Spanish Relativity Meeting 2010@Granada

Osaka City University 130th 都市で学び 夢をつかむ

# Stable Bound Orbits around Black Rings


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arXiv: **1006.3129** [hep-th]


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[Plan of the talk]

I. Introduction
II. geometry of black ring
III. particle motion around a black ring

# Introduction

# *Introduction*

- Higher-dim. spacetime
  - ✓ inspired by unified theories
  - ✓ brane world model

  } Large extra dimensions $\ell \lesssim 0.1\mathrm{mm}$

- Higher-dim. Black Holes
  - ✓ Extra dims is "seen" by gravity
  - ✓ BH production in LHC ?
    - ➢ Asym flat BH solution gives a good approximation

$$\ell^{(D)}_{\mathrm{Plank}} \ll R_{\mathrm{BH}} \ll \ell$$

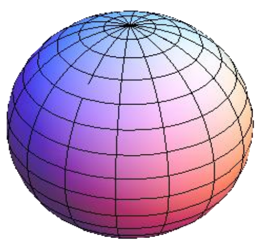
# Black Objects

- variety of horizon topology in $D \geq 5$

✓ $D = 4$

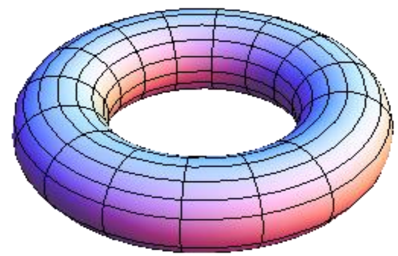
$(M, a)$ --uniquely--→ Kerr BH ($S^2$ horizon)

✓ $D = 5$



$S^3$

Black Hole
Myers & Perry (1986)



$S^2 \times S^1$

Black Ring
Emparan & Reall (2002)

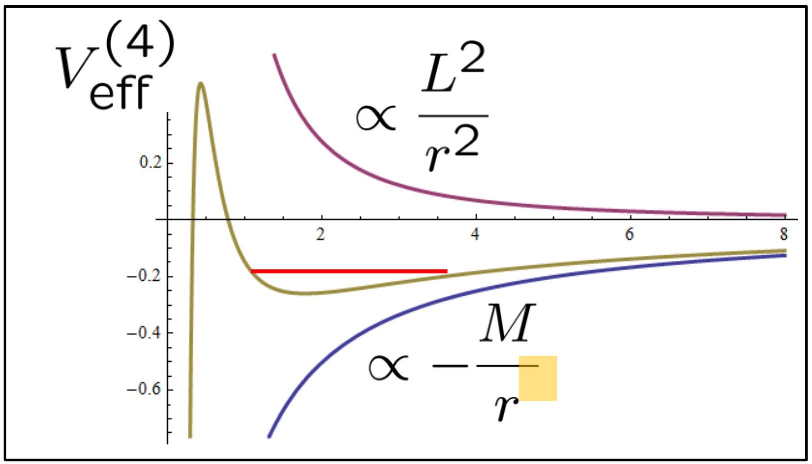
➢ Can we distinguish black objects by particle motion?

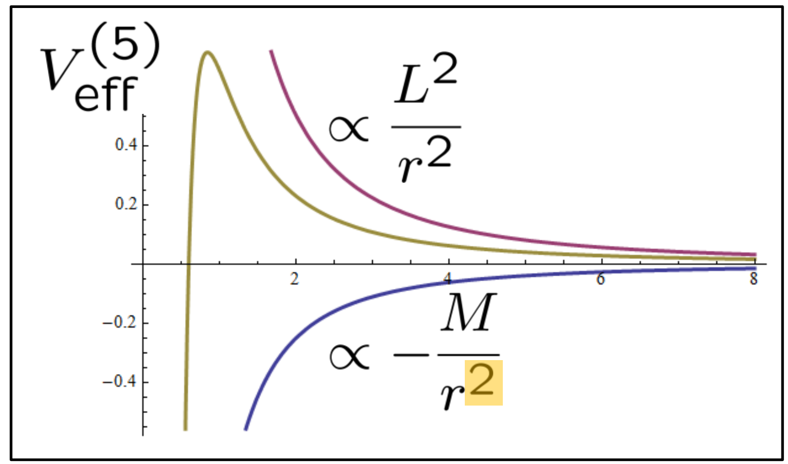
# *Particle Motion around a Black Hole*

■ D−dim. Schwarzschild BH case

$$V_{\text{eff}}^{(D)}(r) = \frac{L^2}{r^2} - \frac{M}{r^{D-3}} + O\left(\frac{1}{r^{D-1}}\right)$$

