「Thermal Equilibrium of Black Di-rings」

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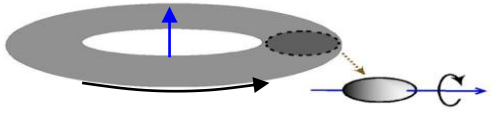
This talk is based on the work done in collaboration
with Hideo Iguchi (CST Nihon Univ.) arXiv:1008.4290v2 [hep-th]

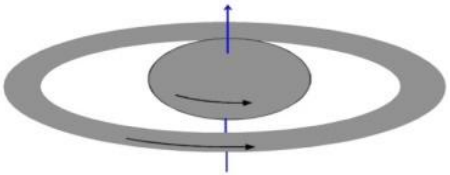
I. Introduction

- **Since the discovery of S^1 rotational black ring by Emparan and Reall several 5-dimensional black hole systems have been obtained using solitonic methods.**

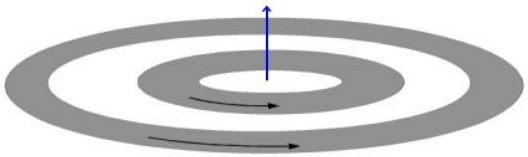
（asymptotically Minkowski cases ）



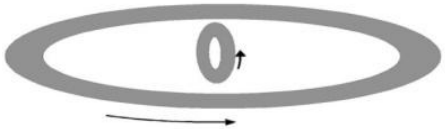
Black rings (full rotation)



Black Saturn



Black di-ring



Orthogonal black di-ring /Bi-ring

Black lense

• • • •

- **Previously we succeeded to superimpose two S^1-rotating black rings in concentric and regular way as simple multi-BH systems.**
  We call the solutions black di-rings.

<The spacetime considered here >

- **metric (** $\det G = -\rho^2$ **)**

$$ds^2 = G_{tt}(dx^0)^2 + 2G_{t\psi}dtd\psi + G_{\psi\psi}(d\psi)^2 + G_{\phi\phi}(d\phi)^2 + e^{2\nu}\left(d\rho^2 + dz^2\right)$$

(**Metric coefficients are the functions of** ρ **and** z **.**)

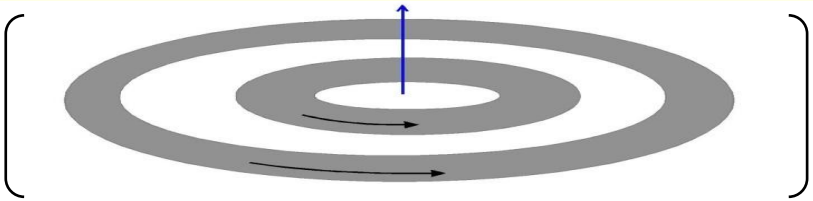
■ **Assumptions**

c1 (five dimensional spacetimes)
c2（the solutions of vacuum Einstein equations）
c3（three commuting Killing vectors including time-translational invariance） ----> $\left[R \times U(1)^2\right]$
c4（any type of angular momentums for ϕ**-rotation are zero）**
c5（asymptotic Minkowski)

Here we concentrate the study on

black dirings

(5 dim. concentrically superimposed two S^1-rotating BRs)



< Two different solution-sets of black di-rings >

(diring I)

the Backlund-like transformation.
(Kramer-Neugebauer's Method, …)

I&M: hep-th/0701043
Phys. Rev. D75, 064018 (2007)

(diring II)

Inverse Scattering Method (ISM)
(Belinsky-Zakharov technique)

