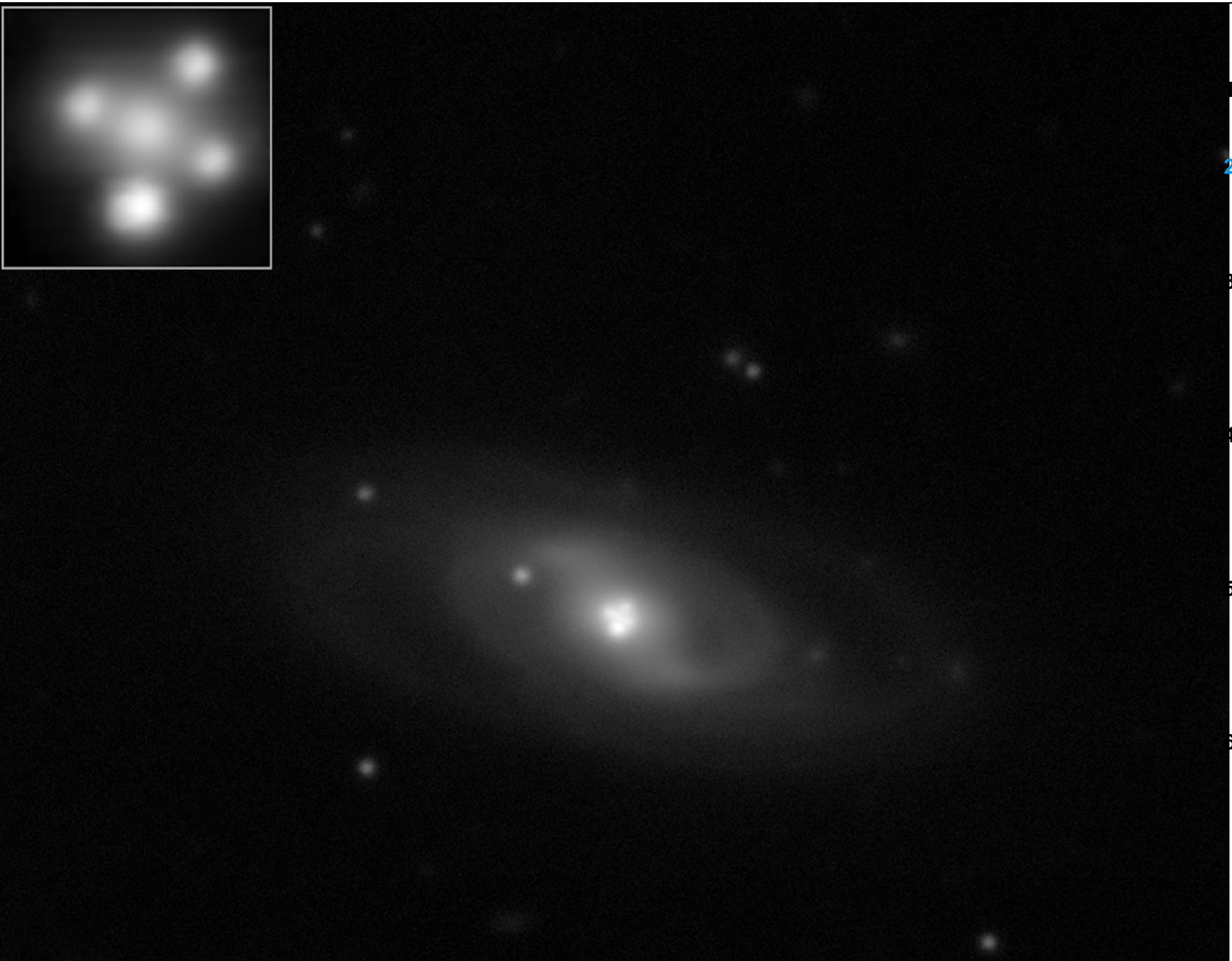


2. Strong Lensing



SDSS J1004+4112

2. Strong Lensing



1. Microlensing

2. Strong lensing

3. Problems in extragalactic microlensing

4. Measuring extragalactic microlensing

5. Detection of extragalactic MACHOs

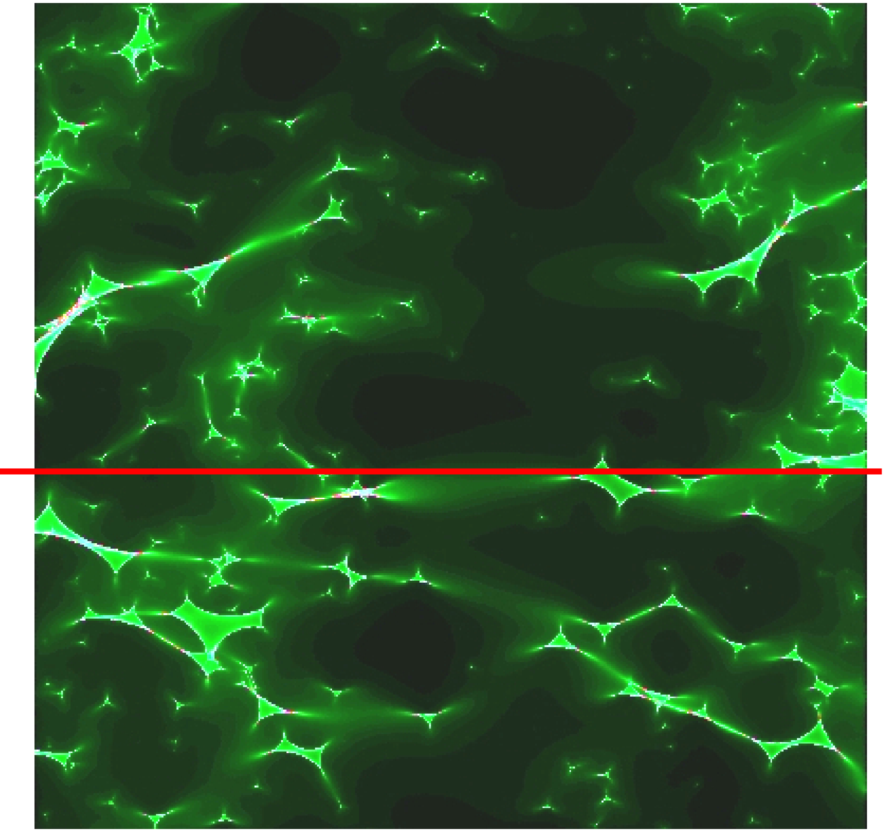
6. Quasar disk size and structure

3. Extragalactic microlensing: difficulties

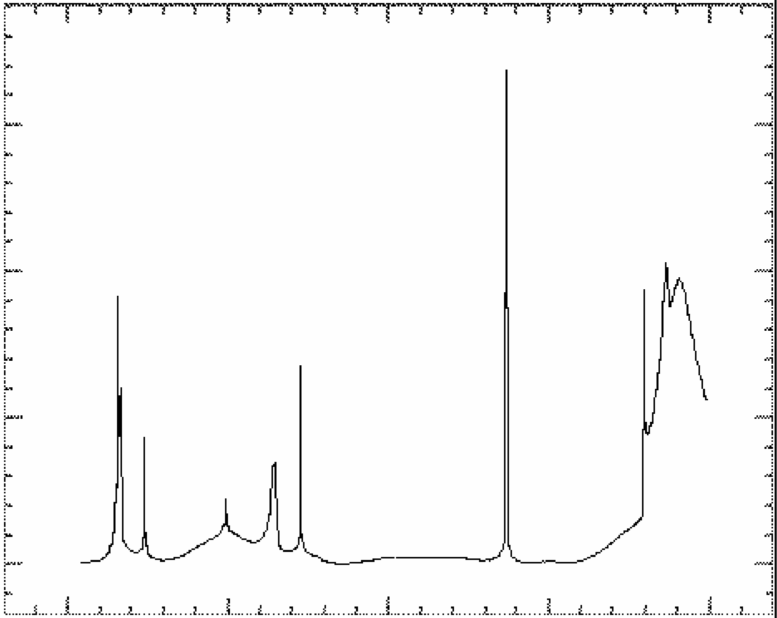
3. Extragalactic microlensing: difficulties

Detecting extragalactic microlensing events is not straightforward:

1. Unknown distribution of multiple deflectors make light curve complex and difficult to interprete (big degeneration).
2. Timescales too long (months, even years)



Multiple point-deflector

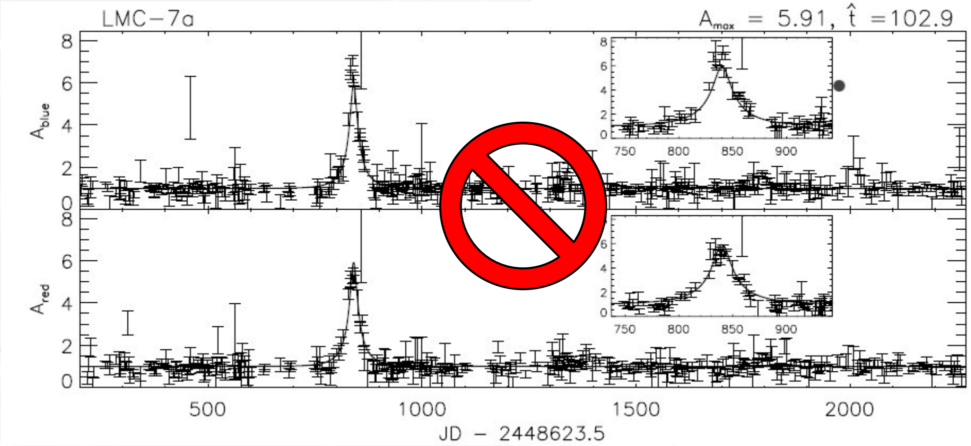


3. Extragalactic microlensing: difficulties

Detecting extragalactic microlensing events is not straightforward:

3. Exact macrolens amplification is unknown, since the exact mass distribution in the lens galaxy/ cluster is unknown. We don't know original source flux either.

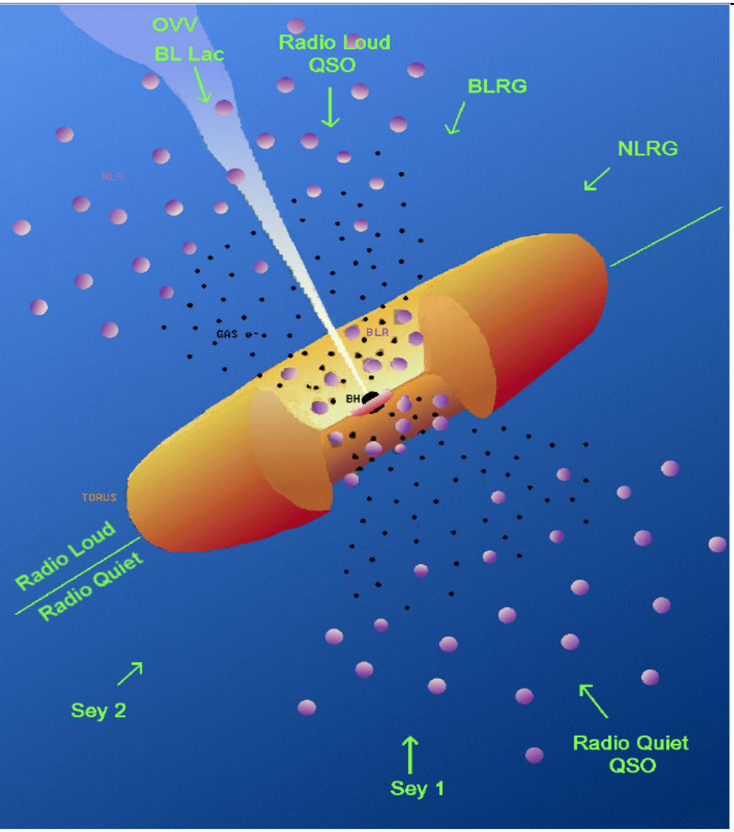
Therefore, we lack the baseline of no microlensing amplification.



Detecting (1) and getting information (2) from extragalactic microlensing require a different approach

4. Measuring extragalactic microlensing

4. Measuring extragalactic microlensing

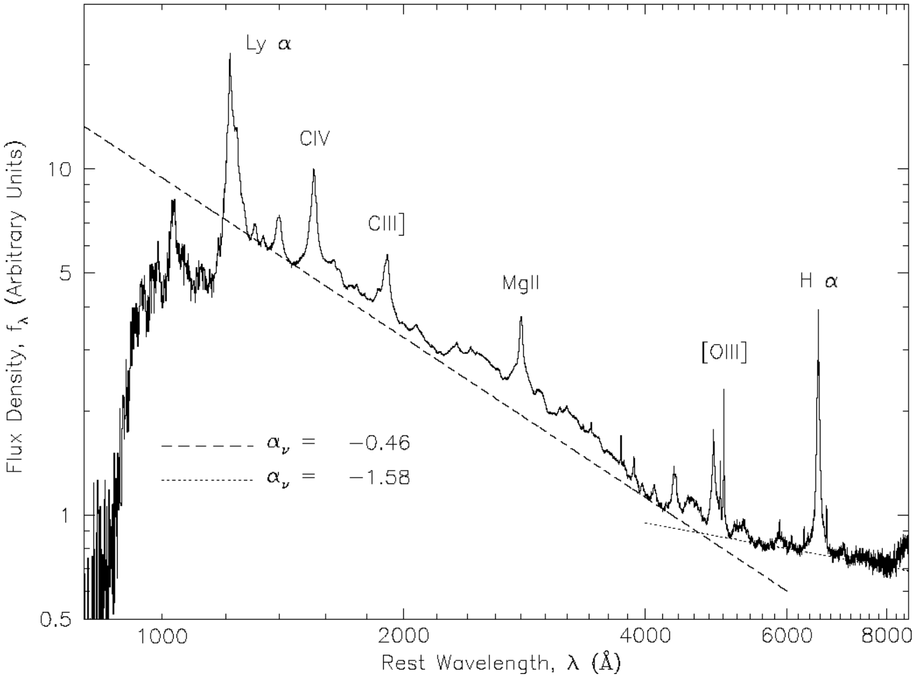


Why QSOs are so good for microlensing

- NEL originate in large regions

  They are not affected by ML
- Continuum source is a small,

  plays the role of source star.