centrifugal force　Newton gravity




✓ No stable bound orbit for $D \geq 5$

■ Black Ring case

*Today's topic*

# *Black Ring*

# *geometry*

- metric by ring coord. $x$ and $y$

$$ds^2 = -\frac{F(y)}{F(x)}\left(dt - CR\frac{1+y}{F(y)}d\Psi\right)^2 + \frac{R^2}{(x-y)^2}F(x)\left[-\frac{G(y)}{F(y)}d\Psi^2 - \frac{dy^2}{G(y)} + \frac{dx^2}{G(x)} + \frac{G(x)}{F(x)}d\Phi^2\right]$$

$$F(\xi) = 1+\lambda\xi, \quad G(\xi) = (1-\xi^2)(1+\nu\xi), \quad C = \sqrt{\lambda(\lambda-\nu)\frac{1+\lambda}{1-\lambda}}$$

- 2-parameter sol.

$R$ : ring radius

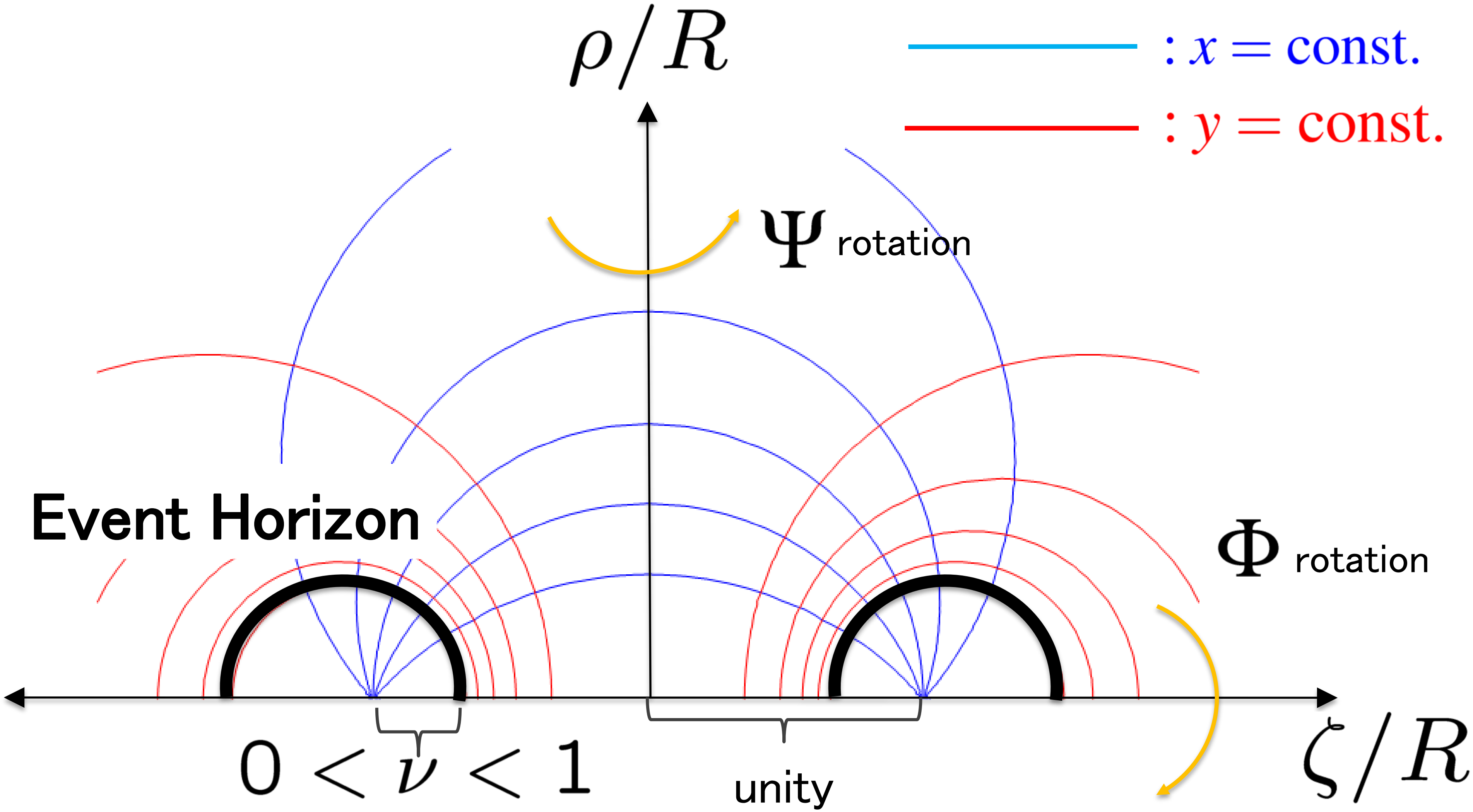
$\nu$ : thickness (thin $0 < \nu < 1$ fat)