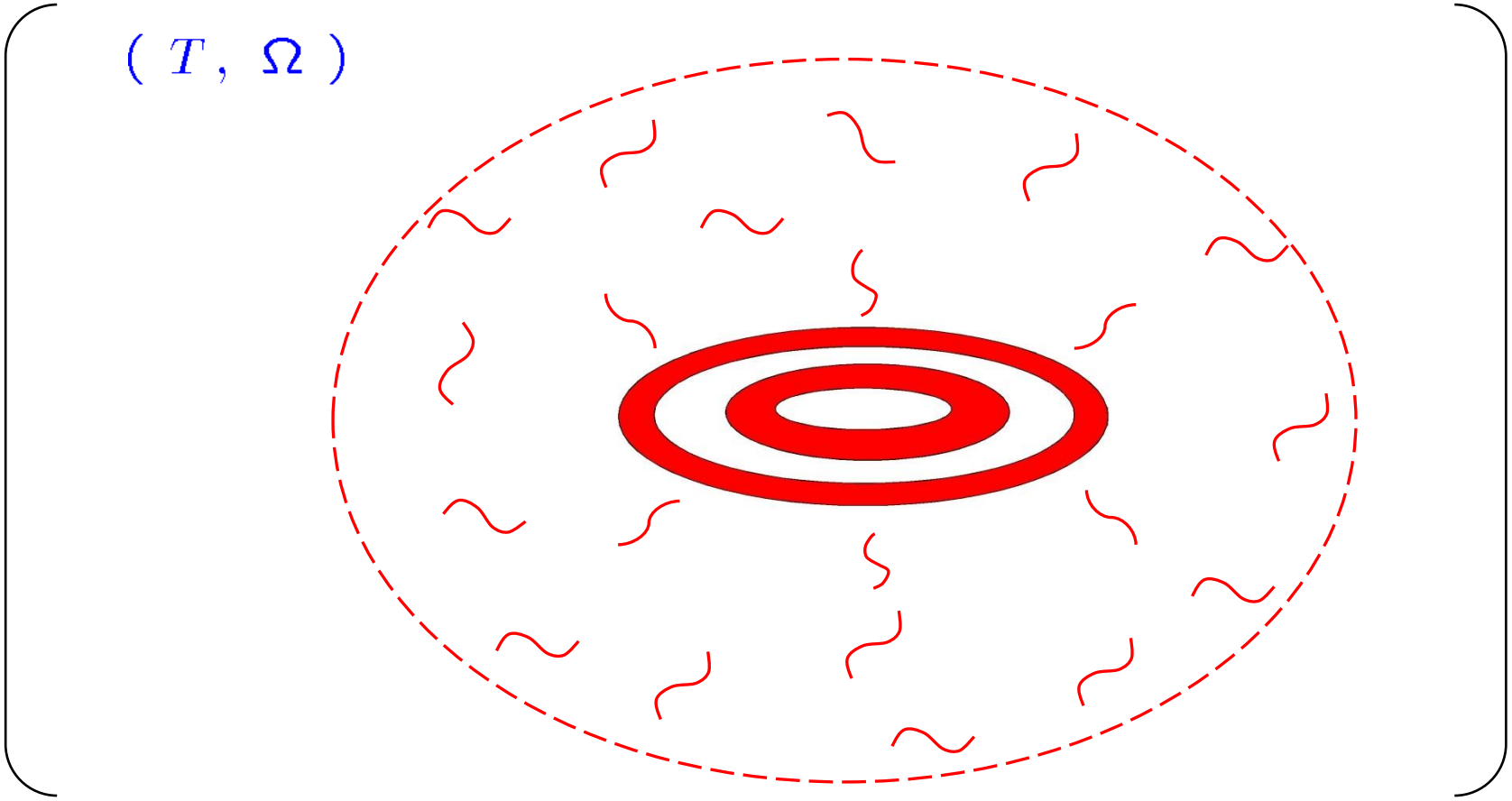
Evslin & Krishnan: hep-th/0706.1231
CQG26:125018(2009)

- **In the recent work, we confirmed the above two solution-sets are equivalent.**
  (arXiv:1008.4290v2[hep-th])

< Purpose of this talk >

We show some physical properties of the di-ring systems,

especially the existence and properties of thermodynamic regular black di-ring systems . (globally defined thermal states of regular black di-rings)



II. Existence of Regular thermodynamic black di-rings

Here we omit the explanation of the solution-generating methods and the expression of the di-ring solutions, and just show the results.

For more details,

See Phys.Rev.D75,064018(2007) hep-th/0701043 (I&M)

CQG26:125018(2009) hep-th/0706.1231 (E&K)

