Thermal radiation from Lorentzian wormholes

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- ***Brief review***
  - *Phantom energy and wormholes*
  - *Accretion onto black- and worm- holes.*
  - *2+2 formalism and applications.*
- ***Wormholes characterization.***
- ***Wormholes thermal radiation and thermodynamics.***
- ***Conclusions and further comments.***

The current Universe is undergoing a period of accelerating expansion

General Relativity $+$ accelerating expansion $\Longrightarrow$ **Dark energy** $w < -1/3 \;\left(w = p/\rho\right)$

It seems even possible that **w<-1**: **phantom energy**.

•**Phantom thermodynamics:** $T_{ph} < 0$

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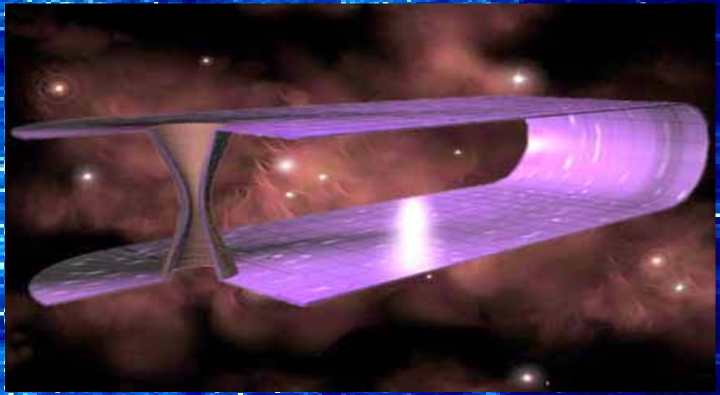
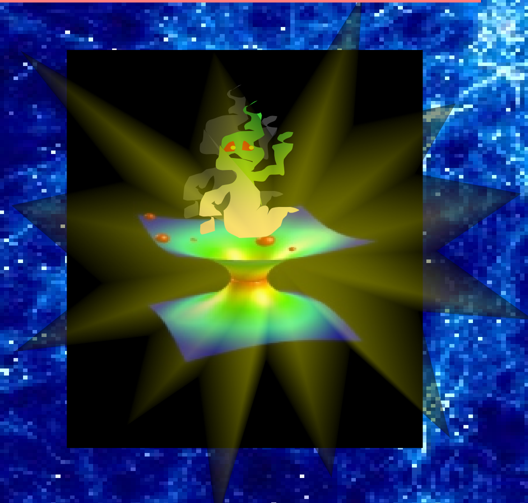


- **Wormholes** must be supported by some kind of "**exotic matter**", characterized by violating the null energy condition, in order to be **traversable**.

It has been shown that an inhomogeneous version of **phantom energy** can be the exotic stuff which is required to support **wormholes.**



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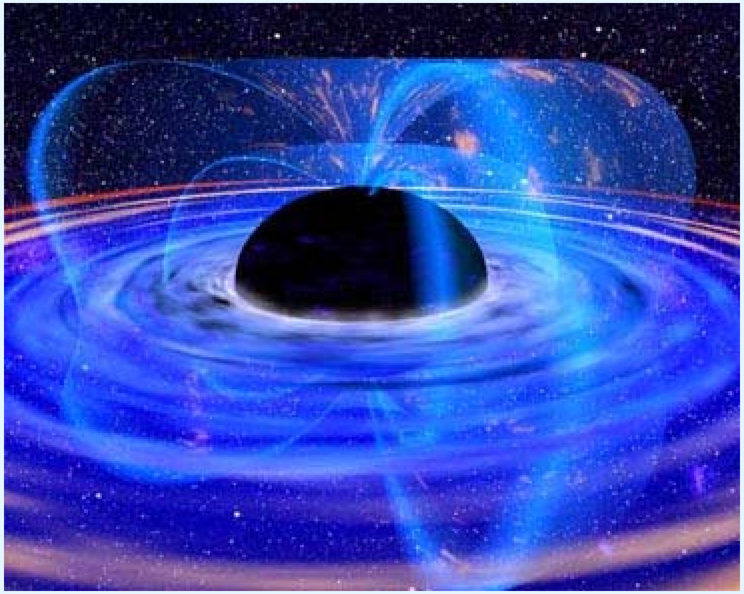
S. V. Sushkov, Phys. Rev. D **71**, 043520 (2005); F. S. N. Lobo, Phys. Rev. D **71**, 084011 (2005).