( $\lambda(\nu)$ : rotational velocity )

- 3 Killing vectors $\partial_t, \partial_\Phi, \partial_\Psi$

- horizon topology $S^2 \times S^1$

■ diagram of a black ring




✓ $(t, \Phi, \Psi)$ are suppressed.

✓ $(\rho, \Phi), (\zeta, \Psi)$ : polar coordinates on two independent planes, respectively.

# *Particle Motion around Black Ring*

- geodesic eq. [J. Hoskisson (2009)]

  ✓ not separable $\Longrightarrow$ We use "effective potential"

- constants of motion

  $E$ : energy $\ell_\Phi,\ \ell_\Psi$ : angular momenta per unit energy

- Hamiltonian constraint (on-shell condition)

$$H = g^{ij}p_i p_j + E^2\left[U_{\rm eff}(\zeta,\rho;\ell_\Phi,\ell_\Psi) + \frac{1}{E^2}\right] = 0$$

$(i,j=\zeta,\rho)$

effective potential

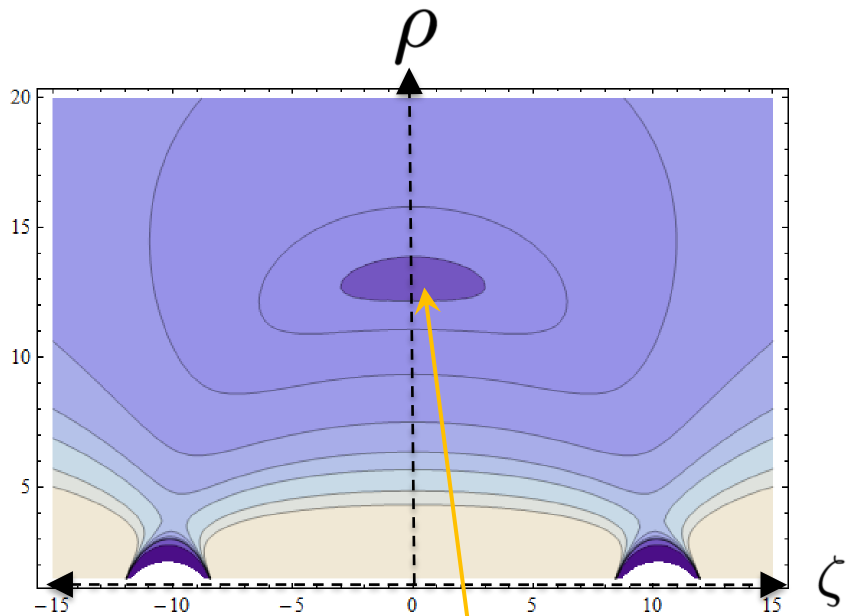
- extreme value problem of $U_{\rm eff}$