Composite SDSS QSO spectrum

2001AJ....122..549V

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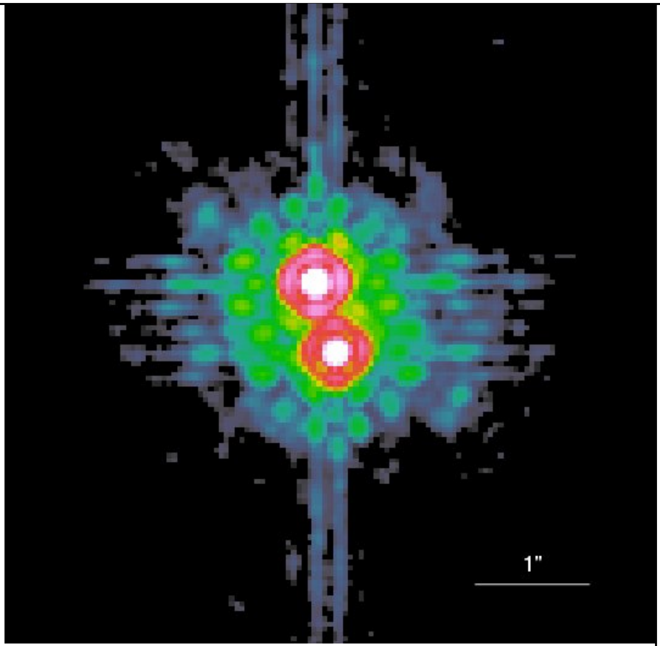
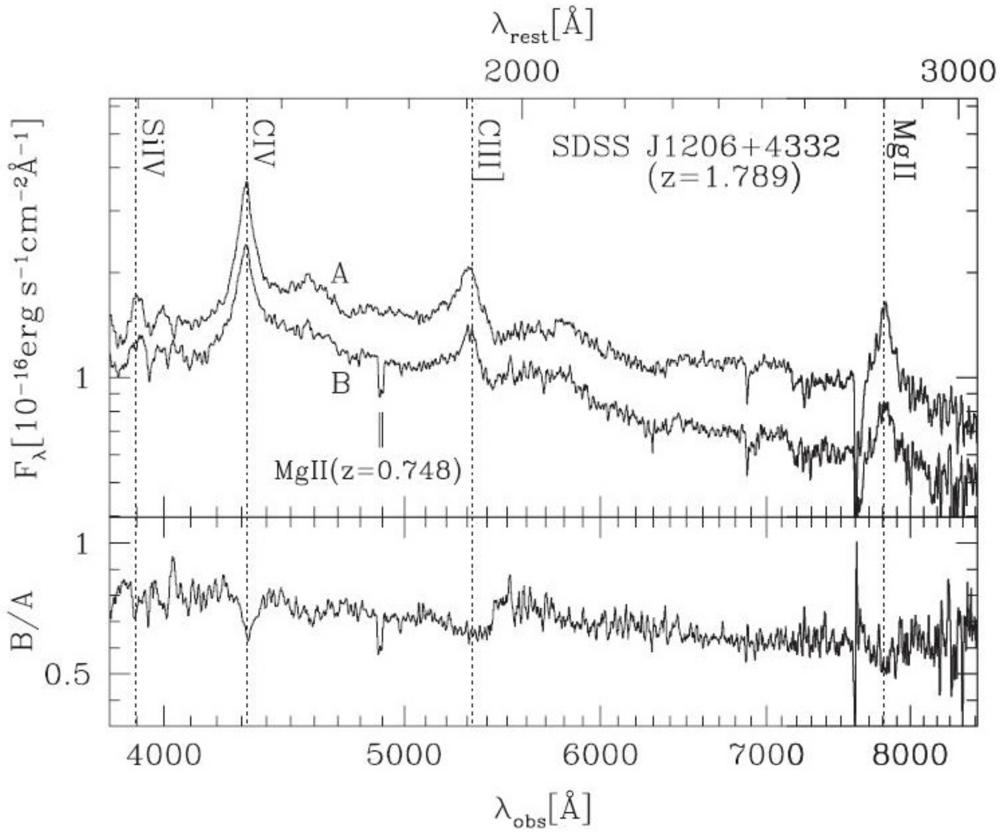
4. Measuring extragalactic microlensing

Therefore the clue for an ongoing microlensing event is finding different flux ratios for lines and continua between two images, sinde only continua are affected by microlensing.

- NEL region provides baseline of no microlensing amplification.



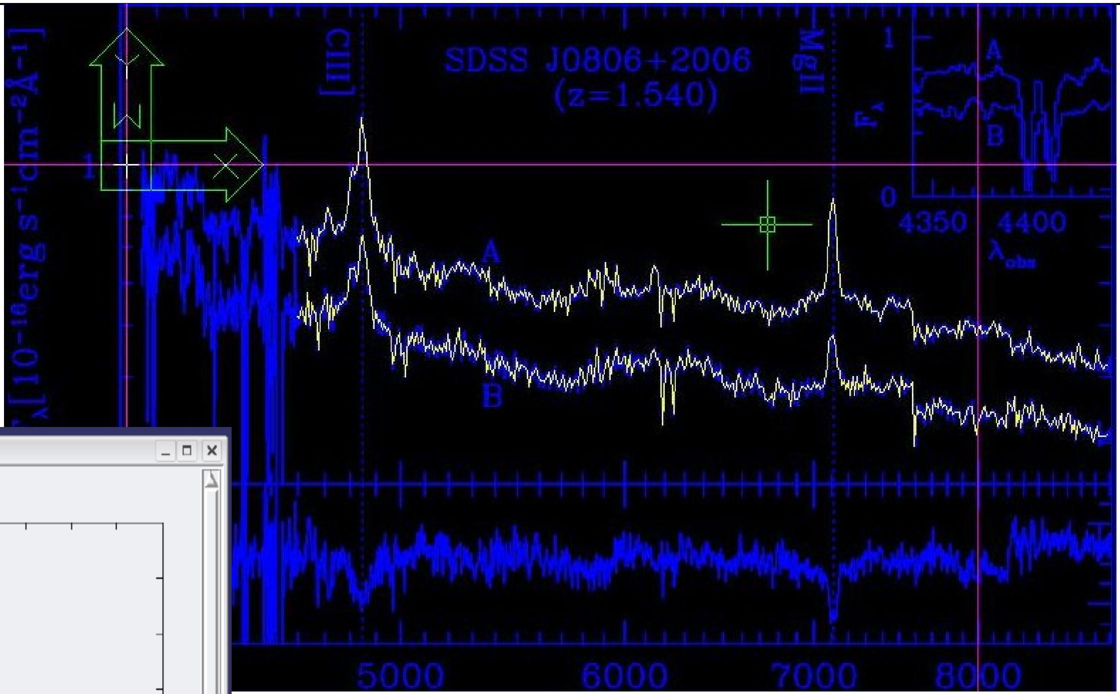


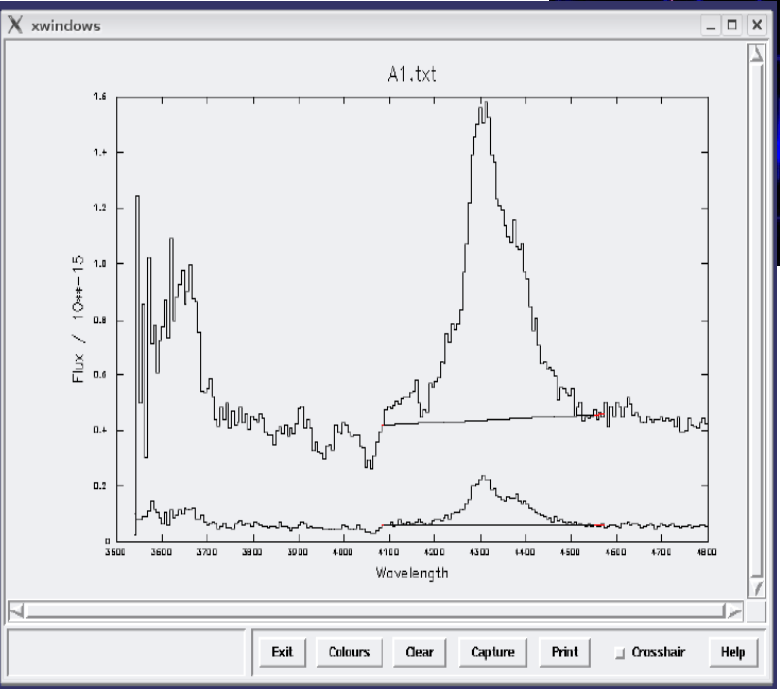
However, only magnification differences between images will be measured

4. Measuring extragalactic microlensing

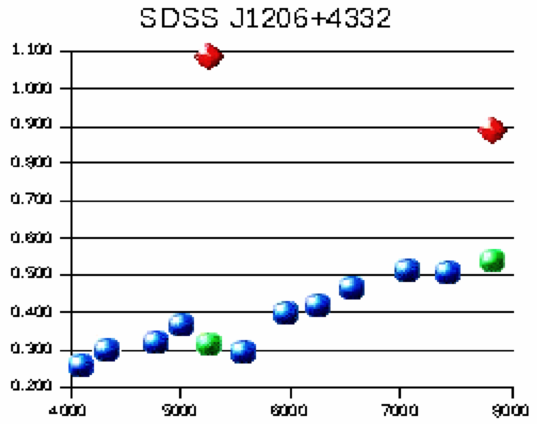
Measuring

Up to now, there are separated spectra for ~ 30 image pairs seen through 20 lens galaxies





After *local* continuum subtraction is performed, we do calculations for flux ratios among the continuum spectrum and the different lines



