TransverseDiff gravity is to scalar-tensor as unimodular gravity is to General Relativity.

ERE2010 (Granada, 2010)

J.J. Lopez-Villarejo

Departamento & Instituto de Física Teórica

Universidad Autonoma de Madrid (UAM)

Outline

1. Motivation & Formalism
    - Motivation for these theories: UG and TDiff
    - Motivation / inspiration for the theorem(s) of equivalence
2. The results

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A quantum perspective of GR

GR $\Longrightarrow$ Linearized gravity in flat space-time, Fierz-Pauli Lagrangian

Fierz-Pauli action

- stands for the quantum theory of a non-interacting massless spin-2 particle
- presents a symmetry under the group of (infinitesimal) diffeomorphisms, Diff group: $x^\mu \to x^\mu + \xi^\mu(x)$

$$\mathscr{L} = \mathscr{L}^I + \beta \mathscr{L}^{II} + a\mathscr{L}^{III} + b\ \mathscr{L}^{IV}$$

$$\mathscr{L}^I \equiv \frac{1}{4}\partial_\mu h^{\nu\rho}\partial^\mu h_{\nu\rho}, \mathscr{L}^{II} = -\frac{1}{2}\partial_\mu h^{\mu\rho}\partial_\nu h^\nu_\rho, \mathscr{L}^{III} = \frac{1}{2}\partial^\mu h \partial^\rho h_{\mu\rho}, \mathscr{L}^{IV} = -\frac{1}{4}\partial_\mu h \partial^\mu h$$

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Massless spin-two particles in QM

TDiff Symmetry.

- Gauge invariance is a requirement for a consistent description of massless particles in Relativistic QM.
- For spin-1 particles, say a photon, one invokes the gauge transformations:

$$A_\mu \to A_\mu + \partial_\mu \alpha$$

- For spin-2 particles, the corresponding gauge transformations are:

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

  with $\xi_\mu(x)$ restricted by $\partial_\mu \xi^\mu = 0$

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A linearized gravitational theory for TDiff

- The restriction $\partial_\mu \xi^\mu = 0$ basically affects the transformation rule of the trace: $h \to h + \partial_\mu \xi^\mu = h$. Invariant!
- There are two routes to implement the TDiff symmetry:
  1. "Unimodular gravity": restrict the configuration space [historical approach: Bij, Dam & Ng]
     $\Rightarrow$ same terms of the F-P Lagrangian with h fixed to zero
  2. **"TDiff gravity":** consider the most general Lagrangian for a symmetric rank-two tensor $h_{\mu\nu}$, compatible with TDiff group
     $\Rightarrow$ more general Lagrangian than F-P *[motivation]*

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At non-linear level, TDiff symmetry group keeps the same form:

$$TDiff\ (infinitesimal):\ x^\mu \to x^\mu + \xi^\mu(x),\ with\ \partial_\mu \xi^\mu = 0$$

$$\Downarrow$$

$$TDiff\ (finite):\ x^\mu \to y^\mu(x),\ with\ J \equiv \left\| \frac{\partial y}{\partial x} \right\| = 1$$

1. "Unimodular gravity": Einstein-Hilbert action with restriction on the determinant of the full metric $g_{\mu\nu}$: typically $g = 1$.

$$S_{UG}[\widehat{g}_{\mu\nu}] = -\frac{1}{2\kappa^2} \int_{\mathcal{M}} d^4x R[\widehat{g}_{\mu\nu}],\ \ with\ \widehat{g} = 1$$

2. **"TDiff gravity":** g behaves as a TDiff-scalar, so that it gets into the Lagrangian.

$$S_{G,TDiff} = -\frac{1}{2\kappa^2} \int_{\mathcal{M}} d^4x \sqrt{g} \left[f(g)R + 2f_\lambda(g)\Lambda + \frac{1}{2} f_k(g) g^{\mu\nu} \partial_\mu g \partial_\nu g \right]$$

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(Classical) correspondence of theories

unimodular gravity is GR with a restriction on the metric field

$$\delta S_{UG} \equiv \delta|_{\sqrt{g}=1} S_{EH} = 0$$

the classical solutions of GR contain a redundancy that can be gauge-fixed

$$\delta S_{EH} = 0 \underset{ph.eq.}{\Longleftrightarrow} [\delta S_{EH} = 0]_{\sqrt{g}=1}$$

TDiff gravity is scalar-tensor with a restriction

$$\delta S_{TDiff} \equiv \delta|_{\sqrt{g}-1=\phi} S_{ST} = 0$$

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$$\delta S_{ST} = 0 \underset{ph.eq.}{\Longleftrightarrow} [\delta S_{ST} = 0]_{\sqrt{g}-1=\phi}$$

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Restricted variation vs. variation+restriction

$$a)\ \delta|_{f(q_1,\dots,q_n)=0} S[q_1(t),\dots,q_n(t)] = 0$$

vs.

$$b)\ [\delta S[q_1(t),\dots,q_n(t)] = 0]_{f(q_1,\dots,q_n)=0}$$

- In general, every solution in b) is also solution in a):

  $$\left\{(q_1(t),\dots,q_n(t))\,|\,\delta|_{f(q_1,\dots,q_n)=0} S[q_1,\dots,q_n] = 0\right\} \supseteq \{(q_1(t),\dots,q_n(t))\,|\,\delta S[q_1,\dots,q_n] = 0\}_{f(q_1,\dots,q_n)=0}$$

  the converse is not true!