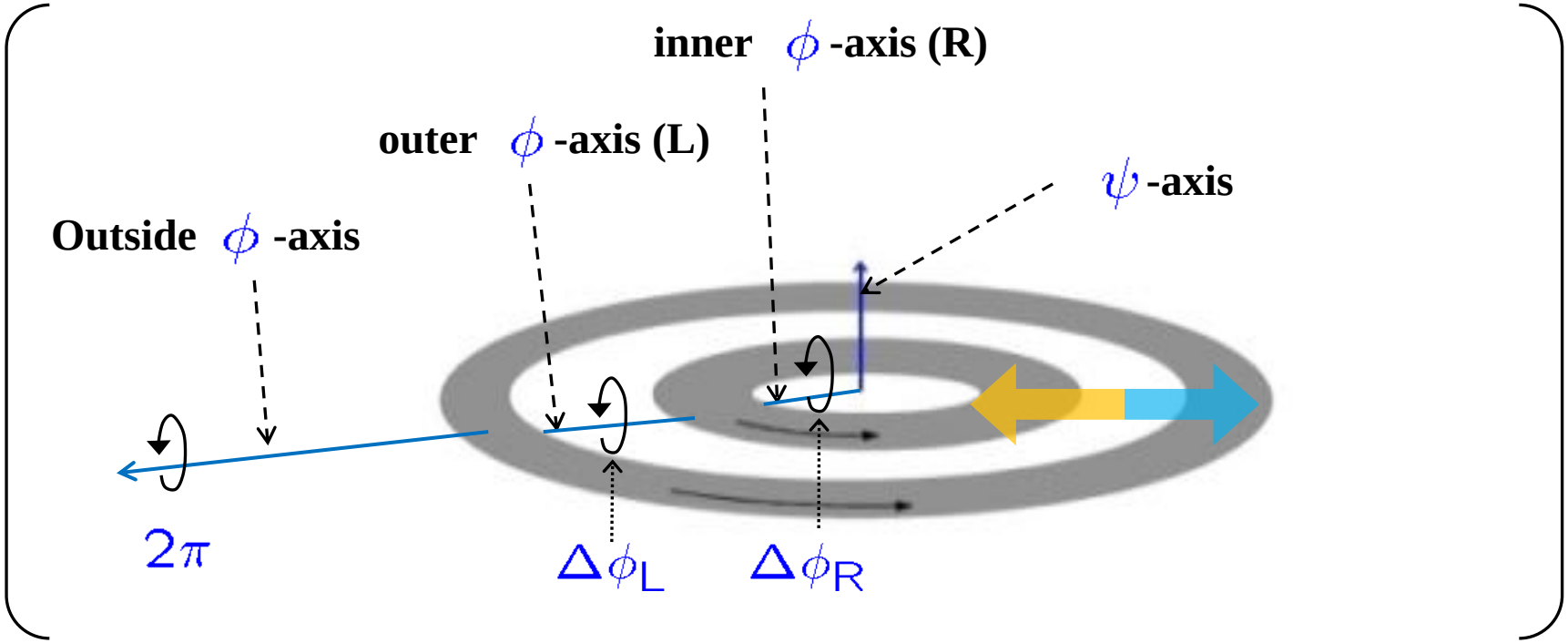
arXiv:1008.4290v2[hep-th] (I&M)

1. regular di-rings

The condition of conical singularity-free :

(= the balance condition of gravitational attraction and centrifugal force)

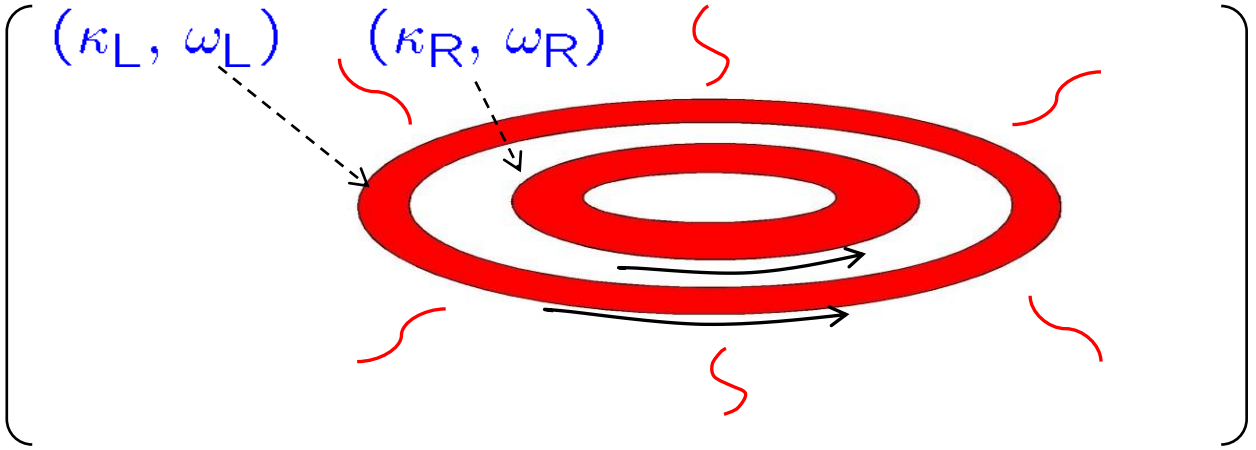
$$(1)\ \Delta\phi_L = 2\pi, \qquad (2)\ \Delta\phi_R = 2\pi$$