•**Babichev** et al. used a **test-fluid approach** to study the evolution of the horizon area of a Schwarzschild **black hole** induced by the accretion of dark energy, obtaining

$$\frac{dM}{dt} = 4\pi D M^2 (p + \rho)$$

where D is a constant of order unity.

$$\left.\begin{array}{l} p + \rho > 0 \\ p + \rho = 0 \\ p + \rho < 0 \end{array}\right\} \Rightarrow \text{black hole mass} \left\{\begin{array}{l} \text{increases} \\ \text{remains constant} \\ \text{decreases} \end{array}\right.$$

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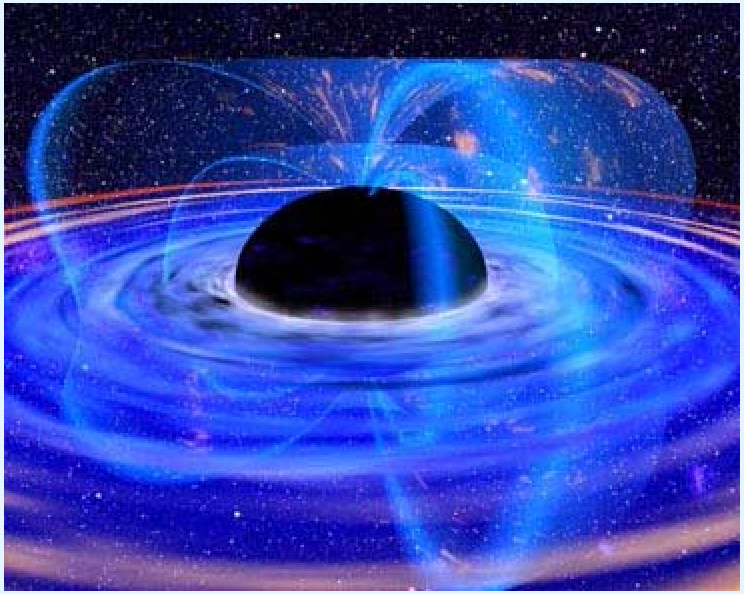

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•That procedure applied to Morris-Thorne **wormholes** leads

$$\frac{dm}{dt} = -4\pi Q m^2 (p+\rho)$$

with Q a positive constant.

$$\left.\begin{matrix} p+\rho>0 \\ p+\rho=0 \\ p+\rho<0 \end{matrix}\right\} \Rightarrow \text{wormhole size} \left\{\begin{matrix} \text{decreases} \\ \text{remains constant} \\ \text{increases} \end{matrix}\right.$$

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2+2 formalism and applications

Spherically symmetric spacetime $$ds^2 = 2g_{+-}d\xi^+ d\xi^- + r^2 d\Omega^2_{(2)}$$

$\xi^\pm$ are double-null coordinates, $r = r(\xi^+, \xi^-)$ is the areal radius.

$r = \sqrt{A/4\pi}$, A: area of the spheres of symmetry

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Expansions $\Theta_\pm = \frac{2}{r}\partial_\pm r$

The sign of $\Theta_+\Theta_-$ is an invariant

If $\begin{Bmatrix} \Theta_+\Theta_- > 0 \\ \Theta_+\Theta_- = 0 \\ \Theta_+\Theta_- < 0 \end{Bmatrix}$ the sphere is $\begin{cases} \text{trapped} \\ \text{marginal} \\ \text{untrapped} \end{cases}$

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A marginal sphere with $\Theta_+ = 0$ can be $\left[\begin{matrix} \Theta_- < 0 \text{ future} \\ \Theta_- = 0 \text{ bifurcating} \\ \Theta_- > 0 \text{ past} \end{matrix}\right.$ $\left[\begin{matrix} \partial_-\Theta_+ < 0 \text{ outer} \\ \partial_-\Theta_+ = 0 \text{ degenerate} \\ \partial_-\Theta_+ > 0 \text{ inner} \end{matrix}\right.$

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Trapping horizon: hypersurface foliated by marginal spheres.

The trapping horizon has the same classification as the marginal spheres.

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•In a spherically symmetric spacetime, one can intruduce the **Kodama vector**

$$k = \mathrm{curl}_2 r \Rightarrow k = -g^{+-}\left(\partial_+ r \partial_- - \partial_- r \partial_+\right)$$

with $k^a \nabla_{[a} k_{b]} = \kappa k_b$ on a trapping horizon.

κ is the **generalized surface gravity** $\kappa = \frac{1}{2}\mathrm{div}_2 \mathrm{grad}_2 r \Rightarrow \kappa = g^{+-}\partial_+ \partial_- r$

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•**Generalized first law of thermodynamics**

$$L_z E = \frac{\kappa L_z A}{8\pi} + \omega L_z V$$

$$\Downarrow$$

entropy $S \propto A$

where $L_z = z \cdot \nabla$ and $z = z^+ \partial_+ + z^- \partial_-$

$E = \frac{r}{2}\left(1 - \partial^a r \partial_a r\right)$ is the Misner-Sharp energy

$$\omega = -\frac{1}{2}\mathrm{trace}_2 T$$

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•**Thermal radiation for dynamical black holes:** $T_H = \frac{\kappa}{2\pi}$

