Causality and entropic arguments pointing to a null Big Bag hypersurface

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Spanish Relativity Meeting (ERE) Granada, September 10, 2010

Cosmological principle and FLRW metric

$$ds^2 = \mathrm{d}t^2 - a(t)^2[\mathrm{d}\chi^2 + \Sigma(\chi)^2 \mathrm{d}\Omega^2]$$

with $\mathrm{d}\Omega^2 = \mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2$ metric element of S^2 and $\Sigma(\chi) = \sin\chi$ for $k = +1$, $\Sigma(\chi) = \chi$ for $k = 0$, $\Sigma(\chi) = \sinh\chi$ for $k = -1$.

The homogeneity/isotropy problem

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Friedmann equation and conservation of energy

$$H^2 + \frac{k}{a^2} = \frac{\Lambda}{3} + \frac{8\pi G}{3}\rho$$

$$\dot{\rho} = -3\frac{\dot{a}}{a}(\rho + p) \qquad (\textit{e.g. from } \mathrm{d}E = -p\mathrm{d}V, V \propto a^3)$$

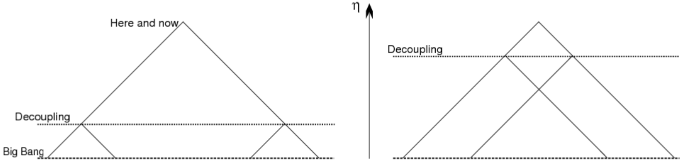
Equation of state $p(\rho)$

Let $p = \alpha\rho$, $\alpha > -1/3$ then for $t \to 0$ the cosmological constant term and the k term do not matter and $a \sim t^{\frac{2}{3(1+\alpha)}}$.

Cosmic microwave background

Approximate black body radiation at 2.7K, variations of order 10^{-5} for angles as large as 80°.

Light moves at 45° in spacetime diagrams with conformal time $\eta = \int_0^t \frac{\mathrm{d}t}{a(t)}$.



Inflation by acting only on the dependence $a(t)$ pushes up the decoupling time (up to rescaling). Inhomogeneities have time to interact and smooth out (thermalization). Note: according to this solution entropy from Big Bang to decoupling increases so the Big Bang has to be more "special" then the universe at decoupling.

Two opposite views on isotropy

Chaotic cosmology (Misner)

Universe had an irregular and chaotic beginning. Homogeneity was produced through dissipative effects, such as particle collisions. Requires that we live in a sufficiently late stage of evolution.

Quiescent cosmology (Barrow)

Universe had a very regular beginning close to FLRW universe. It tends to evolve away from regularity because of gravitational attraction. We see homogeneity because we are in an early stage of evolution.

It depends on what we are talking about, one need to distinguish matter and radiation from gravity.

How special is the Big bang?

Interesting evolution is possible only if the initial entropy is small. A system already at maximum entropy is dynamically trivial. The second law of thermodynamics holds because in the coarse graining we start from a small cell of phase space.

Entropy problem (Penrose's argument)

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A clarification

The entropy increases because the universe expands and so the maximum entropy increases? If so a contracting universe would be physically impossible (the area of black holes seems related to entropy and cannot decrease). Also in every physical system the phase space is always one and the same, it is not the union of phase spaces each at a single cosmic time.

Entropy problem (Penrose's argument) II

Estimate of the maximum entropy of the universe

Use Bekenstein-Hawking formula $S_{BH} = \frac{kc^3}{4G\hbar}A$, and assume that every baryon will fall into a black hole. Comparison with entropy at decoupling gives that the cell at decoupling is about one part of $10^{10^{123}}$ than that at end of universe.

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In which way could the Big Bang be special? The role of gravity

Gravitational degrees of freedom had small entropy. Homogeneity evolves into *clumping* as gravitation is attractive. Thus small entropy is related to gravitational degrees of freedom rather that to the smallness of the universe.

