Gravitational wave background in modified gravity dark energy models

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1. Dark Energy models

Recent cosmological observations
(type Ia supernovae, WMAP, BAO, weak lensing...)
have revealed:

Dark Energy (DE) – 70 %
Dark Matter (DM) – 25 %

LambdaCDM model gives a successful description

But theoretical difficulties:
- Physical substance of DE and DM
- Cosmic coincidence
  i.e. discrepancy of many orders of magnitude
  between theoretical and observed values of Lambda

Many possible models of Dark Energy

- Quintessence
- K-essence with a non-canonical scalar field
- Phantom field with a negative kinetic term
- (Generalized) Chaplygin gas
  a fluid model unifying DE and DM $P = -A\rho^{-\alpha}$
- Dirac-Born-Infeld model
- $f(R)$-modified gravity
- $f(G)$-modified gravity ...

Modification of gravity theory will cause observational consequences in gravitational phenomena such as:

- solar-system experiments
- cosmic expansion
- large-scale structure formation
- cosmological GW background *(LISA, DECIGO,...)*

May be helpful to make more stringent constraints on DE models

We consider here:

GW background caused by gravity modification in dark energy context

both tensor and scalar perturbations

2. Basic equations in *f(R)*-modified gravity

Action:

$$S = \frac{1}{16\pi G}\int f(R)\sqrt{-g}\, d^4x \quad + S_{\mathrm{m}}$$

Field equation:

$$F(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} - F(R)_{;\mu\nu} + g_{\mu\nu}F(R)^{;\alpha}_{\ ;\alpha} = 8\pi G T^{(\mathrm{m})}_{\mu\nu}$$

$$F(R) := f'(R)$$

In the case of Einstein's gravity with Λ

$$f(R) = R - 2\Lambda\,, \quad F(R) = 1$$

In the case of *f(R)*-modified gravity

$$f(R) = R - \xi(R)\,, \quad F(R) = 1 - \xi'(R)$$

Equations for a background universe:

$$3H^2 = 8\pi G\rho_\mathrm{b} + \frac{1}{2}\xi(R_\mathrm{b}) + 3H[\xi'(R_\mathrm{b})]^{\cdot} - 3\xi'(R_\mathrm{b})(\dot{H}+H^2)$$