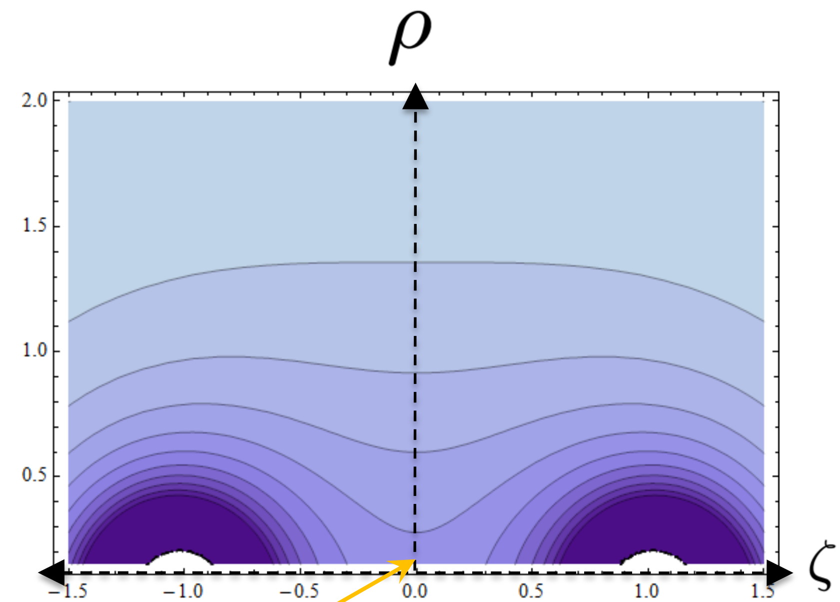
local minimum point = stable bound orbit

# *Results*

# *Typical Shapes of* $U_{\text{eff}}$

■ $\ell_\Psi = 0$ (no barrier at $\zeta = 0$ )