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Penrose's Weyl tensor hypothesis

The Weyl tensor

$$C_{ab}{}^{dc} = R_{ab}{}^{cd} - 2R_{[a}{}^{[c}g_{b]}{}^{d]} + \frac{1}{3}Rg_{[a}{}^{c}g_{b]}{}^{d}$$

is small for homogeneous matter distributions thus it is expected to be small at the beginning of the universe.

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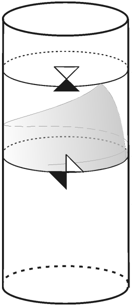
Yet the problem remains: If the Big Bang is made by causally unrelated events on the boundary how it could be special?

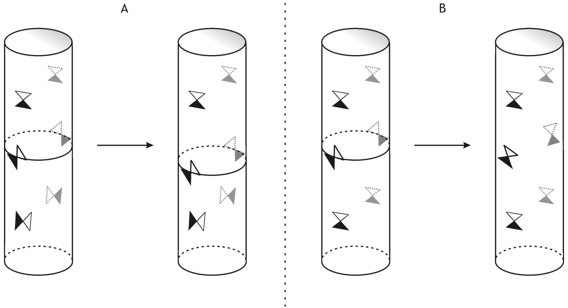
A simple causality solution: The Big Bang hypersurface is null

Inflation solves the problem of homogeneity/isotropy inducing a modification of the conformal factor $a(t)$, here we just tilt the cones so that the whole null hypersurface is in the past of every point of the last scattering hypersurface.



Exact solutions of Einstein equations with similar causal behavior exist, e.g. Taub-NUT and λ-Taub-NUT.
Between "Big Bang" and decoupling there is no thermalization so there is no such increase of entropy as in the inflationary case, a fact which made the problem of the small entropy at Big Bang even worse in the inflationary scenario.

This mechanism for homogeneity in order to be stable under small perturbations of the metric requires that behind the Big Bang null hypersurface there be a chronology violating region.



The matching could be conformally smooth but metrically singular.

Compatibility with singularity theorems

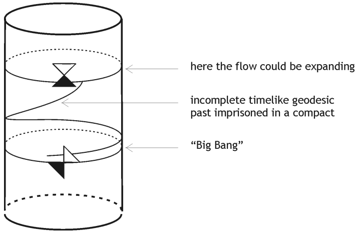
Most singularity theorems assume that strong causality conditions hold. The only exception is

Hawking's (1967) singularity theorem

Spacetime is not geodesically complete if:

- $R_{ab}K^aK^b \geq 0$ for every causal vector K (positive energy condition);
- there exist a compact spacelike hypersurface $\mathscr{S}$ (without edge);
- the unit normals of $\mathscr{S}$ are everywhere diverging.

But a common misunderstanding is that every incomplete geodesic must escape every compact i.e. go to the boundary. This is false!



Why this scheme gives a solution to the entropy problem?

The role of rigidity

The solution stand on the achronality of the boundary given by the Big Bang hypersurface. This hypersurface can be proved to be generated by achronal inextendible lightlike geodesic. This fact implies *rigidity*: the metric is special near the boundary e.g. its transverse section gives the Euclidean flat metric. As the metric is special, gravitational degrees of freedom are frozen, exactly the way out to the entropy problem that Penrose suggested.

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Mathematically this has still to be proved in generality but consider the prototype rigidity result:

Galloway's (2000) null splitting theorem

Suppose M is an asymptotically simple vacuum spacetime which contains a null line. Then M is isometric to Minkowski space.

or use Penrose's plane wave limit.

The beginning of time

A point belongs to the chronology violating region if there passes a closed timelike curve. A chronology violating class $[p]$ is made by those points such that
$q \ll p \ll q$.
A time function is a function $t : M \to \mathbb{R}$, such that $x < y \Rightarrow t(x) < t(y)$.