These terms play a role of **effective dark energy**

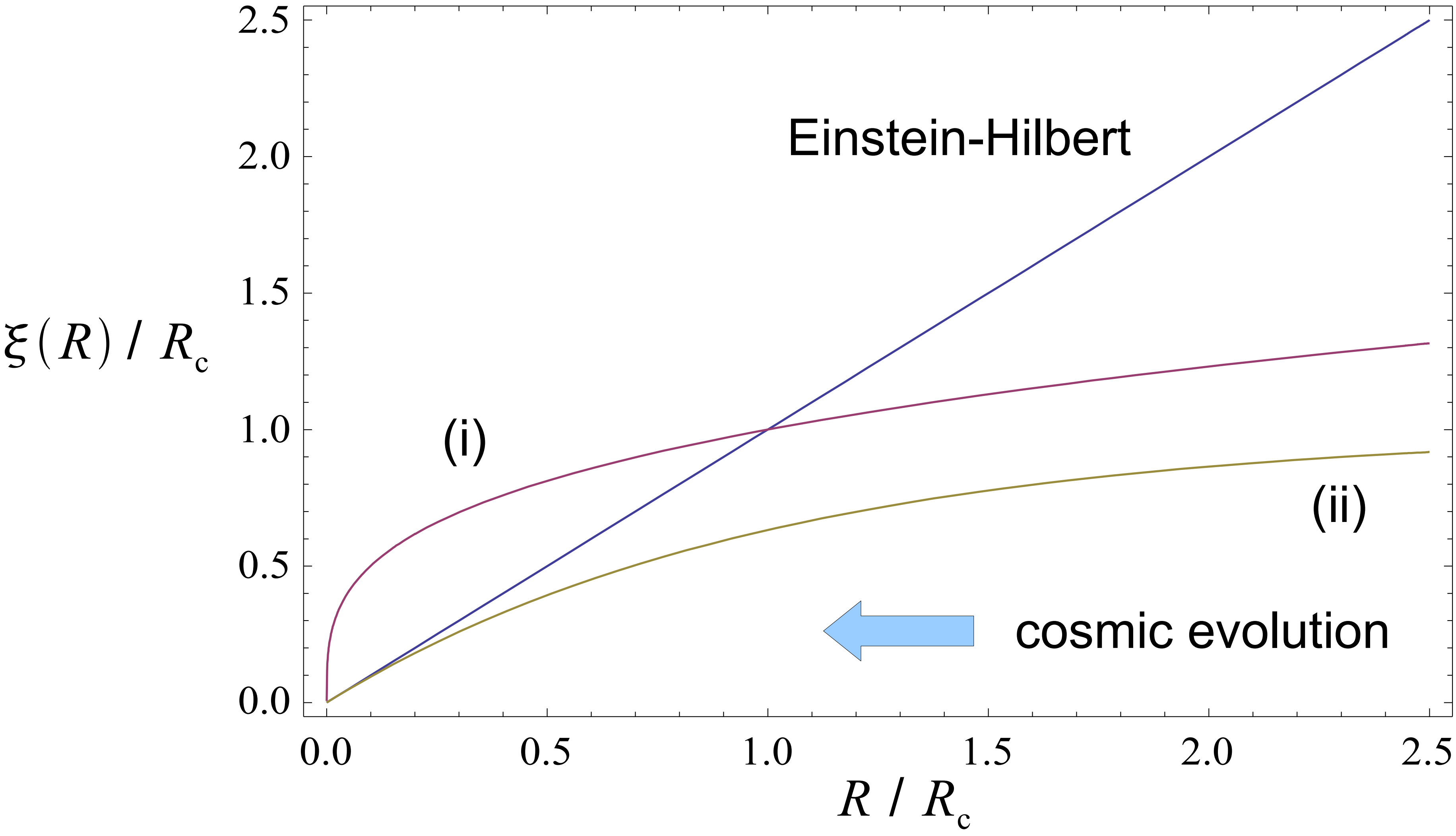
$$2\dot{H} = -8\pi G\rho_\mathrm{b} + 2\xi'(R_\mathrm{b})\dot{H} + [\xi'(R_\mathrm{b})]^{\cdot\cdot} - H[\xi'(R_\mathrm{b})]^{\cdot}$$

$$H := \frac{\dot{a}}{a}, \quad R_\mathrm{b} = 6\dot{H} + 12H^2$$

ρ_b : energy density of pressureless matter of a background universe (DM and baryon)

Cosmic evolution is quite complicated because the DE terms are functions of R

(i) $\xi(R) = \lambda R_c \left(\frac{R}{R_c}\right)^{\alpha} \quad (0 < \alpha < 1)$ (ii) $\xi(R) = \lambda R_c \left(1 - e^{-R/R_c}\right)$



3. Linear perturbations in *f(R)*-modified gravity

Comoving synchronous gauge conditon

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + {}^{(3)}g_{ij}\,\mathrm{d}X^i\,\mathrm{d}X^j$$
$$= -\mathrm{d}t^2 + a(t)^2(\gamma_{ij} + h_{ij})\,\mathrm{d}X^i\,\mathrm{d}X^j, \quad u^\mu = (1, \mathbf{0})$$

for irrotational dust matter $T^{\mu\nu}_{(\mathrm{m})} = \rho u^\mu u^\nu$

Metric perturbation ${}^{(3)}g_{ij} = a(t)^2(\gamma_{ij} + h_{ij})$

$$F(R) = F(R_\mathrm{b}) + F'(R_\mathrm{b})R_1 \quad =: F_\mathrm{b} + F_1$$
$$= 1 - \xi'(R_\mathrm{b}) - \xi''(R_\mathrm{b})R_1$$

$$R_1 = \ddot{h} + 4H\dot{h} + \frac{1}{a^2}\left(h^k{}_{l,k}{}^{,l} - \nabla^2 h\right), \quad h := \gamma^{ij}h_{ij}$$