$\ell_\Phi = 0$

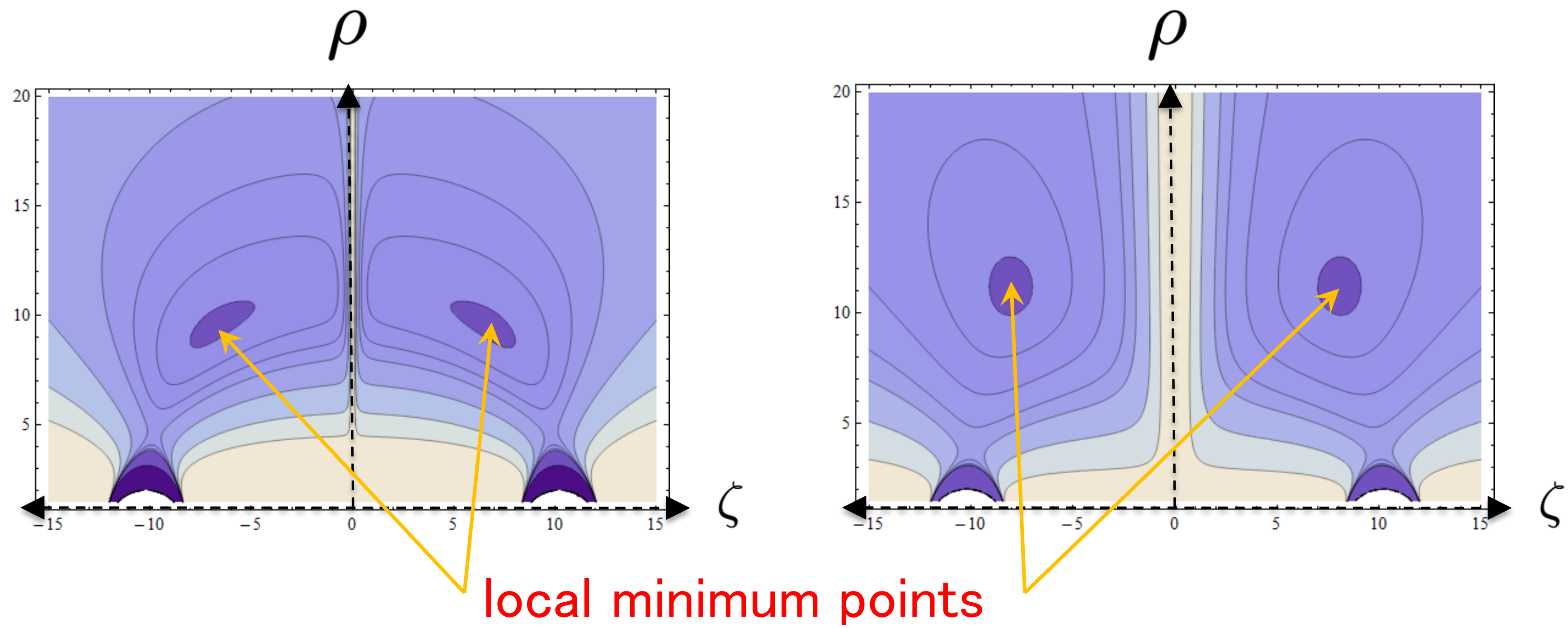
local minimum point

unstable

✓ On the ring axis, $\zeta = 0$, there exist stable bound orbits

✓ No stable bound orbit exist at the ring center.

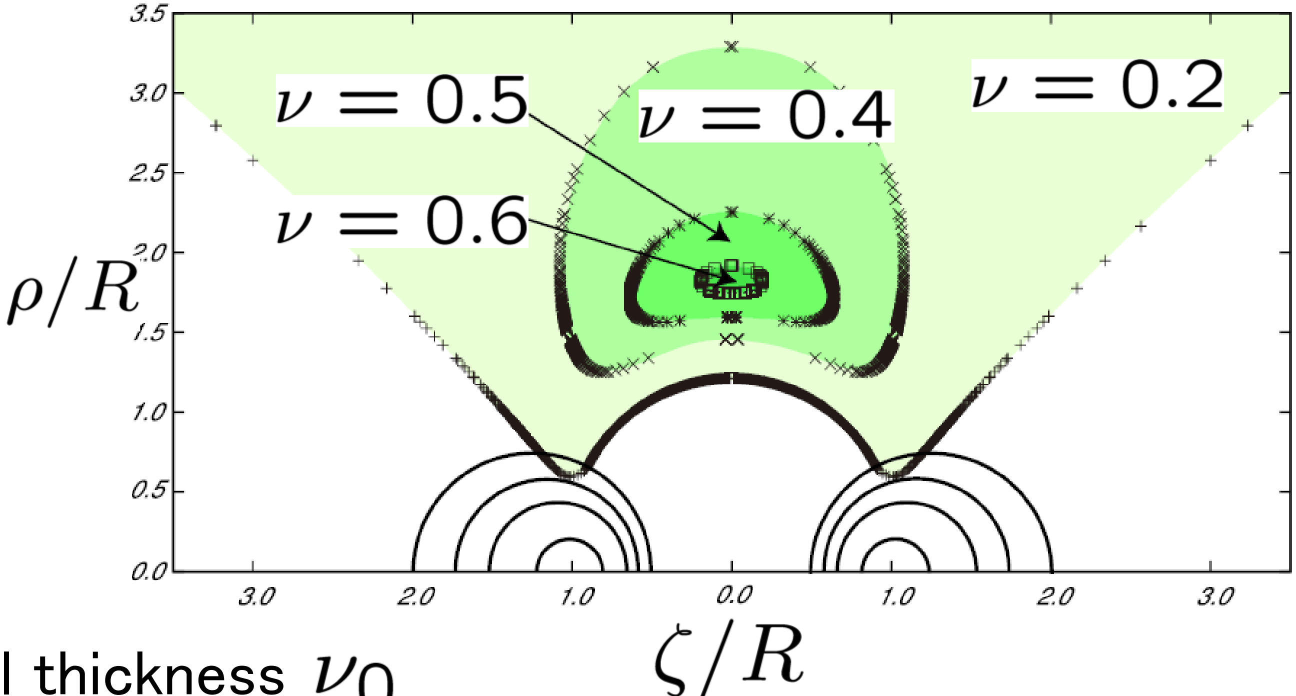
■ $\ell_\Psi \neq 0$




✓ Near the ring axis, there exist stable bound orbits.