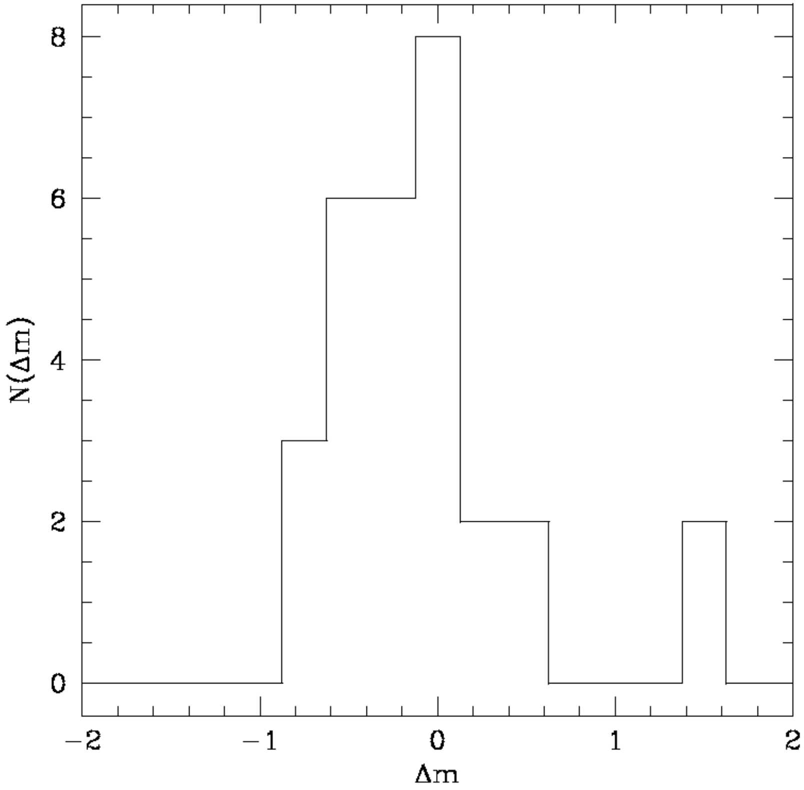
- Available MID-IR data for some systems confirm the reliability of the optical line flux ratios as baseline

4. Measuring extragalactic microlensing

Histogram for the measured *differential* microlensing magnifications:

- It peaks close to no differential magnification

- It is highly concentrated below 0.6 mag



This histogram is a realisation. We must compare it with a set of a-priori simulated distributions from which we can get statistical estimators -> Bootstrap

5. Detection of extragalactic MACHOs

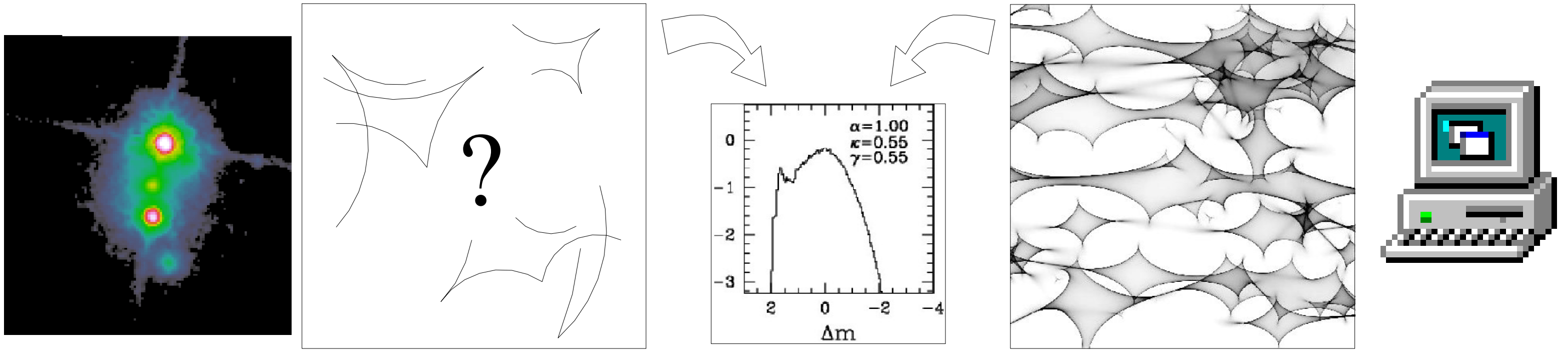
Main idea: modelling realistic magnification difference histograms for a wide range of compact objects densities and comparing them with the observational histogram

Section 5 describes this method and the results obtained. It is a (limited) summary of the work by Mediavilla, E. et al. published in ApJ under the title "*Microlensed-based Estimate of the Mass Fraction in Compact Objects in Lens Galaxies*" *(2009ApJ…706.1451M)*

5.a: Modelling probability distributions

Starting point:

We cannot know how the "real" magnification maps are, but a simulated map with the same local conditions should have the same magnification histogram as the "real" one.



Every map is dependent on 3 dimensionless numbers: the mean surface density κ, the surface density in stars $\kappa\ast$, and the (tidal) shear γ

The first two parameters are set by means of a macrolens model for each system, from which to obtain the local conditions.

The third parameter, mass fraction in stars (compact objects) is needed for computing the maps, so we have to **generate a set of possible values** and somehow choose the value that best matches the real data (the observational histogram)

1. Microlensing
2. Strong lensing
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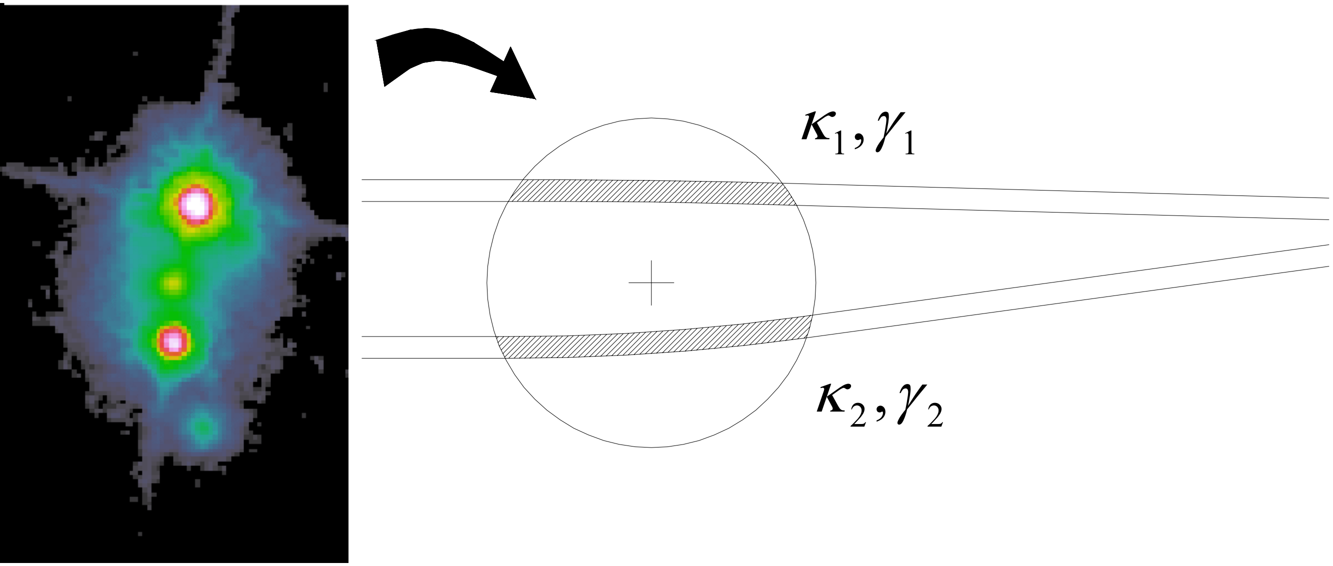
5.a: Modelling probability distributions

By fitting image positions in a singular isothermal sphere plus external shear (SIS+γ_e) macrolens model we obtain projected matter density κ and shear Y for every image in every system