2. Global thermal equilibrium of black dirings

In the thermal system, a temperature and a angular velocity must be globally defined. $(T \sim \kappa)$ (Ω)

$$(3)\ \kappa = \kappa_L = \kappa_R, \quad (4)\ \omega = \omega_L = \omega_R$$



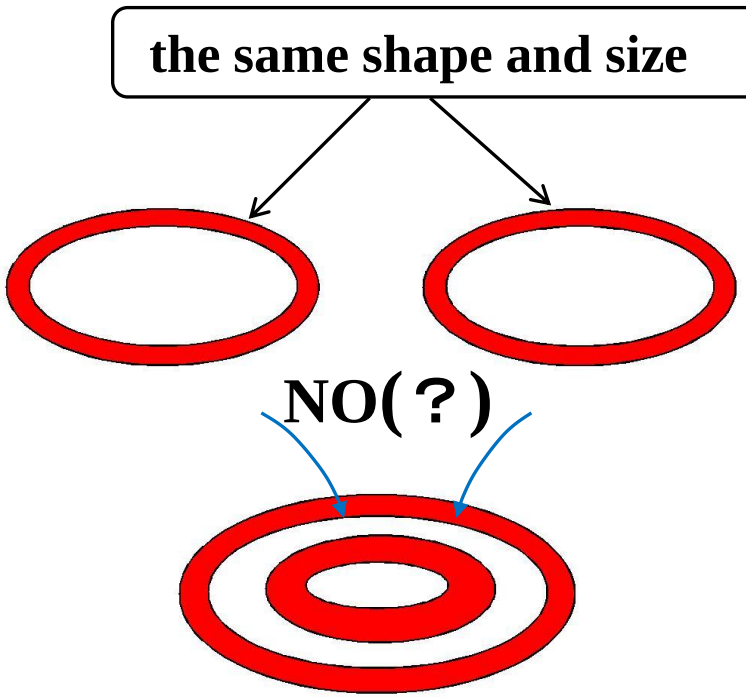
The first important problem is to determine

whether the thermodynamic regular black diring systems exist or not.

< Existence is not so trivial >

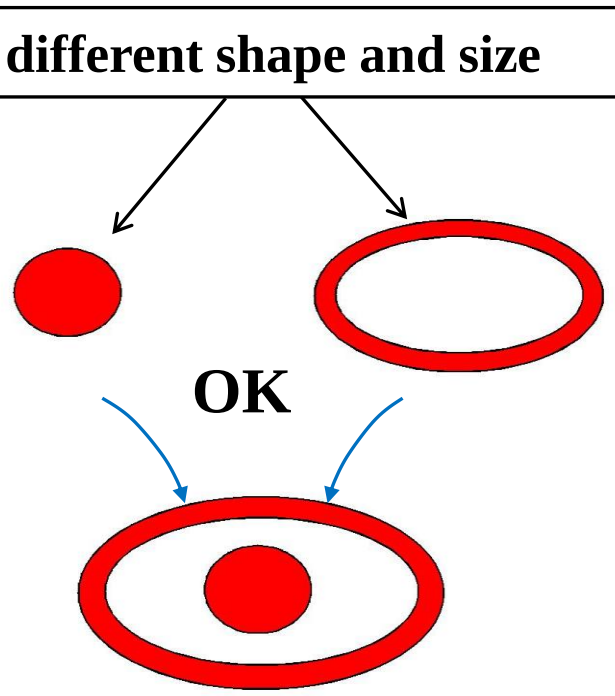
- some plausible suggestion … (Emparan et al.)

If two black rings have the same temperature and angular velocity, …



(Black Di-ring)

cf.

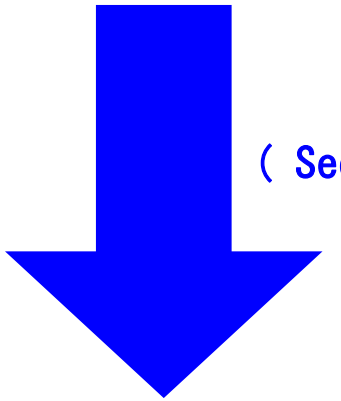


(Black Saturn)

If the answer is yes, some mechanism is necessary … ?

