Quantum Gowdy Model within the new LQC Improved Dynamics

Mercedes Martín-Benito

In collaboration with Luis J. Garay and Guillermo A. Mena Marugán

Instituto de Estructura de la Materia, CSIC, Spain

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Motivation

- Introduction of inhomogeneities in Loop Quantum Cosmology
- Gowdy T^3 model with linearly polarized gravitational waves: simplest case
- Simplest possibility: hybrid quantization
  - Loop quantization for the homogeneous sector (zero modes = d.o.f. of Bianchi I)
  - Fock quantization for the inhomogeneities
- This approach explores the effects on quantum geometry on the homogeneous gravitational sector
- It may suffice to cure the classical initial singularity

Gowdy T^3 model with linear polarization

- Vacuum spacetime, with 3-torus spatial topology and two hypersurface orthogonal spatial Killing fields $(\partial_\sigma, \partial_\delta)$

  Coordinates $(t, \theta, \sigma, \delta)$, with $\theta, \sigma, \delta \in S^1$

  Metric components: functions of (t, θ)

Reduced phase space

- Homogeneous sector: Bianchi I phase space
- Inhomogeneous sector: nonzero modes of a field and its momentum

Remaining global constraints

- Generator of S^1 translations: $C_\theta = 0$

  (It only involves the inhomogeneous sector)

- Zero-mode of the Hamiltonian constraint:

$$C_{\mathsf{G}} = C_{\mathsf{BI}} + C_\xi = 0$$

C_{BI}: Hamiltonian constraint of the Bianchi I model

C_ξ: coupling term

It involves an interaction term H_{int} which creates and annihilates pairs of particles

Inhomogeneous sector: Fock quantization

- Variables associated with the free massless scalar field

$$a_m,\ a_m^* \rightarrow \hat{a}_m,\ \hat{a}_m^\dagger$$

Unique Fock quantization with unitary dynamics and a natural unitary implementation of the remaining gauge group of S^1-translations

- Generator of S^1-translations: $\widehat{C}_\theta$

  Defined on n-particle states: $|\mathfrak{n}\rangle := |..., n_{-2}, n_{-1}, n_1, n_2, ...\rangle$.

  The n-particle states annihilated by $\widehat{C}_\theta$ provide a basis for a proper Fock subspace $\mathcal{F}_p$.

Homogeneous sector: Loop quantization

- Bianchi I phase space in Ashtekar formalism:

$$\{c_i, p_j\} = 8\pi G\gamma\delta_{ij}, \qquad i, j = \theta, \sigma, \delta$$

- Configuration algebra: generated by holonomies matrix elements

$$\mathcal{N}_{\mu_j}(c_j) = \exp\left(\frac{i}{2}\mu_j c_j\right) := |\mu_j\rangle$$

$\mu_j \sim$ coordinate length of the holonomy in the j-direction.

- Homogeneous sector of the kinematical Hilbert space: closure w.r.t the **discrete inner product** for each fiducial direction

- Basic operators: $\hat{p}_i|\mu_i\rangle \propto \mu_i|\mu_i\rangle, \quad \hat{\mathcal{N}}_{\mu_i'}|\mu_i\rangle = |\mu_i + \mu_i'\rangle$

Improved dynamics prescription

- New scheme for anisotropic situations:
  Minimum fiducial length of the holonomies: $\bar{\mu}_i = \sqrt{\frac{|p_i|\Delta}{|p_j p_k|}}$
  - Requirement: the exponents of the holonomy elements $\mathcal{N}_{\bar{\mu}_i}(c_i)$ have a fixed constant Poisson bracket with the variable

$$v := \text{sgn}(p_\theta p_\sigma p_\delta) \frac{\sqrt{|p_\theta p_\sigma p_\delta|}}{2\pi\gamma l_{\text{Pl}}^2 \sqrt{\Delta}}, \quad (\propto \text{volume})$$

  - Adapted to the volume: Suitable scaling properties
- Reparametrization of the states:
  - $|\mu_\theta, \mu_\sigma, \mu_\delta\rangle \rightarrow |v, \lambda_\sigma, \lambda_\delta\rangle$, with $v = 2\lambda_\theta\lambda_\sigma\lambda_\delta$
  - $\hat{\mathcal{N}}_{\bar{\mu}_i}$ multiplies λ_a by a function of v and/or shifts v one unit

Quantum Hamiltonian constraint

- Out of the basic operators $\hat{p}_i$, $\hat{\mathcal{N}}_{\pm\bar{\mu}_i}$, $\hat{a}_m$, and $\hat{a}_m^\dagger$ we obtain the Hamiltonian constraint operator $\widehat{C}_{\mathsf{G}}$
- We adopt a particular symmetric factor ordering:
  $\widehat{C}_{\mathsf{G}}$ is well defined in suitable **superselection sectors**, spanned by $|v, \lambda_\sigma, \lambda_\delta\rangle \otimes |\mathfrak{n}\rangle$ with $v, \lambda_\sigma, \lambda_\delta > 0$ and such that:
  - The values of v run along a semilattice of step 4 contained in $\mathbb{R}^+$, with a minimum value $\varepsilon \in (0, 4]$
  - λ_a runs along a numerable set which is dense in $\mathbb{R}^+$
- The constraint provides a difference equation in $v \geq \varepsilon$
  - Kinematical resolution of the classical singularity
  - No-boundary description

Physical Hilbert space

- The constraint can be regarded as an evolution equation w.r.t the internal time v.
- Although the solution is formal (owing to $\hat{H}_{\text{int}}$), it is completely determined by the data at the initial section $v = \varepsilon$:
  well-posed initial value problem
- Reality conditions in a complete set of observables acting on the initial data $\rightarrow$ physical inner product

$$\mathcal{H}_{\text{Phys}} = \mathcal{H}_{\text{Phys,BI}} \otimes \mathcal{F}_p$$

We recover the standard QFT for the inhomogeneities, on the loop quantized Bianchi I background

Conclusions

- Hybrid loop/Fock approach to deal with the quantization of inhomogeneous cosmologies
- The loop quantization of the homogeneous sector suffices to cure the classical initial singularity
- While the volume is discretized in a lattice of constant step, the values of the anisotropies, being numerable, are dense in $\mathbb{R}^+$.
- Quantization completed
- We recover the standard quantum field theory for the inhomogeneities, that can be regarded as degrees of freedom which propagate in a Bianchi I background