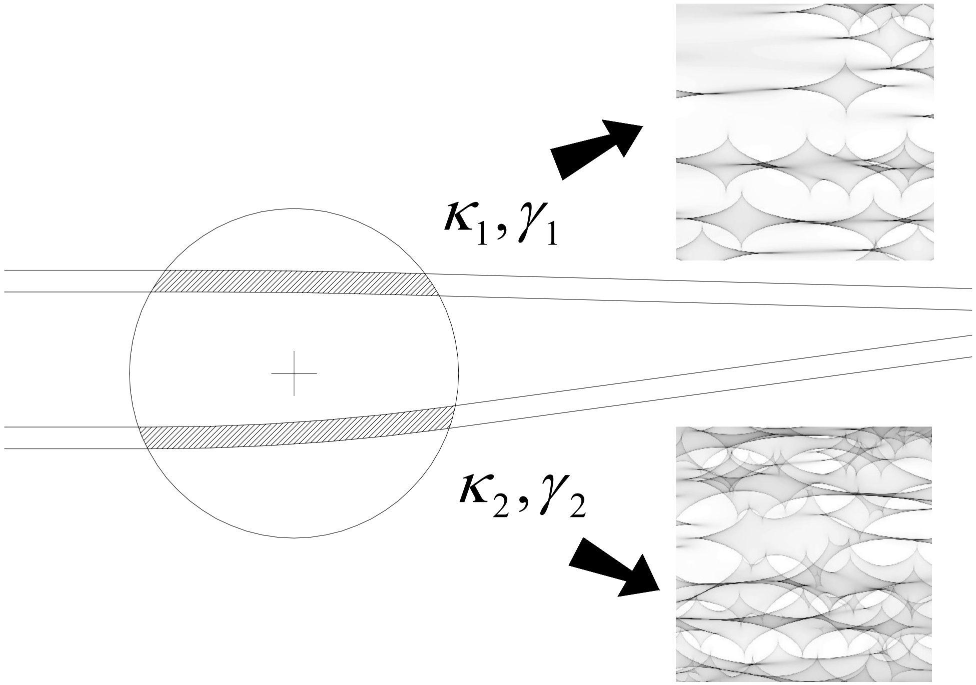


We used the "lensmodel" code by Keeton (2001)

http://www.cfa.harvard.edu/castles/

5.a. Modelling probability distributions

For every par of values *and a given mass fraction in point-deflectors*, a magnification map is computed



- To compute the magnification maps we use the inverse polygon mapping method (Mediavilla et al. 2006)

(!) To account for the extended (though small) nature of the source we blur every map by means of convolution with a 2D gaussian profile

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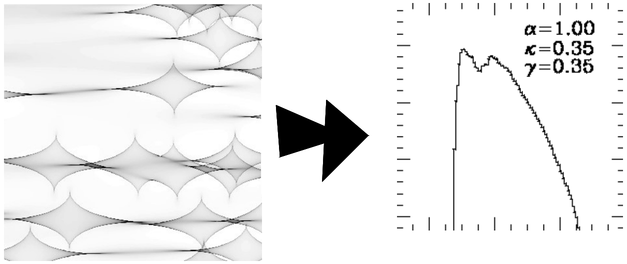
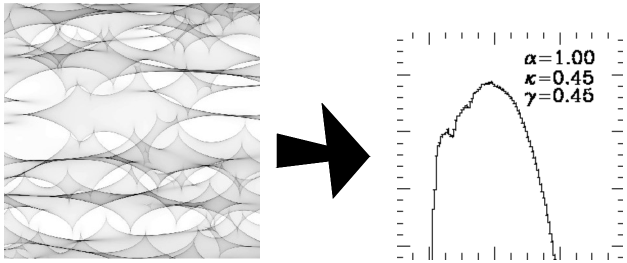
5.a. Modelling probability distributions

Every magnification map results in a histogram of magnifications



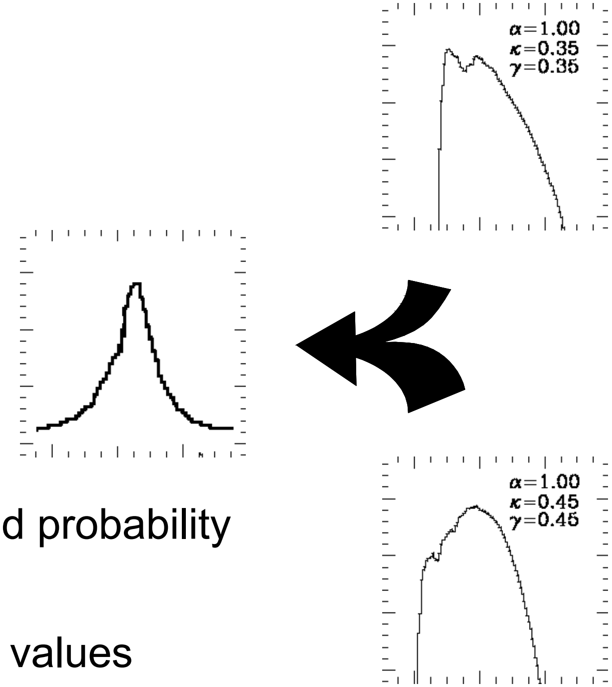


The microlensing magnification at a given pixel is obtained as the ratio of the magnification in the pixel to the average magnification.

This histograms give the frecuency distribution of microlensing magnifications.

5.a. Modelling probability distributions

Since we are interested in the frequency distribution of the difference in microlensing magnification between pairs, we do a final crosscorrelation of the magnification histograms:



$$f_{\alpha\kappa_1,\alpha\kappa_2,\kappa_1,\kappa_2,\gamma_1,\gamma_2}(\Delta m) =$$

$$\int f_{\alpha\kappa_1,\kappa_1,\gamma}(\Delta m_1) f_{\alpha\kappa_2,\kappa_2,\gamma_2}(m_1 - \Delta m) dm_1$$

- Everyone of this distributions give the normalized probability for measuring any magnification difference.
- There is one distribution for every set of the five values

$(\alpha, \kappa_1, \gamma_1, \kappa_2, \gamma_2)$ (α = mass fraction of compact objects)

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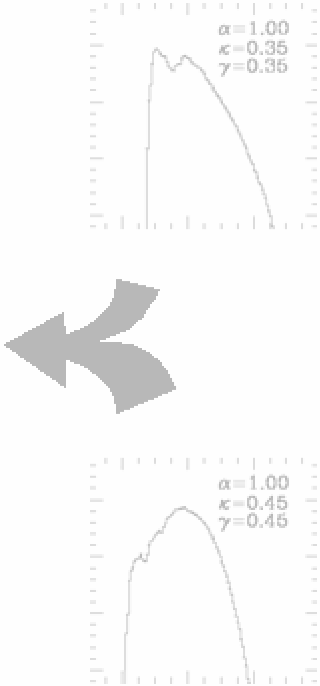
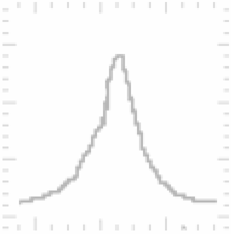


$$f_{\alpha\kappa_1,\alpha\kappa_2,\kappa_1,\kappa_2,\gamma_1,\gamma_2}(\Delta m) =$$

$$\int f_{\alpha\kappa_1,\kappa_1,\gamma}(\Delta m_1) f_{\alpha\kappa_2,\kappa_2,\gamma_2}(m_1 - \Delta m)\, dm_1$$