After describing the above four constrains of thermodynamic regular BD by appropriate four independent moduli-parameters, Solve the constraints:

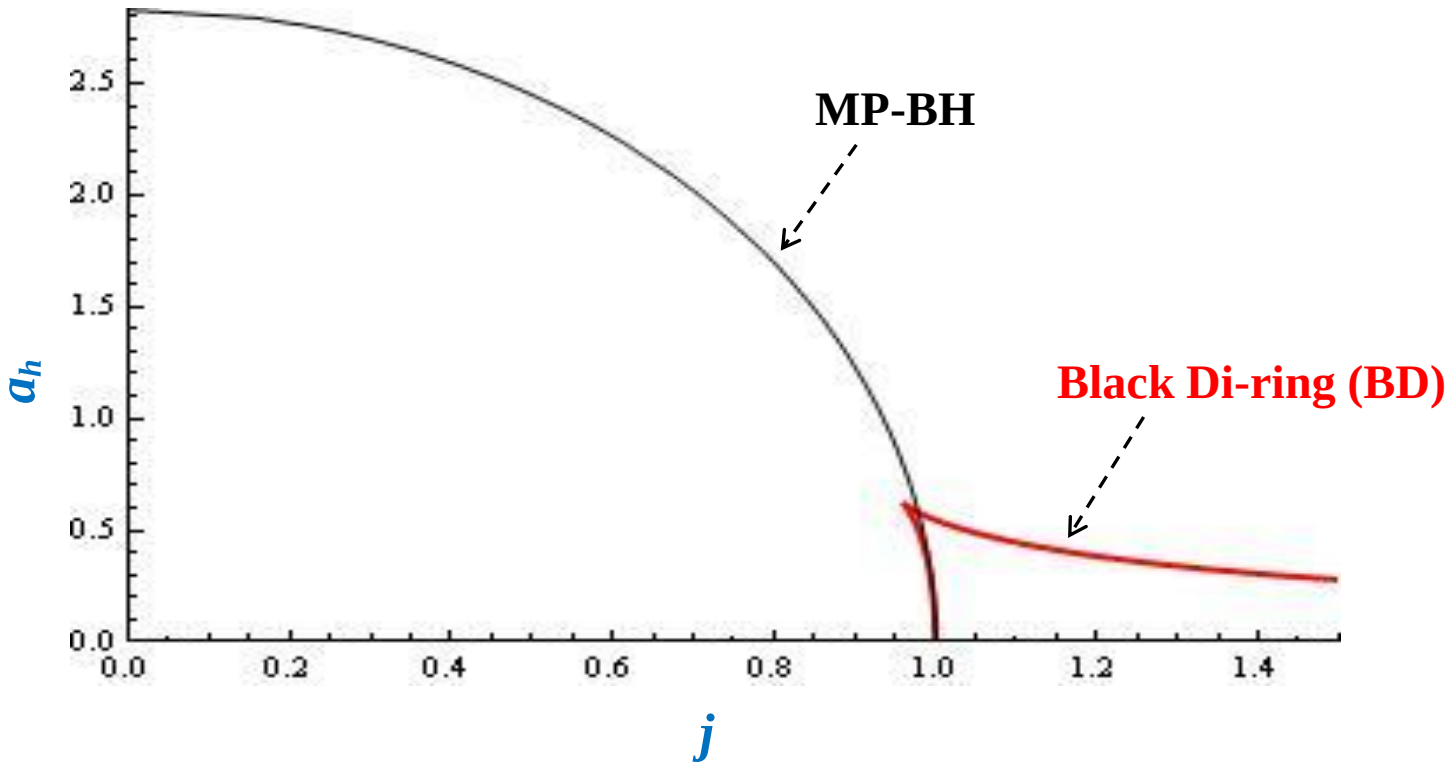
$$\left\{ \left(\frac{\Delta\phi_L}{2\pi}\right)^2 = 1,\ \left(\frac{\Delta\phi_R}{2\pi}\right)^2 = 1,\ \kappa_L = \kappa_R,\ \omega_L = \omega_R \right\}$$



(See arXiv:1008.4290v2[hep-th])