Field equations yield the evolution equation for h_{ij}

$$\ddot{h}_{ij}+\left(3H+\frac{\dot{F}_{\mathrm{b}}}{F_{\mathrm{b}}}\right)\dot{h}_{ij}+\frac{2}{a^2}\left({}^{(3)}R_{ij}[h]-\frac{1}{4}\,{}^{(3)}R[h]\gamma_{ij}\right)$$

$$+\frac{1}{F_{\mathrm{b}}}\left[\ddot{F}_1+2H\dot{F}_1-(\dot{H}+3H^2)F_1\right]\gamma_{ij}-\frac{2}{F_{\mathrm{b}}a^2}F_{1,ij}=0$$

Mode decomposition of metric perturbations

$$h_{ij}=\underline{2\Phi\gamma_{ij}+2B_{,ij}}+\underline{h_{ij}^{\mathrm{TT}}}$$

scalar mode — tensor mode

(No vector mode because of no vorticity of matter considered)

For tensor-mode perturbations with wavenumber k

$$\ddot{h}_{ij}^{\mathrm{TT}} + \left(3H - \frac{\dot{\xi}'(R_{\mathrm{b}})}{1-\xi'(R_{\mathrm{b}})} \right) \dot{h}_{ij}^{\mathrm{TT}} + \frac{k^2}{a^2} h_{ij}^{\mathrm{TT}} = 0$$

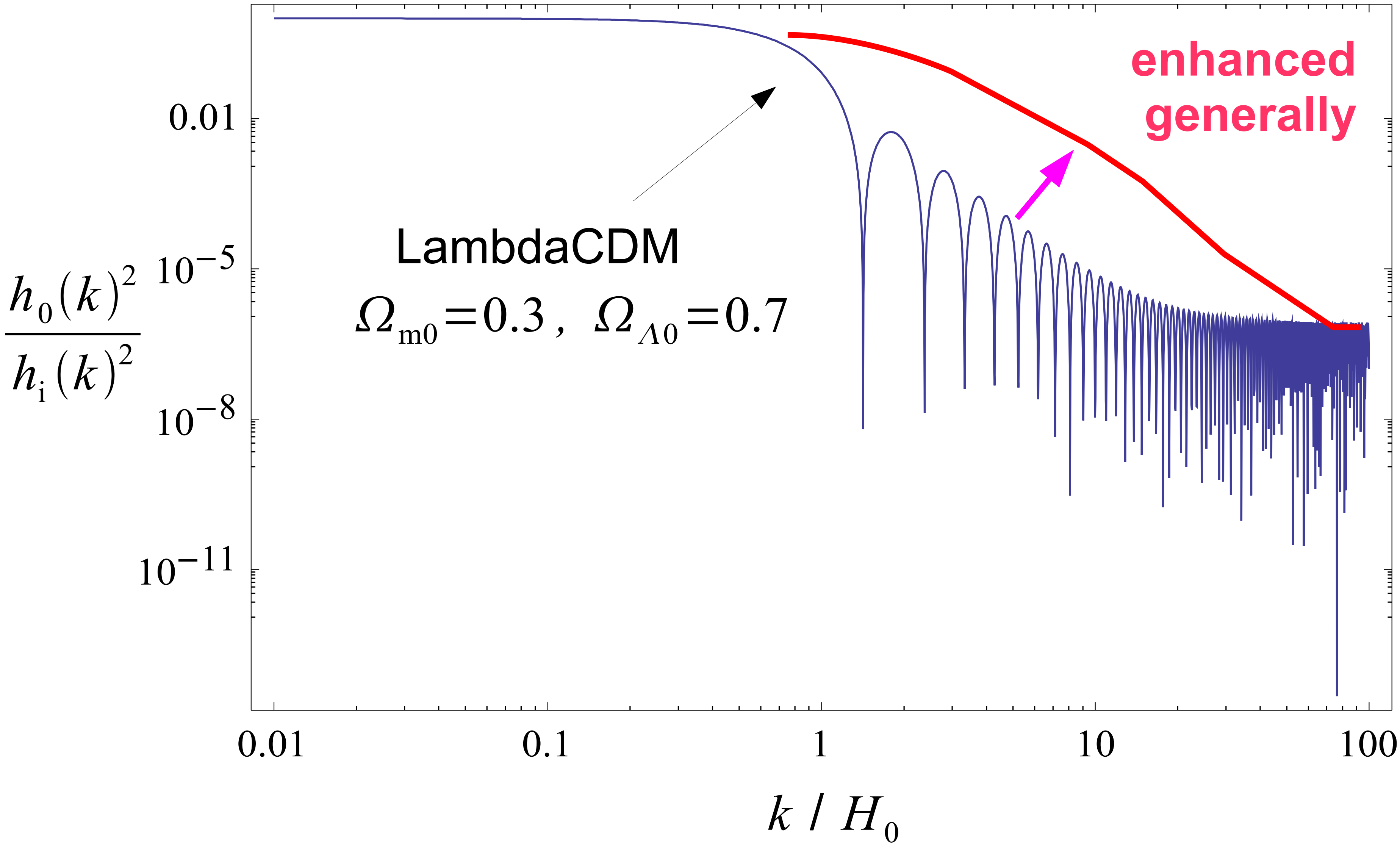
The effect of gravity modification appears only in the friction term