# Domains of stable bound orbits

- numerical analysis
  - ✓ allowed region shows a set of local minimum points




- critical thickness $\nu_0$

$$\begin{cases} 0 < \nu \leq \nu_0 & : \text{There exist stable bound orbits} \\ \nu_0 < \nu \leq 1 & : \text{no stable bound orbit exists} \end{cases}$$

# *Critical Value of $\nu$ (analytically)*

$$\nu_0 = \frac{13}{2} + \frac{1}{2}\left(145 - 24\left(\frac{2}{3+\sqrt{41}}\right)^{1/3} + 6\left(4(3+\sqrt{41})\right)^{1/3}\right)^{1/2}$$
$$- \left[\frac{145}{2} + 6\left(\frac{2}{3+\sqrt{41}}\right)^{1/3} - 3\left(\frac{3+\sqrt{41}}{2}\right)^{1/3}\right.$$
$$\left. + \frac{1783}{2}\left(145 - 24\left(\frac{2}{3+\sqrt{41}}\right)^{1/3} + 6\left(4(3+\sqrt{41})\right)^{1/3}\right)^{-1/2}\right]^{1/2}$$

$$= 0.65379\cdots$$

# *Potential Analysis on the Ring axis*

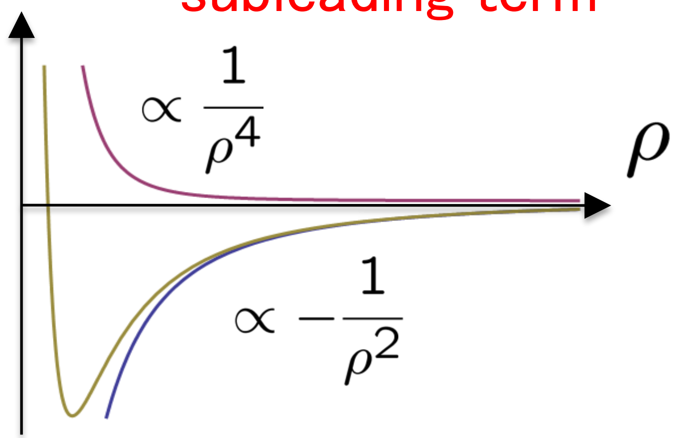
- asymptotic form $(\rho \to \infty)$

$$U_{\rm eff}\Big|_{\zeta=0} \simeq -1 + \left[-\frac{4\nu R^2}{(1-\nu)^2} + \frac{\ell_\Phi^2}{1-\nu}\right]\frac{1}{\rho^2} + \frac{2\nu R^2(2R^2-\ell_\Phi^2)}{(1-\nu)^2}\frac{1}{\rho^4}$$

Newton gravity
&
centrifugal force

subleading term

✓ exist a stable point at infinity ?




- local minimum point at $\rho \to \infty$

$$\rho_{\rm st} = +{\rm const.} \times \frac{\frac{1}{3}-\nu}{\frac{4\nu R^2}{1-\nu} - l_\Phi^2} \sim +\infty \quad \left(l_\Phi^2 \to \frac{4\nu R^2}{1-\nu}\right)$$

※ $0 < \nu < \nu_\infty \equiv \frac{1}{3}$

# Summary

We analyzed particle orbits around a black ring.

- Stable bound orbits (SBO) exist on and near the ring axis

  ✓ unique property of BR not BH in 5D

- critical thickness $\nu_0$

$$\begin{cases} 0 < \nu \leq \nu_0 & : \text{There exist SBOs} \\ \nu_0 < \nu \leq 1 & : \text{No SBO exists} \end{cases}$$

$$\nu_0 = 0.65379\cdots$$

- existence of SBOs with infinite radius in the case

$$0 < \nu < \nu_\infty = \frac{1}{3}$$