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Time begins just outside the chronology violating set at the beginning of the universe

Theorem. Let (M, g) be a spacetime that admits no chronology violating class but one which generates the whole universe, $M = I^+([r])$. Assume that the spacetime is null geodesically complete and satisfies the null convergence condition and the null genericity condition on the null geodesics intersecting $M \backslash \overline{[r]}$. Then the spacetime $M \backslash \overline{[r]}$ is stably causal and hence admits a time function.

The first assumption is related to the *chronology protection conjecture*.
Recall
Null genericity: On every lightlike geodesic $n^c n^d n_{[a} R_{b]cd[e} n_{f]} \neq 0$ for some tangent vector.
Null convergence: $R_{ab} n^a n^b \geq 0$.

Augustine of Hippo (Thagaste 354 - Hippo 430), romanized berber philosopher.

Augustine's time studies. Confessions book XI

"What, then, is time? If no one ask me, I know; if I wish to explain to him who asks, I know not."

Connection with Augustine of Hippo's philosophy II

A few assumptions on God's nature

1. There is an entity which we call God that satisfies the following points.
2. God has created the world.
3. God cannot be wrong.

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Augustine's solution

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In other words Augustine separates temporality from causality! Something usual in GR but philosophically it was a very non-trivial argument.

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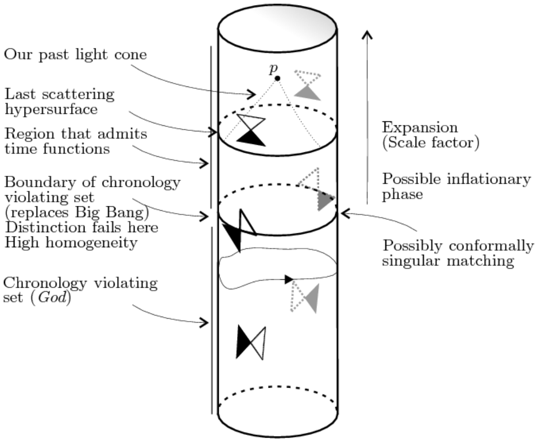
The connection

Assume God is represented by the chronology violating class at the beginning of the universe. A closed timelike curve cannot represent an observer (conflicts with free will) but can represent a God since its will is eternal thus confirms previous path (choices). The null Big Bang represents the creation of world and time.

Conclusions

I have advocated that a picture for the beginning of the universe whose causal aspects are as in the figure can solve the homegeneity/isotropy problem and the entropy problem and fits nicely Augustine's considerations on the beginning of time.



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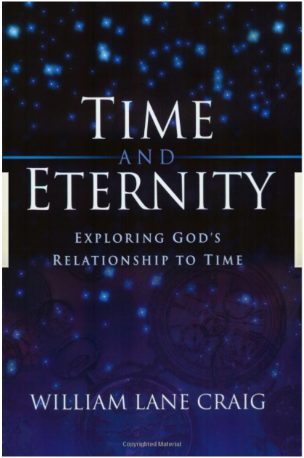
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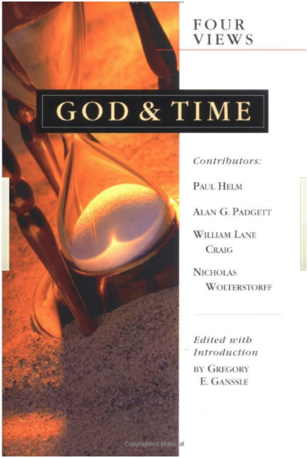
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GOD & TIME
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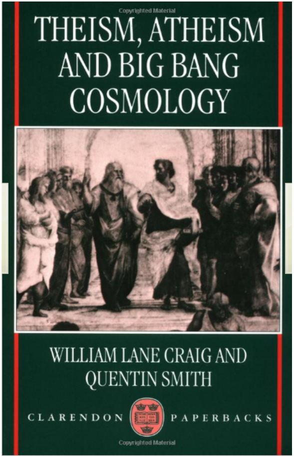
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