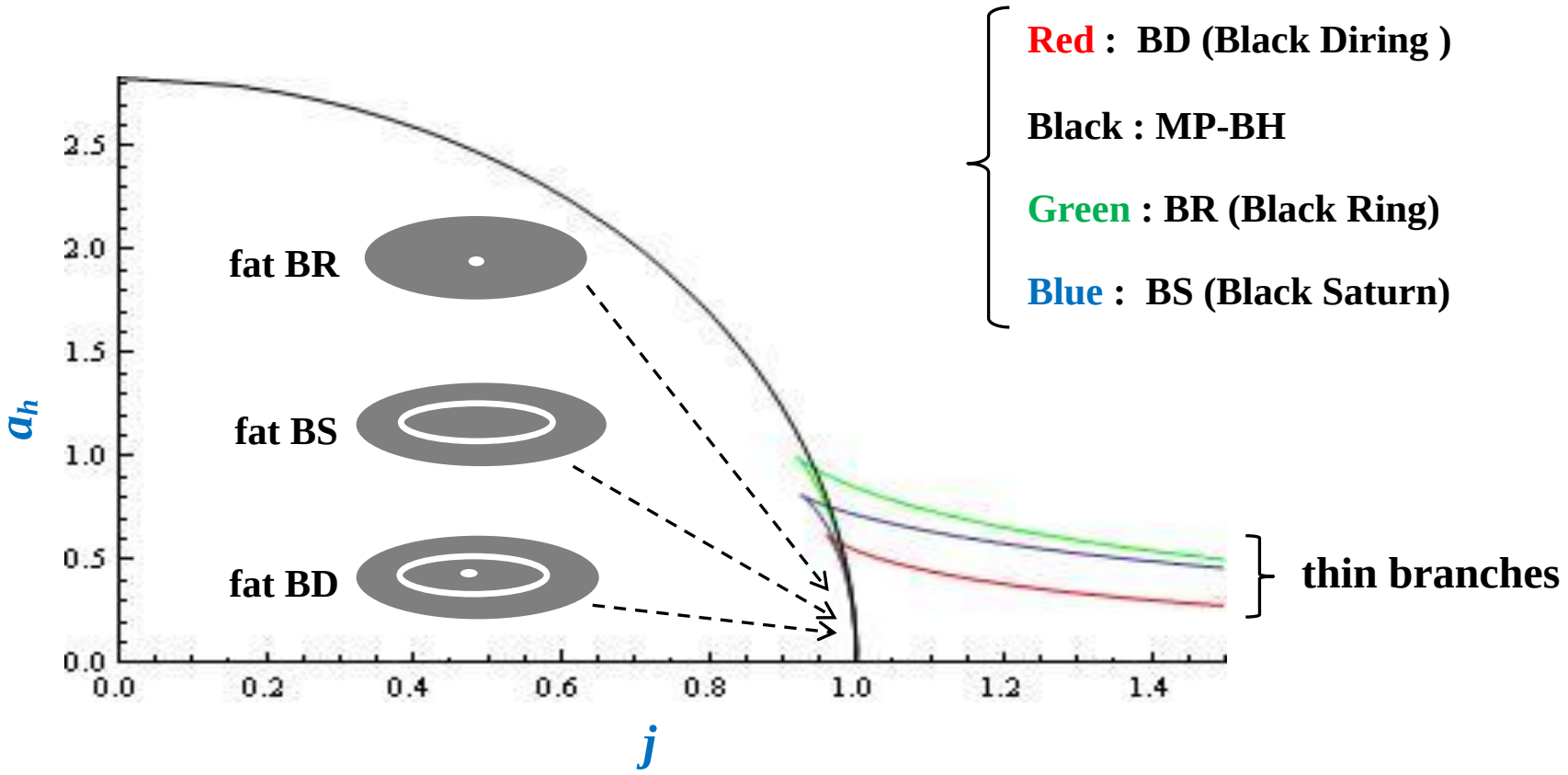
< Numerical result >

Phase diagram of total areas (a_h) against angular momentum (j)