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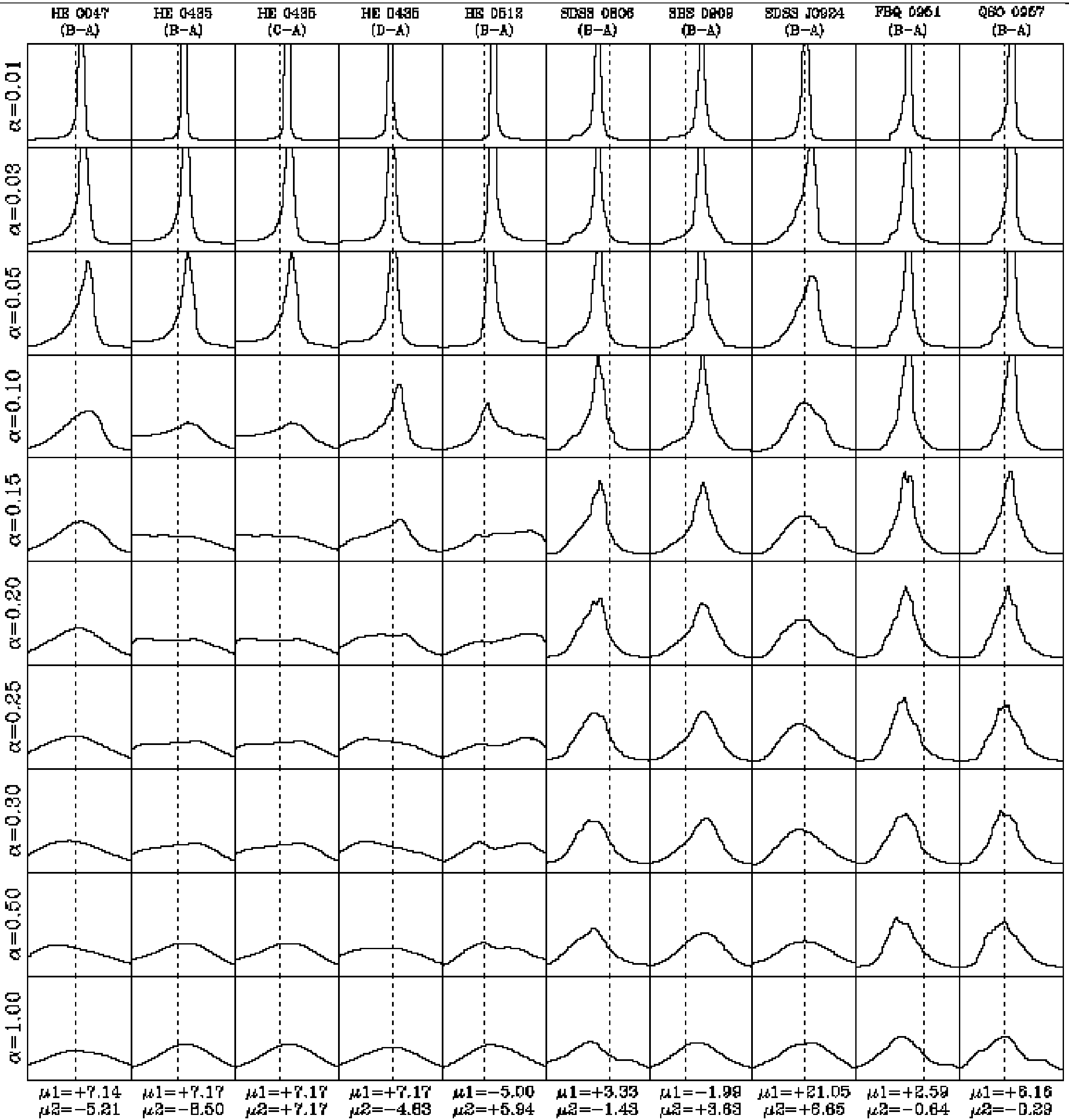
$(\alpha, \kappa_1, \gamma_1, \kappa_2, \gamma_2)$ (α = mass fraction of compact objects)

<u>Summary:</u>

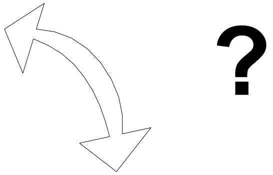
Through computer modelling and simulation, we are able to infer the probability distribution of differences in microlensing for each system, with the mass fraction of compact objects as an input parameter.

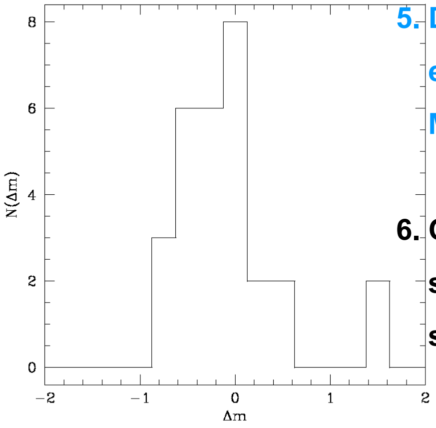
5.b. Chi square test



Standard tool in statistics for a comparison between probability functions.

Which ones best match the observational histogram ?





1. **Microlensing**
2. **Strong lensing**
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5.b. Chi square test

- This test tries to find the value for α for which the probability distributions most resemble the observational histogram
- For each value of α, the sum of the cuadratic distances between between modeled and measured values in the observational histogram is computed. The minimum value identifies the best candidate.

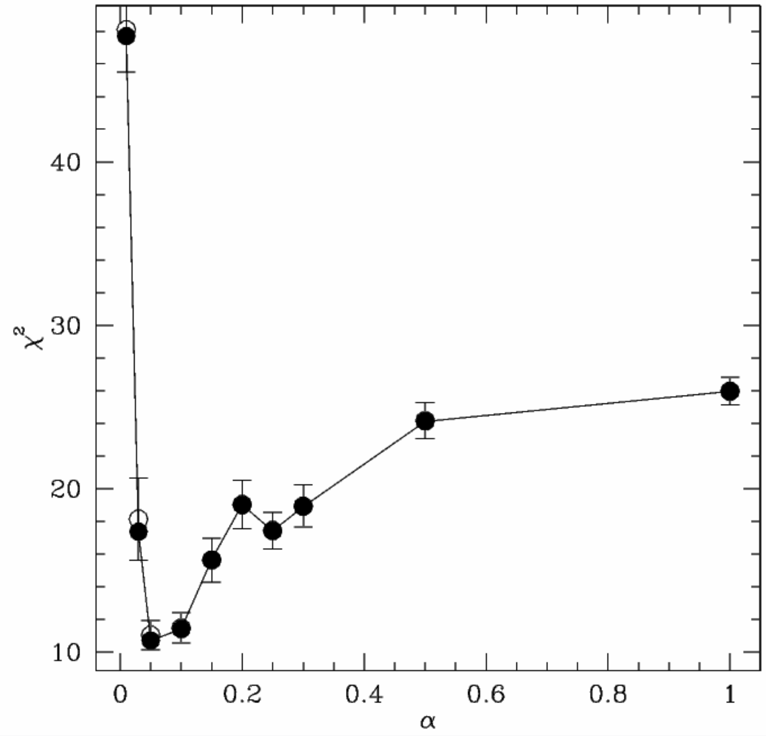
$$\chi^2_\alpha = \sum_i \left(\frac{f_\alpha(\Delta m_i) - f_{obs}(\Delta m_i)}{\sigma_i} \right)^2,$$

The best match corresponds to

α = 5% aprox

of halo mass in compact objects

Errorbars result from a montecarlo algorithm based on permutations of the system values



5.c. Maximum Likelihood Analysis

Our 29 microlensing measurements are a specific realization of the prediction made by the computed distributions. We may ask: *how similar* to the predicted most likely set of values is our realization?

We search the value of α for which that "similarity" is maximum.

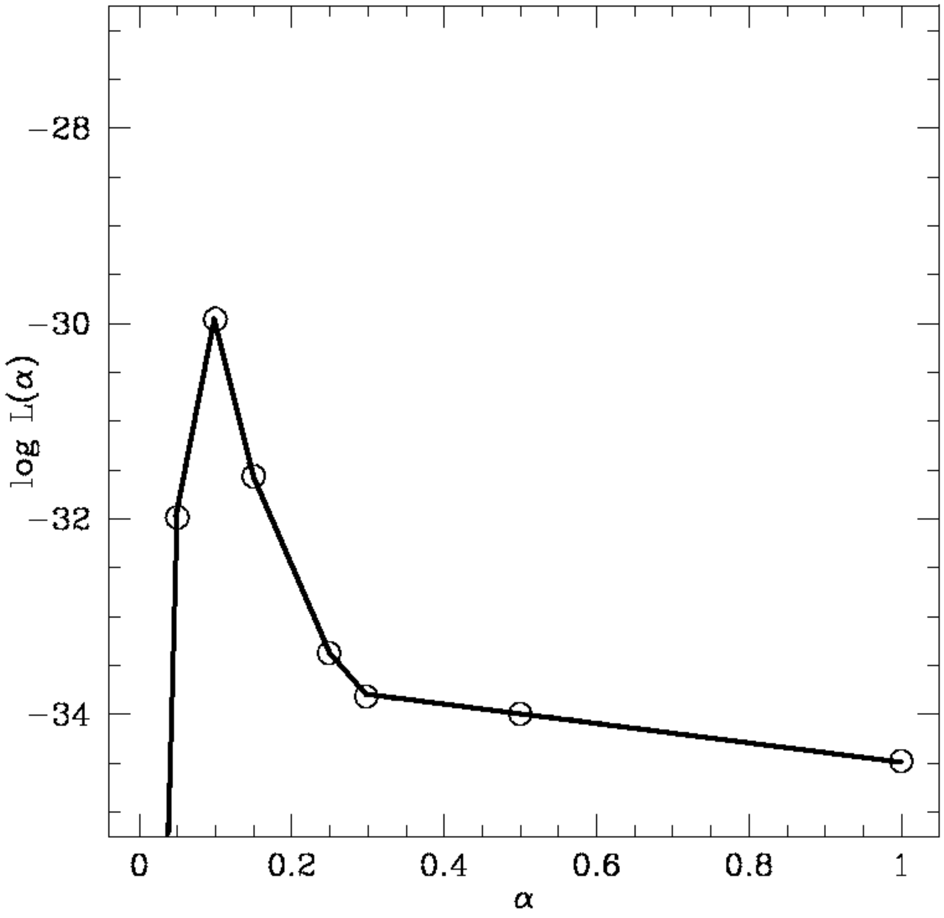
- We get from the distributions which frequency corresponds to the observed magnification difference in each system,

$$f_{\alpha\kappa_1,\alpha\kappa_2,\kappa_1,\kappa_2,\gamma_1,\gamma_2}(\Delta m)$$

