Numerical evolution of spinning black holes and the detection of their gravitational wave signals

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arXiv:0909.2867: **P Ajith, M Hannam, SH**, Y Chen, B Brügmann, N Dorband, D Müller, F Ohme, D Pollney, C Reisswig, L Santamaria, J Seiler
arXiv:1005.3306: **L Santamaria, F Ohme, P Ajith**, B Brügmann, N Dorband, **M Hannam, SH**, P Mösta, D Pollney, C Reisswig, E Robinson, J Seiler, B Krishnan
arXiv:1007.4789 **M Hannam, SH, F Ohme**, D Mueller, B Brüegmann
arXiv:1008.2961, **M Hannam, SH, F Ohme, P Ajith**

ERE 2010, Granada, September 2010

Simulations performed at Mare Nostrum-BSC, CESGA-Santiago de Compostela, LRZ Munich, Vienna Scientific Cluster, ICHEC Dublin, Kraken/Teragrid

General Relativity is becoming observational science!

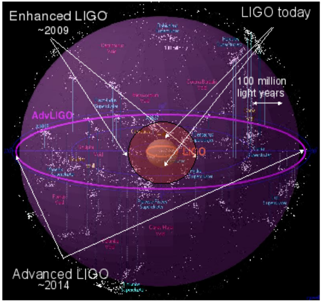
Second long science run (enhanced LIGO/Virgo) close to finishing.

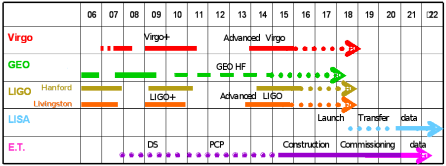
First & routine GW detection expected 2015-2017 with advanced detectors.

GW astronomy ≈ 2 PhD theses away. . .

Are theorists ready for GW detection & GW science?

NS-NS binaries: expect PN theory is sufficiently accurate to detect inspirals & determine physical parameters.





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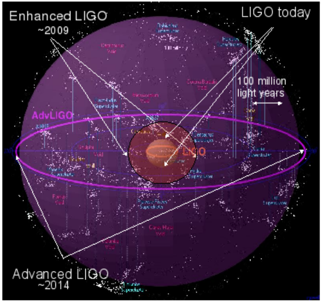
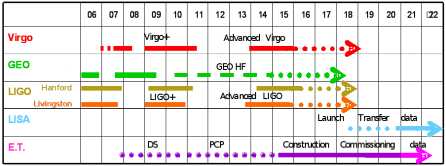




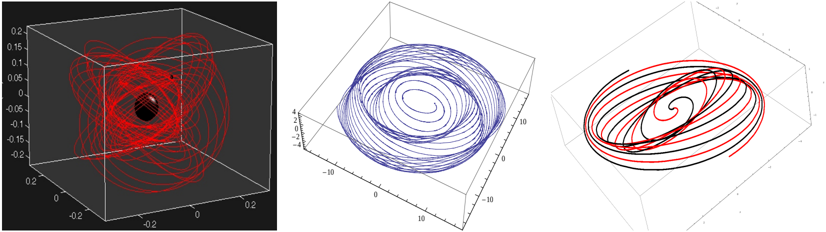
For BH-BH (BH-NS) binaries ($\gtrsim 10 M_\odot$) this is far from clear!

How to search for BBH coalescence?

- For comparable mass BBH spiral/merger, negligible eccentricity events are expected to be the “bread and butter” for GW observations.
- Unequal masses and large spins are expected to be typical, precession quite possibly typical.
- Current search templates mostly based on frequency domain versions of PN WFs for quadrupole mode from nonspinning binaries.
- How accurate is PN theory? PN does not provide intrinsic error control!
- NR faces a vast parameter space, computational cost depends steeply on mass ratio, spin magnitude and initial frequency.
- Searches for the full spinning parameter space currently not feasible – false alarm rate rises with size of parameter space.

Modeling spinning BBHs

Dynamics of precessing binaries can become extremely complicated: EMRIs!



Very simple dynamics is possible when the spins are aligned with the orbital momentum – no precession, but inspiral rate and angular velocity dominated by

$$\vec{\chi}_i \cdot \vec{L}, \qquad \vec{\chi}_i = \vec{S}_i / M_i^2$$

Our approach: model the simple family $\chi_1 = \chi_2 = \chi$ (only 1 spin-parameter):

- Construct hybrid WFs by connecting PN and NR, represent hybrids by phenomenological analytical expression.
- What are the (dominant) sources of error in our analytical waveform family?
- How far do we get in modeling general configurations?