- **Thermodynamic black di-ring systems exist !**

< Comparing thermodynamic black di-ring with other thermodynamic objects >



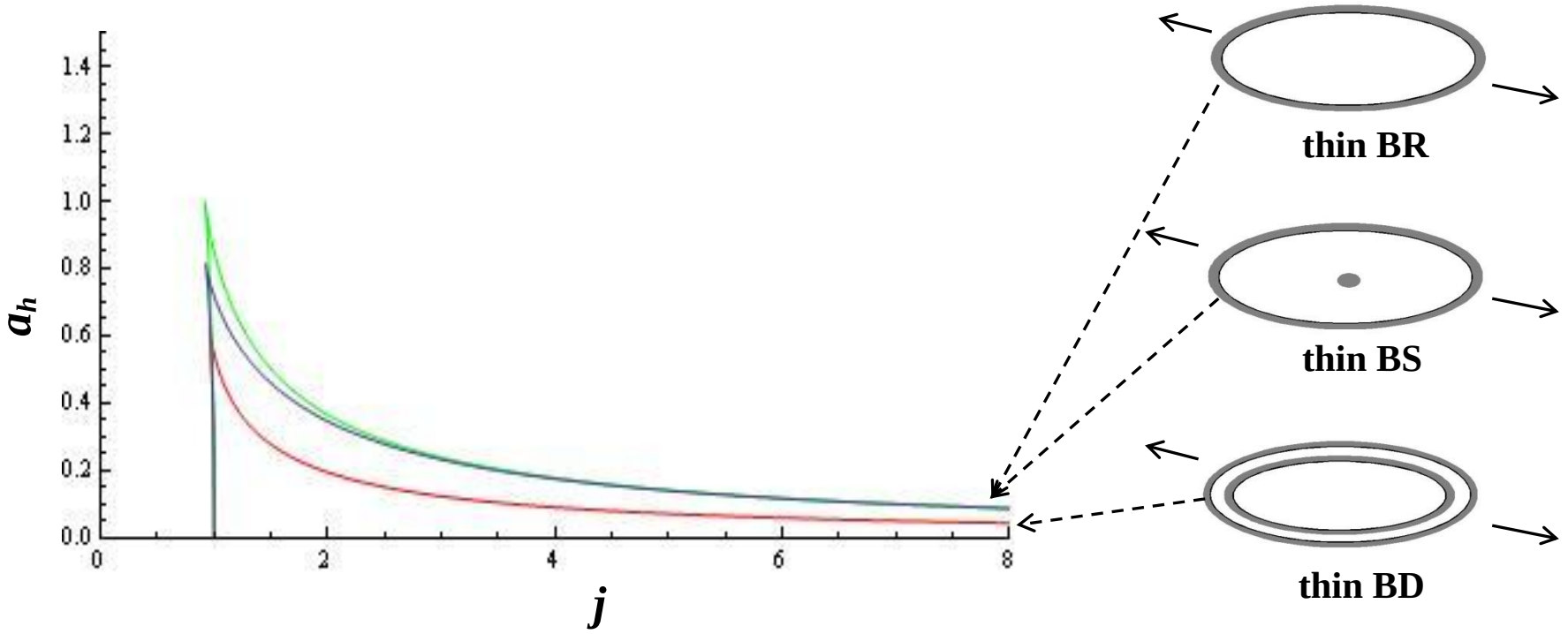
- The phase of thermodynamic diring has a similar behavior to the BR or BS system.

a ‘fat ring ’ branch and a ‘thin ring’ branch appear.

III. Some peculiarities of thermodynamic black di-rings

To clarify the property of TBD, we contrast TBD with TBS .

1. Behavior of total areas in the thin branch (large j)



- As j increases, BS immediately approaches BR, while BD does not approach the BR relatively.

- As j increases, in the black Saturn case the central black hole is just left, in the black di-ring case the inner ring always runs after the outer ring.