- Then we obtain the likelihood function for the 29 measurements of the sample:

$$\log L(\alpha) = \sum_{i=1}^{29} \log f^{i}{}_{\alpha\kappa^{i}{}_1,\alpha\kappa^{i}{}_2,\kappa^{i}{}_1,\kappa^{i}{}_2,\gamma^{i}{}_1,\gamma^{i}{}_2}(\Delta m^{i})$$

5.c. Maximum Likelihood Analysis

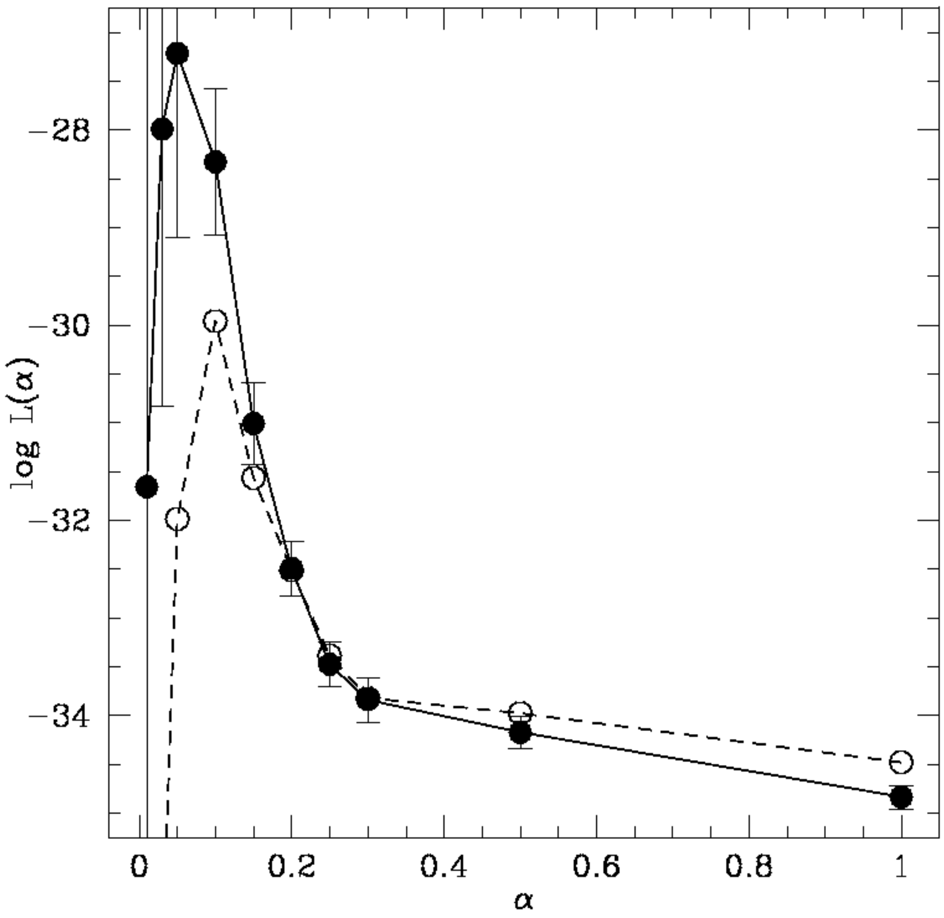


The likelihood function peaks at a value of

$\alpha = 0.10 \pm 0.04$ at 90% confidence interval

using the $\log L(\alpha \pm n\sigma_\alpha) \sim \log L_{max} - n^2/2$ criterion

5.c. Maximum Likelihood Analysis



By considering each microlensing measure as a normal distribution of σ=0.20 we account for realistic errors in the detemination of the microlensing differences.

In that case, the analysis yields a value of 0.05 for the mass fraction in MACHOs

5.d. Fixing the size of the source

$r_{s0} = 0.8 \times 10^{15}\, cm$

🟩 $(0.3\, ld)$

$r_{s0} = 3.9 \times 10^{15}\, cm$

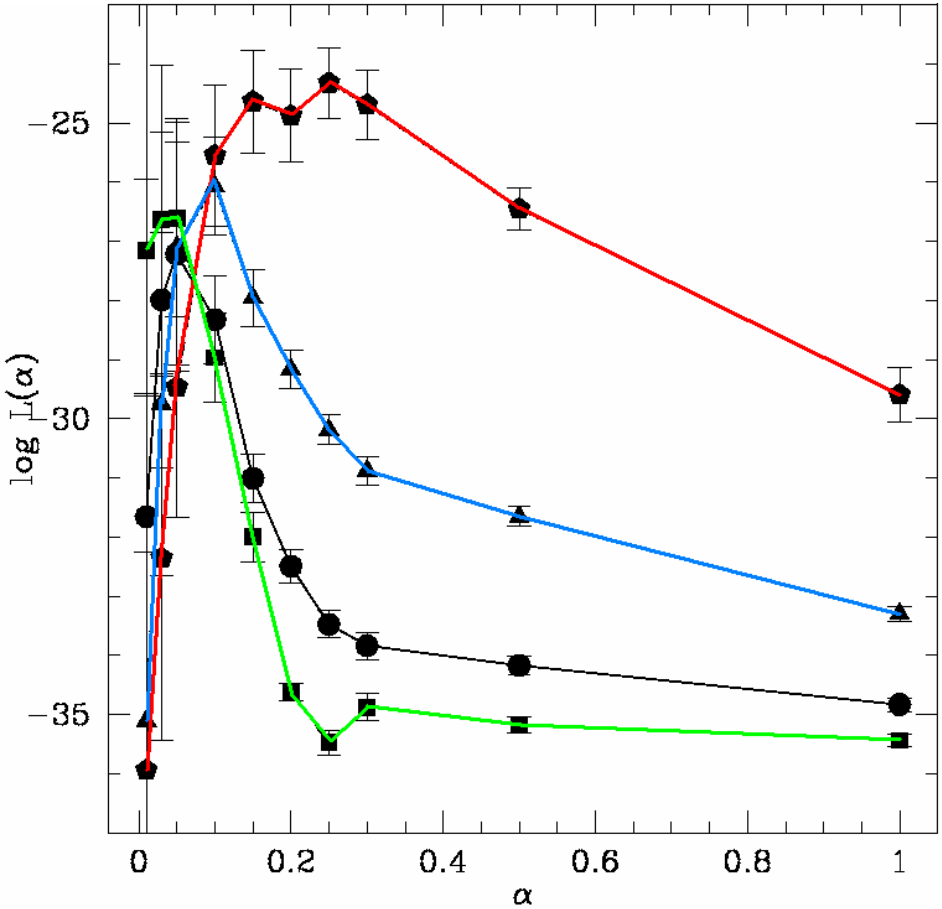
⬛ $(1.5\, ld)$

$r_{s0} = 12.4 \times 10^{15}\, cm$

🟦 $(4.8\, ld)$

$r_{s0} = 40 \times 10^{15}\, cm$

🟥 $(15\, ld)$



Changing the source pixel size or increasing the gaussian representing the continuum source affects by blurring the magnification maps and therefore the probablility distributions. We have chosen to model four sizes for the source plane deprojected size parameter.

Accretion disk size determined by Morgan et al. (2007) and Pooley et al. (2007) matches our range of results for α between 0.05 and 0.10

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5.d. Conclusions about extragalactic MACHOs

- We have extended up to the **extragalactic domain** the local (LMC/ LMC/ M31) use of microlensing to probe the properties of the galactic halos.