Summary of non-precessing simulations

q	S_i/M_i^2	D/M	p_t/M	$-p_r/M(\times 10^{-4})$	e	N_{GW}	t_{max}/M	a_f/M_f
1	-0.85	13.0	0.084542	5.247	0.0025	16	1868	0.412
1	-0.75	13.0	0.084057	5.060	0.0016	17	2036	0.446
1	-0.50	12.5	0.085124	5.258	0.0029	18	2065	0.531
1	-0.25	12.0	0.086312	5.623	0.0025	18.5	1955	0.609
1	0	12.0	0.085035	5.373	0.0018	19	1939	0.686
1	0.25	12.0	0.083813	0	0.0061	21.5	2129	0.760
1	0.50	11.0	0.087415	0	0.0061	20	1739	0.832
1	0.75	10.0	0.091435	0	0.0060	19	1432	0.898
1	0.85	10.0	0.090857	0	0.0050	20	1492	0.915
2	0	10.0	0.085599	7.948	0.0023	12.5	1069	0.623
3	0	10.0	0.072408	5.802	0.0016	14.5	1240	0.540
4	0	10.0	0.061914	4.333	0.0038	17	1461	0.471

Numerical accuracy - I

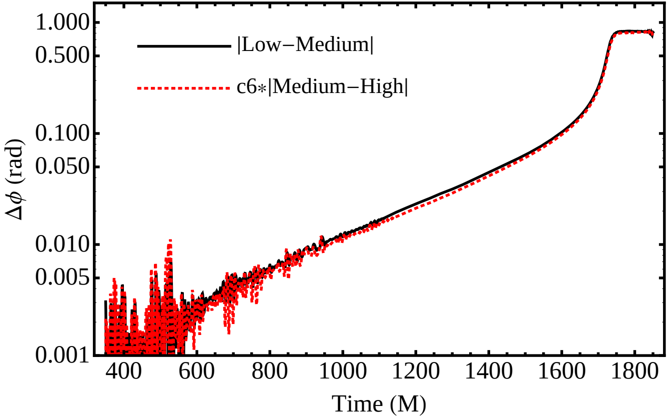


Figure: 6th-order convergence of the phase as a function of time for the $\chi_i = 0.5$ case.

Numerical accuracy - II

Table: Estimates of uncertainty in phase and amplitude.

Case	Phase uncertainty (radians)			Amplitude uncertainty (percentage)		Mismatch ($\times 10^{-4}$)
	inspiral	merger	complete	inspiral	merger	
$q = 1$						
$\chi_i = +0.85$	0.1	2.10	10	0.25	5.0	2.8
$\chi_i = +0.50$	0.05	0.75	1.0	0.5	1.0	1.0
$\chi_i = -0.50$	0.1	0.80	10	1.0	4.0	0.8
$\chi_i = -0.85$	0.1	0.75	15	0.5	2.0	0.7
$q = 2$	0.05	0.2	5.0	0.2	1.0	0.3
$q = 3$	0.05	0.3	10	0.4	2.0	2.7
$q = 4$(a)	0.1	1.5	15	0.25	4.0	3.2
$q = 4$(b)	0.05	0.8	7.0	0.25	2.0	—

PN phase comparison - I

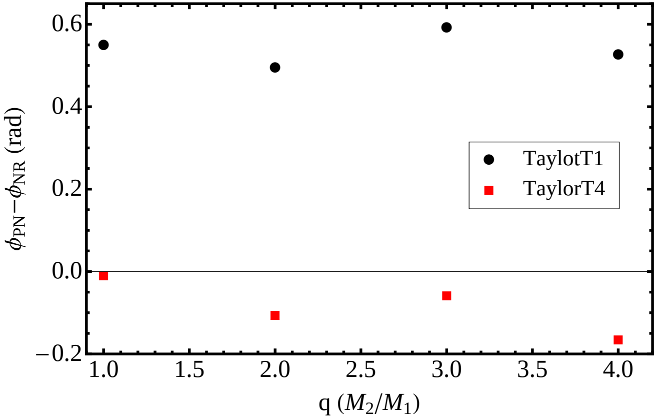


Figure: Non-spinning BBHs: TaylorT4 approximant shows robust accuracy, e.g. clearly outperforms TaylorT1.