The gravity modification generally impairs the friction caused by cosmic expansion

In particular, when $1-\xi'(R_{\mathrm{b}}) \to 0$,

$$\dot{h}_{ij}^{\mathrm{TT}} \approx 0$$

``Transfer function"



Evolution equation for scalar perturbations:

$$\ddot{\varphi}+\left[3H-2\frac{(H+\dot{F}_{\mathrm{b}}/2F_{\mathrm{b}})^{\cdot}}{H+\dot{F}_{\mathrm{b}}/2F_{\mathrm{b}}}+\frac{(\dot{F}_{\mathrm{b}}^{2}/F_{\mathrm{b}})^{\cdot}}{\dot{F}_{\mathrm{b}}^{2}/F_{\mathrm{b}}}\right]\dot{\varphi}+\left(1+\frac{16\pi G\rho_{\mathrm{b}}}{3\dot{F}_{\mathrm{b}}^{2}/F_{\mathrm{b}}}\right)\frac{k^{2}}{a^{2}}\varphi$$

$$=4\pi G\rho_{\mathrm{b}}\left[\dot{h}-2\frac{(H+\dot{F}_{\mathrm{b}}/2F_{\mathrm{b}})^{\cdot}}{H+\dot{F}_{\mathrm{b}}/2F_{\mathrm{b}}}\delta_{\mathrm{m}}\right]$$

$$\varphi:=\Phi-\frac{H}{\dot{F}_{\mathrm{b}}}F_{1}$$

gauge-invariant variable

(Hwang & Noh 1996)

At late time of structure formation, $\rho_{\mathrm{b}} \approx 0$,

$$\ddot{\varphi} + \left[3H - 2\frac{\left(H + \dot{F}_{\mathrm{b}}/2F_{\mathrm{b}}\right)^{\cdot}}{H + \dot{F}_{\mathrm{b}}/2F_{\mathrm{b}}} + \frac{\left(\dot{F}_{\mathrm{b}}^{2}/F_{\mathrm{b}}\right)^{\cdot}}{\dot{F}_{\mathrm{b}}^{2}/F_{\mathrm{b}}} \right] \dot{\varphi} + \frac{k^{2}}{a^{2}}\varphi \approx 0$$

(quantitative estimations still under investigation)

In the case of Einstein's gravity, $F_{\mathrm{b}} \approx 1$,

$$h_{ij} = \text{ const} \cdot \Phi \gamma_{ij} + 2D(t)\,\Phi_{,ij} \qquad \Phi = \Phi(X)$$

no oscillations

linear growth rate of density perturbations

4. Summary and Outlook

- Cosmological GW background has been considered in *f(R)*-modified gravity in the context of dark energy cosmology
- In the tensor-mode perturbations, the effect of gravity modification appears only in the friction term, which may enhance the spectrum of GW background
- In the scalar-mode perturbations, scalar GWs emerge as the effect of gravity modification at late time of structure formation when matter density becomes small enough
- Now studying the relation to the constraints from density perturbations (done by Tsujikawa, Koyama,...)