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Wormholes characterization

Using the Einstein equations, it can be obtained that at an **outer** trapping horizon

$$sign\left(\frac{z^{+}}{z^{-}}\right) = -sign\left[(p+\rho)_H\right] \Rightarrow \text{ if } \left\{\begin{matrix}(p+\rho)_H > 0 \\ (p+\rho)_H < 0\end{matrix}\right\} \text{ the horizon is } \left\{\begin{matrix}\text{space-like} \\ \text{time-like}\end{matrix}\right.$$

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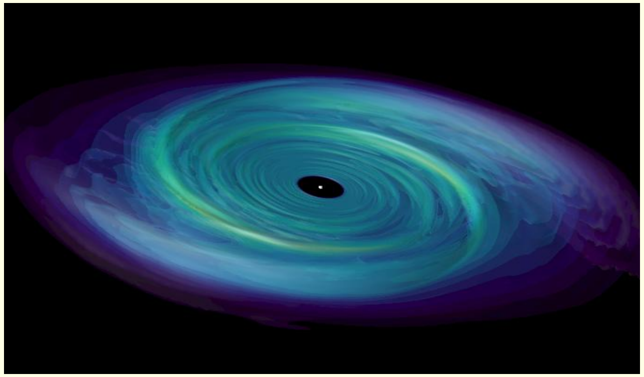
•**Dynamical black hole** (which is characterized by a future outer trapping horizon)

$$\text{If} \begin{cases} p+\rho > 0 \\ p+\rho < 0 \end{cases} \text{the black hole size} \begin{cases}\text{increases} \\ \text{decreases}\end{cases} \Rightarrow$$

Similar results to those obtained by Babichev et al.!!



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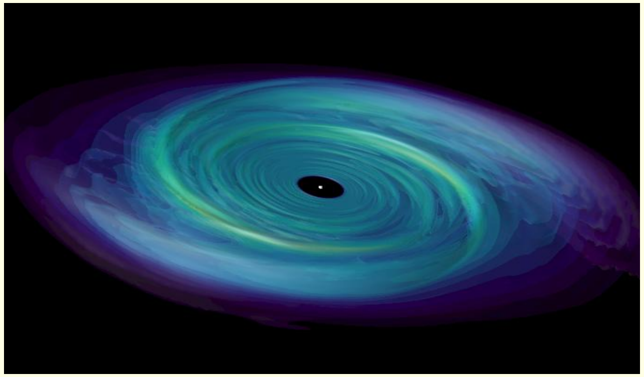
Both methods seems to describe just the same process, which is originated from a flow of the surrounding material into the hole.



$\Downarrow$

•**Wormholes** must be characterized by **past outer trapping horizons** in order to recover the results obtained following the accretion method.

A traversable wormhole possesses a classically allowed trajectory.

The existence of a trapping horizon (with $\kappa \neq 0$) opens the possibility for an additional quantum traversing phenomenon through the wormhole.

The tunneling probability Γ along a classically forbidden trajectory can be considered.

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Considering a massless scalar field, it can be seen that Γ has a thermal form, $\Gamma \propto \exp\left(-\omega_\phi / T_H\right)$, with

$$T_H = -\frac{\kappa}{2\pi} < 0$$

Wormholes radiate matter of the same kind as their surrounding material: phantom energy.

THERMODYNAMIC LAWS FOR WORMHOLES

$$L_z E = -T_H L_z S + \omega \cdot L_z V$$

$$S = \frac{A}{4}$$

- First law: the change in the gravitational energy of the wormhole is equal to the sum of the energy removed from the wormhole and the work done in the wormhole.
- Second law: the entropy of a wormhole, which is given in terms of the throat surface area, can never decrease, when placed in its most natural dominant-energy-condition violating environment.
- Third law: it is impossible to reach the absolute zero for surface gravity by means of any dynamical process.

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- Third law: it is impossible to reach the absolute zero for surface gravity by means of any dynamical process.

If any hypothetical process would be able to change the surface gravity to be zero, then the outer horizon would converts into degenerate.

Conclusions and further comments

•Wormholes posses a well-defined thermodynamics.

•Wormholes thermally radiate phantom energy.

•The radiation process would produce a decrease of the wormhole size, decreasing the wormhole entropy too. This violation of the second law is only apparent, because is the total entropy of the universe what should increase.

•The initial conditions in the action integral could be chosen in such a way that the wormhole would radiate ordinary matter increasing its size in the process. But, if that case would be possible, the thermal radiation would be always thermodynamically forbidden in front of the accretion entropicaly favored process.

•Although this study is a crucial step in the development of wormhole thermodynamics, a lot of work is still necessary to understand some ambiguities of the used method.

Thank you

References:

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