PN phase comparison - II

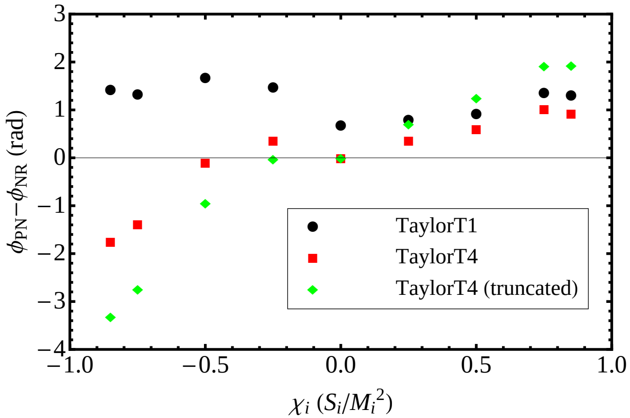


Figure: Equal-mass binaries with non-precessing equal spins: accumulated phase disagreement for the ten GW cycles up to $M\omega_m = 0.1$: TaylorT1 and TaylorT4 show comparable performance.

PN amplitude comparison

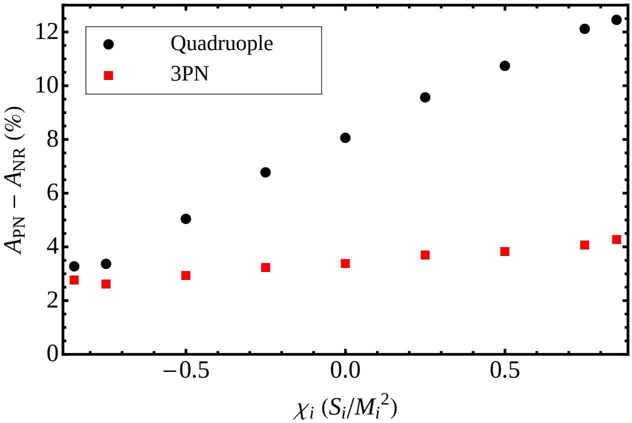


Figure: Average amplitude disagreement between PN and NR results, over the last ten cycles up to $M\omega = 0.1$. When the amplitude corrections are included up to 3PN-order, the PN amplitude error is only 3-4% for all spin values.

Accuracy of PN-NR hybrids: time vs. freq. domain

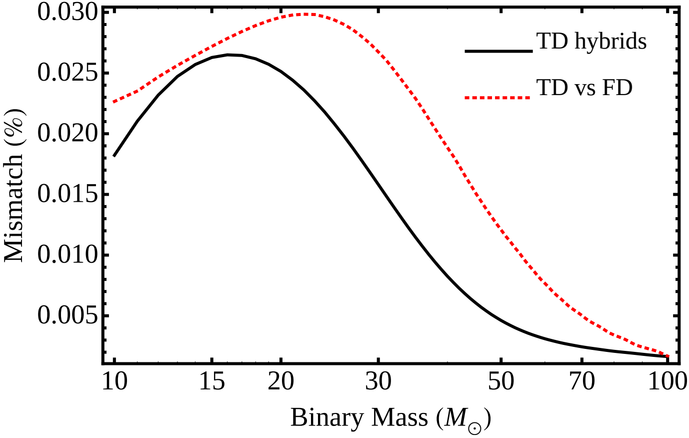


Figure: Mismatch between equal-mass nonspinning T4+NR hybrids constructed in the time-domain with matching frequency $M\omega_m = 0.07$, and in the frequency-domain hybrid matched at $M\omega = 0.079$.

Accuracy of PN-NR hybrids: vary matching frequency

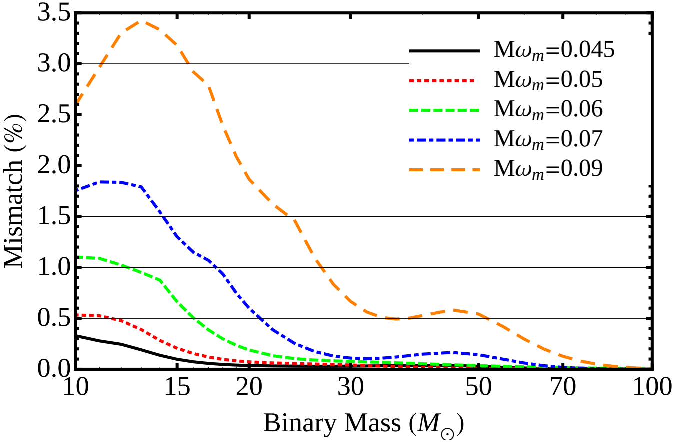


Figure: Mismatches between T1+$\mathrm{NR_L}$ and $\mathrm{NR_L}$ ≡T4+NR hybrids, for matching frequencies $M\omega_m = \{0.045, 0.05, 0.06, 0.07, 0.09\}$.