- Regarding the current controversy about local microlensing DM studies, our work supports the hypothesis of a **very low content in MACHOs (~5%)**

- In fact, QSO microlensing probability arises from the normal star populations and, according to our work, **there is no statistical evidence for MACHOs** in the dark halos.

6. Size and Internal Structure of Quasar Accretion Disks

Main idea: to derive the radial dependence of temperature and size of the accretion disk in the case of SBS 0909+532 by measuring the wavelength dependence of the microlensing magnification detected.

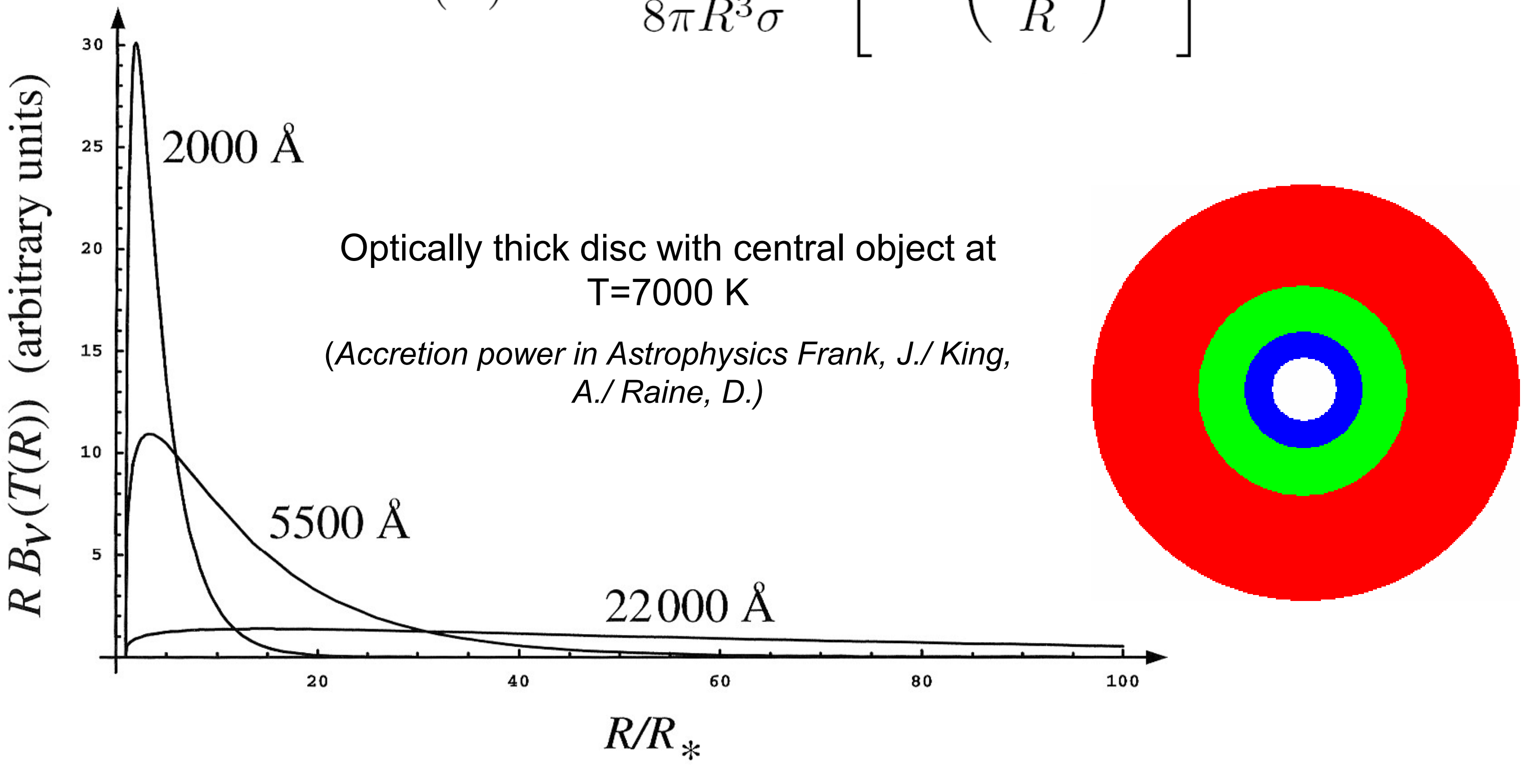
In this section we merely mention the underlying principles which the current work of the group is based upon.

6a. Evidence for thermal structure

What we mean by 'thermal structure'

The standard thin accretion disk model of a quasar (Shakura & Sunyaev 1973) consists of a black hole surrounded by a thermally radiating disk with a temperature profile:

$$T(R)^4 = \frac{3GM_{BH}\dot{M}}{8\pi R^3\sigma}\left[1-\left(\frac{R_{in}}{R}\right)^{1/2}\right]$$

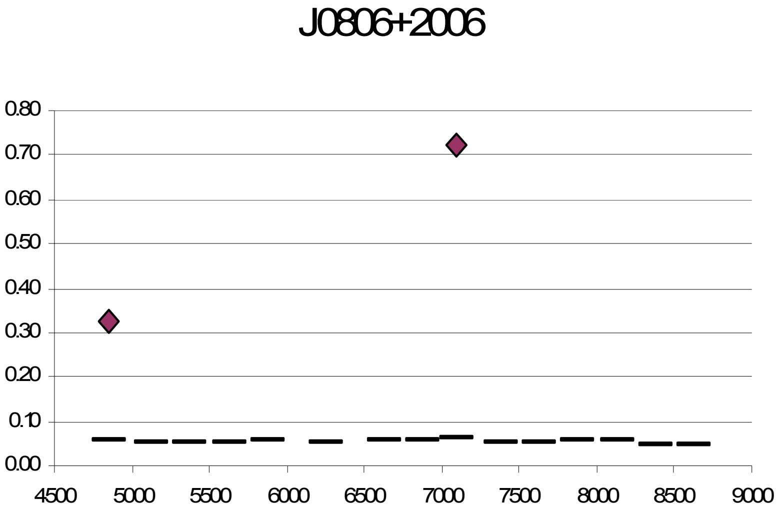


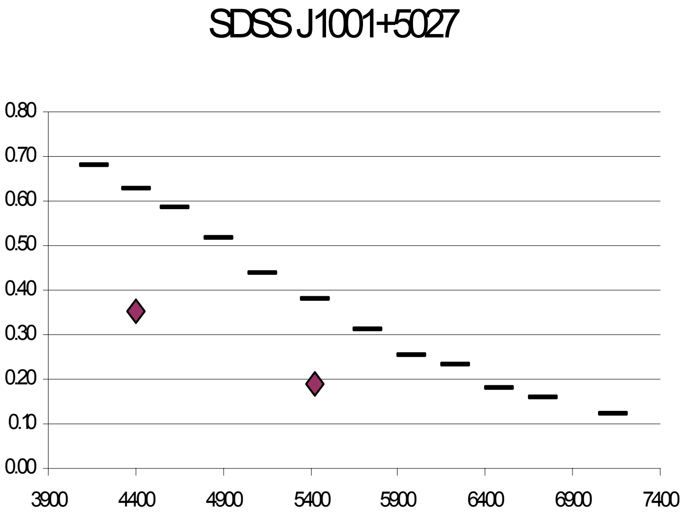
Optically thick disc with central object at T=7000 K

(Accretion power in Astrophysics Frank, J./ King, A./ Raine, D.)

6a. Evidence for thermal structure

Different source sizes (with wavelegth) give different microlensing measures (with wavelenght !)





The smaller the source region the more sensitive to microlensing

Cromaticity in the continuum ratio is the microlensing signature of the thermal structure of the accretion disc

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7. If you want to know more…

Ultra-short introduction to the very basics of microlensing:

Introduction to Gravitational Microlensing

Shude Mao

2008arXiv0811.0441M

Quite complete and rigourous document, yet easy to read at the same time:

Lectures on Gravitational Lensing

Ramesh Narayan - Matthias Bartelmann

1996astro.ph..6001N

About our work with MACHOs and microlensing:

Microlensing-based Estimate of the Mass Fraction in Compact Objects in Lens Galaxies

Mediavilla et al.

2009ApJ...706.1451M / arXiv:0910.3645