- However, what happens if the restriction is actually a trivial one, a gauge-fixing??

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A theorem of equivalence

$$S_{TDiff} = \int \sqrt{g}\mathscr{L}(g_{\mu\nu}, \sqrt{g}-1;\, \psi)$$

formulated in a specific set of coordinates

$$S_{ST} = \int \sqrt{g}\mathscr{L}(g_{\mu\nu}, \phi;\, \psi)$$

A theorem of equivalence between TDiff and scalar-tensor

$$\{(g_{\mu\nu},\psi)|\,\delta S_{TDiff}[g_{\mu\nu};\psi]=0, \mathscr{L}|_{\partial B}=0\}$$

$$\equiv$$

$$\{(g_{\mu\nu},\phi,\psi)|\,\delta S_{ST}[g_{\mu\nu},\phi;\psi]=0, \mathscr{L}|_{\partial B}=0\}\Big|_{\phi=\sqrt{g}-1}$$

JJL-V, *Arxiv:hep-th/0103715 (tomorrow!)*

E. Alvarez, A.F. Faedo & JJL-V, *JHEP 0810:023,2008; JCAP07(2009)002.*

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A theorem of equivalence between TDiff and scalar-tensor

$$\{(g_{\mu\nu}, \psi) |\, \delta S_{TDiff}[g_{\mu\nu}; \psi] = 0, \mathscr{L}|_{\partial B} = 0\}$$

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A technical point in the theorem

The requirement that $\mathscr{L}$ is zero at the boundary is used in two different ways:

1. To make the local gauge-fixing *globally valid* over the whole spacetime (we are fixing a trivial direction in configuration space).
2. To make up for the subtlety that our equations of motion correspond to bounded variations rather than unbounded ones.

Consequences of the theorem

Corollary

- Equivalence between TDiff and a *general* scalar-tensor theory for all solutions that make the Lagrangian vanish at the boundary of spacetime.
- An analogous theorem exists which connects unimodular gravity and GR.
- A TDiff symmetry group on a "rank-two tensor, Lagrangian-based theory" generates a scalar mode.
- Bounds on scalar-tensor theories apply trivially to TDiff gravity.
- The theorem generalizes for an understanding of the differences between applying a gauge-fixing before the extremalization procedure and after (definition).

BACKUP

Gravitational Action for TDiff: Linear case.

Most general LI local Lagrangian for free masless tensor field $h_{\mu\nu}$:

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Linear Diff action for Gravity (Fierz-Pauli)

$$Diff:\ x^\mu \to x^\mu + \xi^\mu(x)$$

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Diff action for gravity (Einstein-Hilbert)

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Compare & Contrast with Unimodular Gravity.

- Unimodular Gravity is based on a reduction of the functional space on which the Einstein-Hilbert action is defined:

$$g_{\mu\nu} \longrightarrow \widehat{g}_{\mu\nu}, \; with\, det(\widehat{g}_{\mu\nu}) = \varepsilon_0$$

$$S_{EH}[g_{\mu\nu}] = -\frac{1}{2\kappa^2}\int_{\mathscr{M}} d^4x\sqrt{g}R[g_{\mu\nu}] \;\longrightarrow S_{UG}[\widehat{g}_{\mu\nu}] = -\frac{1}{2\kappa^2}\int_{\mathscr{M}} d^4x\sqrt{\varepsilon_0}R[\widehat{g}_{\mu\nu}]$$

  (ε_0 is some fixed scalar density, usually taken unity).

  Remaining symmetry: "volume preserving diffs. (VPD)": $g'(x) = g(x)$ (same as TDiff only for $\varepsilon_0(x) = cons.$; usual setting).

- There are 9 e.o.m: $\delta S/\delta\widehat{g}_{\mu\nu} = 0$ (only 6 independent in virtue of VPD-based Bianchi identities), for 9 functions $\widehat{g}_{\mu\nu}$ ($\sim$ GR with one constraint on the metric).
- TDiff Gravity is based upon a symmetry principle on the full space of metrics $g_{\mu\nu}$.
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Quantum divergences

- UV divergences to 1-loop order computed using the background field method.
- New divergences found, as compared to the General Relativity template, due to the extra "g-mode". *Only exception is the WTDiff case (TDiff symmetry enhanced with Weyl conformal invariance).*

JHEP 0810:023,2008

Observational Constraints.

- There is an extra degree of freedom propagating ("g-mode").
- An (almost exact) correspondence can be established with Scalar-Tensor Theory ("g-mode" $\rightarrow$ scalar mode), wherein the scalar may be present in any part of the action, including the matter part $\Longrightarrow$ violations of EP.
- Scalar-Tensor theories overcome this problem by means of the *"metric postulate"* (there exists a frame in which the matter action only depends on a second rank tensor, *the physical metric*). This does not have to be necessarily the case: String Theory, Extra Dimensions, etc.

JCAP07(2009)002; new paper in preparation.

Conclusions

1. Tranverse Diffeomorphisms constitute the necessary and sufficient symmetry group for the consistent quantum description of the massless tensor graviton .
2. Construction of theories based on TDiff does not alleviate the problem of divergences in perturbative Gravity.
3. TDiff theories naturally lead to Scalar-Tensor Gravity, so that existing observational constraints on this one are applicable.