Accuracy of PN-NR hybrids: mismatch summary

Table: Minimum number of numerical GW cycles before merger that ensure mismatches below 3%, 1.5% and 0.5% for masses above 10 $M_\odot$ & lowest mass for which our WFs could be used for searches (mismatch $\leq$ 3%).

Configuration	$\mathcal{M} < 3\%$	$\mathcal{M} < 1.5\%$	$\mathcal{M} < 0.5\%$	$M_{\rm min}/M_\odot$
$\chi = -0.5$	8.0	10.0	19.0	10
$\chi = -0.25$	10.0	15.0	20.0	10
$\chi = 0$	7.0	9.5	15.0	10
$\chi = 0.25$	13.0	18.0	26.0	10
$\chi = 0.5$	20.0	26.0	36.0	15
$q = 2$	8.5	11.5	25.0	10
$q = 3$	11.0	15.5	25.0	10
$q = 4$	15.0	21.0	33.0	10

Accuracy of PN-NR hybrids: WF distinguishability

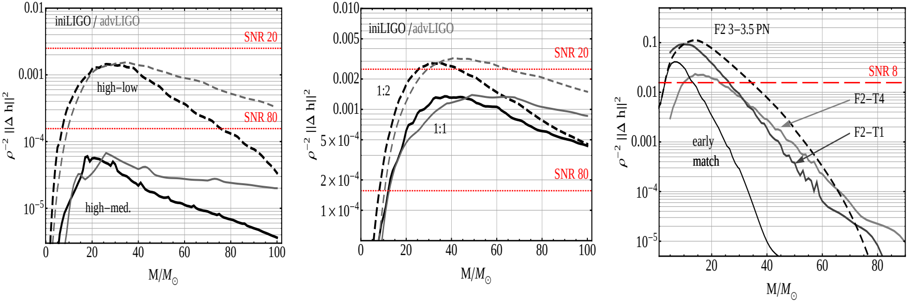


Figure: Left: hybrids from `Llama` equal-mass waveforms at different resolutions. Center: hybrids from `BAM` or `Llama` codes. Right: Initial LIGO's ability to distinguish hybrids constructed from different PN approximants: hybrids are not sufficient for detection at the $\epsilon = 0.03$ level only for a small range of masses.

Construction of phenomenological waveforms

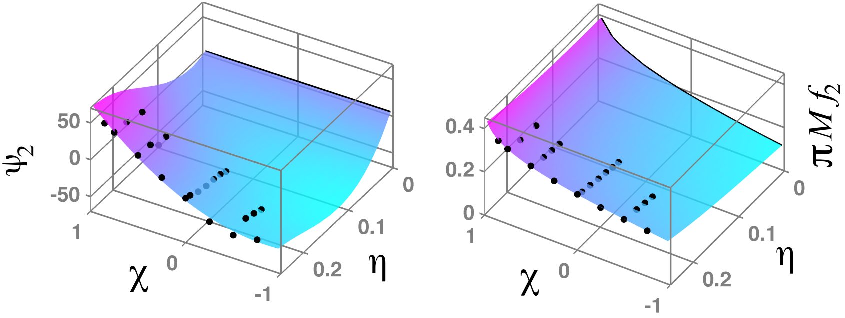


Figure: Phenomenological parameters ψ_2 and f_{ring} computed from *equal-spin* hybrid waveforms (dots). Test-mass limit is indicated by black traces.