2. No thermal meta-stability of the thermodynamic diring system

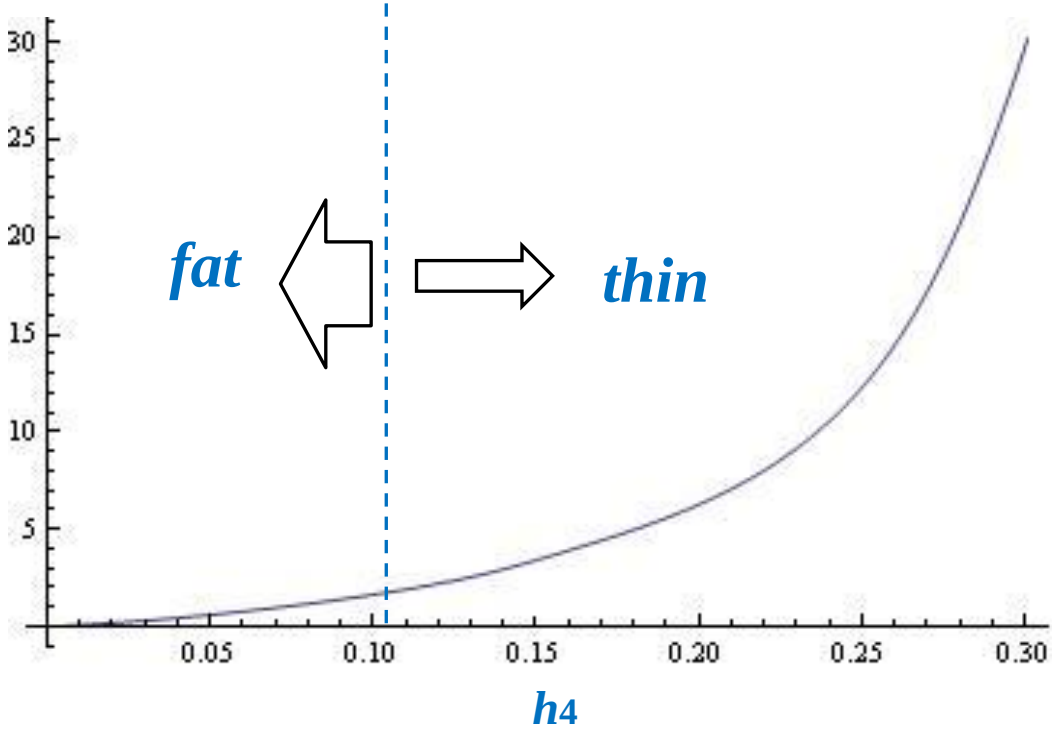
- **Next , as another peculiar property of the di-ring , we consider a certain kind of thermodynamic local stability.**
- **We follow the discussion introduced by Evslin & Krishnan.**
- **They treated black Saturn and found the existence of meta-stable states of black Saturn.**
- **To do this, under the condition of fixed mass and angular momentum , we shall search for local maxima of the corresponding entropy function, which is a function of appropriate moduli-parameters deduced from rod structure.**

- **If maximal eigenvalue of Hessian of the entropy function becomes negative, we can say that meta-stability occurs at this point.**

< Numerical result >

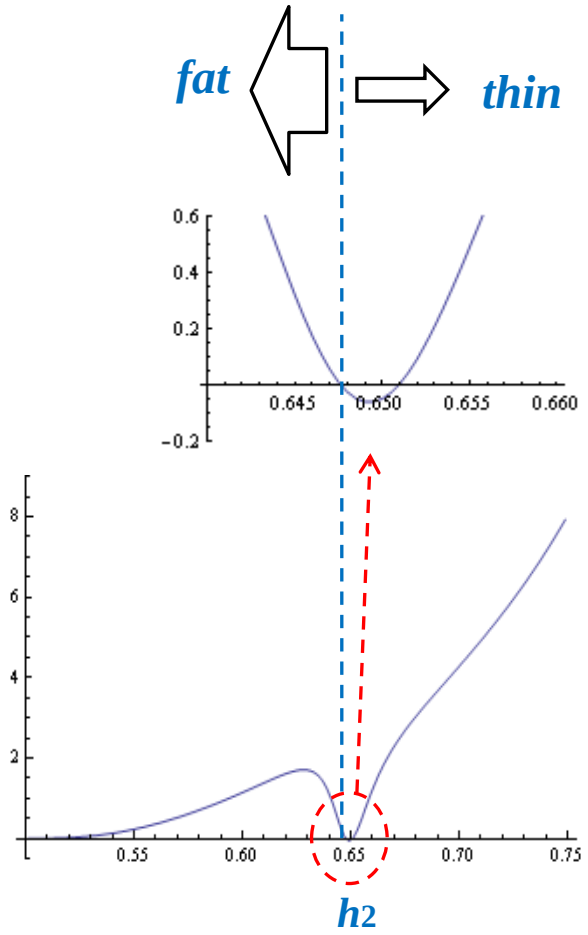
(BD)

Max eigenvalue of Hessian of Entropy



no meta-stability

(cf. BS)



meta-stability

III. Summary

- **The thermodynamic systems of regular di-rings exist.**
- **The phase of thermodynamic di-ring has a ‘fat ring’ branch and a ‘thin ring’ branch.**
- **In the thin branch, the behavior of the phase of black di-ring is very different from that of black Saturn.**
- **No meta-stable state seems to be realized in the thermodynamic black di-ring.**

✓ **See arXiv:1008.4290v2[hep-th] for further results and discussion.**

✓ **Similar results are also obtained by Emparan and Figueras.**
(arXiv:1008.3234[hep-th])