Spinning vs. Nonspinning IMR WFs

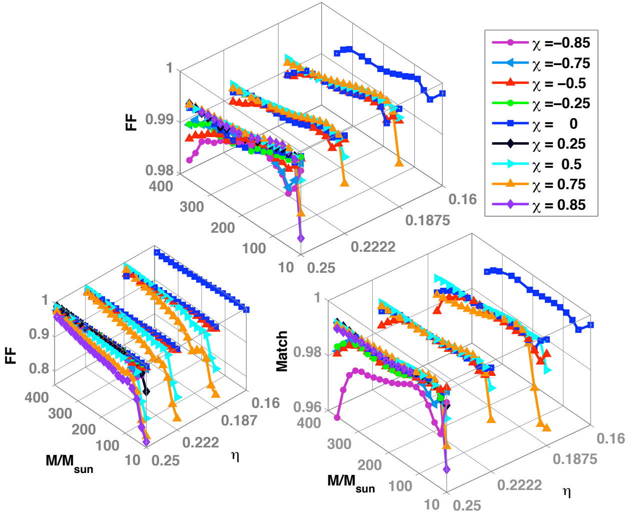


Figure: *Top and right*: Match and FF of analytical IMR templates with equal-spin hybrids. *Bottom left*: FF of *non-spinning* IMR templates with equal-spin hybrids → nonspinning templates loose up to 50% of events.

Equal-spin IMR WFs vs. unequal spin & precessing WFs

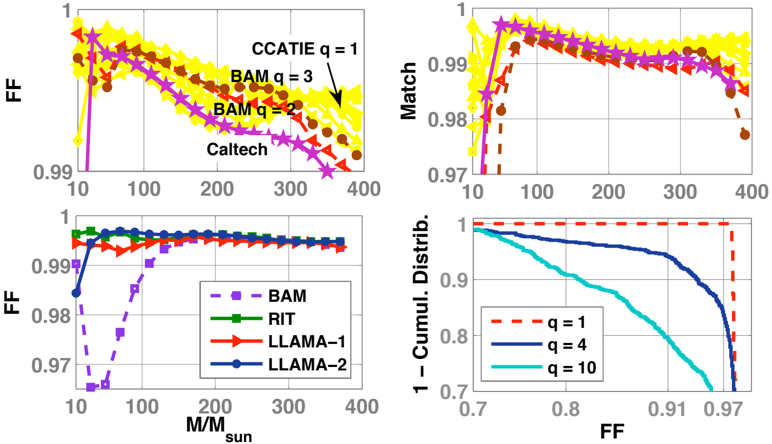


Figure: *Top:* Match & FF of our templates with *unequal-spin* hybrids. *Bottom left:* FF with some *precessing* hybrids. *Bottom right:* Fraction of generic PN waveforms ($M = 20M_\odot$) producing FF with out templates — 100% (84%) 65% of the binaries with $q = 1(4)10$ produce FF > 0.97.

Increased horizon distance with IMR WFs

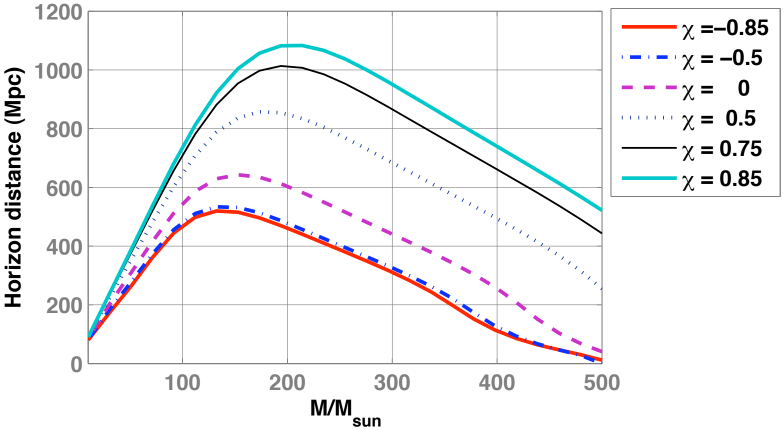


Figure: Distance to optimally-located and oriented- equal-mass binaries with (equal) spin χ producing optimal SNR 8 in Initial LIGO.

Conclusions

- Accuracy of phenomenological template families limited by accuracy of PN-NR hybrids.
- Accuracy of hybrids is determined by PN errors (or shortness of NR WFs).
- Higher mass ratios and larger spins parallel to the orbital angular momentum require longer WFs.
- We are now ready to optimize phenomenological WFs for accuracy.
- Parameter estimation and for larger mass ratios and spins also detection call for higher precision in PN (spin terms) – or *much longer* NR simulations.
- Nonspinning WFs may loose up to 50 % of event rate.
- Equal-spin non-precessing NR-based IMR waveforms are already used in GW detection/signal injection and may be able to detect WFs in a large portion of the BBH parameter space.