New Horizons in Gravity: Dark Energy, & Condensate Stars

Macroscopic Effects of the Trace Anomaly & the Non-Singular Endpoint of Gravitational Collapse

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Review: arXiv: 1008.5006

w. **R. Vaulin**, *Phys. Rev. D* 74, 064004 (2006)

w. **P. Anderson & R. Vaulin**, *Phys. Rev. D* 76, 024018 (2007)

Review Article: w. **I. Antoniadis & Mazur**, *N. Jour. Phys.* 9, 11 (2007)

w. **M. Giannotti**, *Phys. Rev. D* 79, 045014 (2009)

w. **P. Anderson & C. Molina-Paris**, *Phys. Rev. D* 80, 084005 (2009)

w. **P. O. Mazur** Proc. Natl. Acad. Sci., 101, 9545 (2004)

Outline

- Classical Black Holes in General Relativity
- Quantum Effects -- Microscopic & Macroscopic
  - Entropy & the Second Law of Thermodynamics
  - Temperature & the 'Trans-Planckian Problem'
  - Negative Heat Capacity & the 'Information Paradox'
- Effective Theory of Low Energy Gravity
  - New Scalar Degrees of Freedom from the Trace Anomaly
  - Conformal Phase Transition and Running of Λ
  - Near Horizon Boundary Layer
- Gravitational Condensate Stars
- Cosmological Term as Macroscopic Dynamical Condensate

Classical Black Holes

Schwarzschild Metric (1916)

$$ds^2 = -dt^2 f(r) + \frac{dr^2}{h(r)} + r^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2\right)$$

$$f(r) = 1 - \frac{2GM}{r} = h(r)$$

Classical Singularities:

- $r = 0$: Infinite Tidal Forces, Breakdown of Gen. Rel.
- $r \equiv R_S = 2GM \quad (c = 1)$: Event Horizon, Infinite Blueshift, Change of sign of f, h

Trapping of light inside the horizon is what makes a black hole

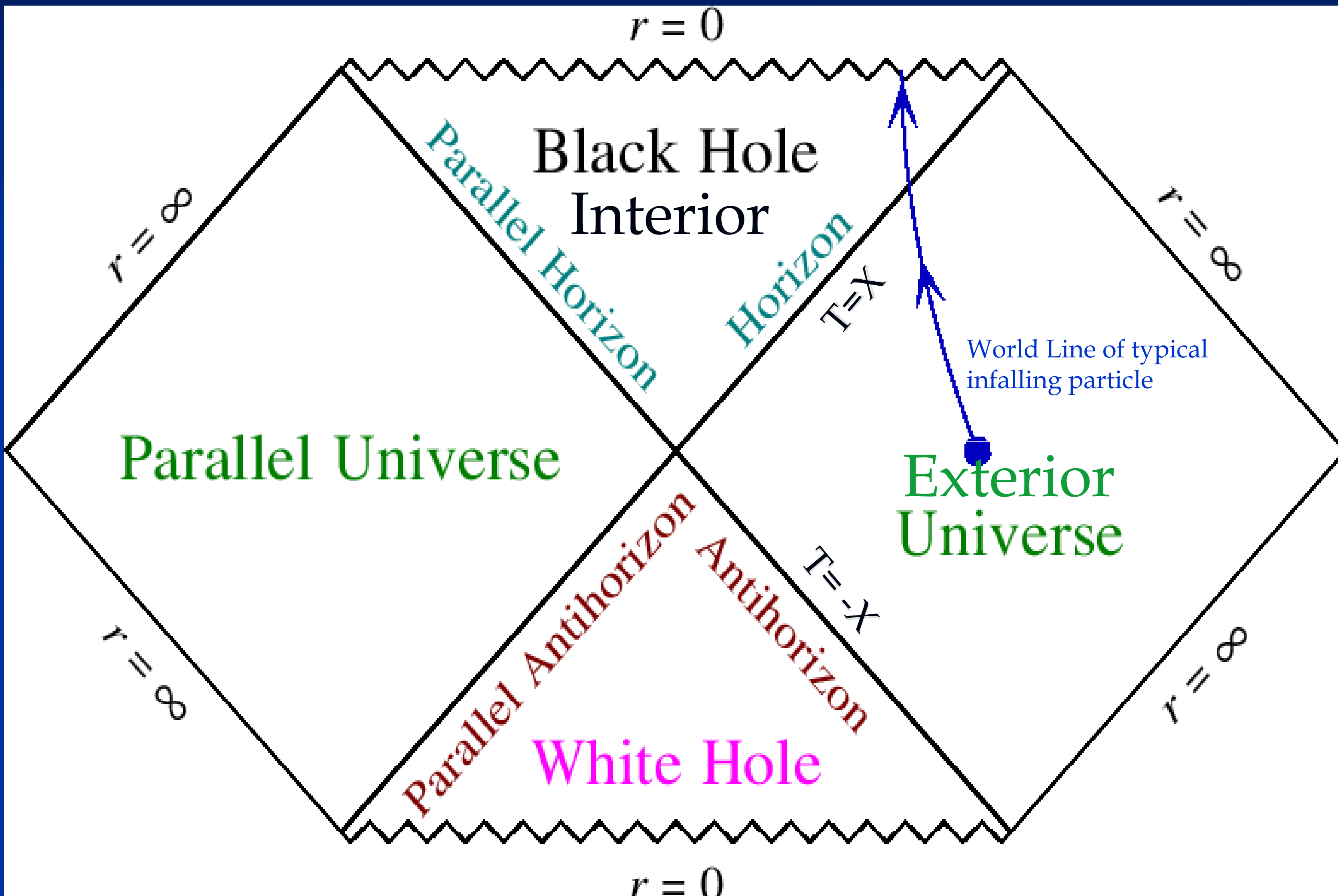
BLACK

The $r = R_S$ singularity is purely kinematic, removable by a coordinate transformation

iff $\hbar = 0$

Schwarzschild Maximal Analytic Extension

Carter-Penrose Conformal Diagram



Black Holes and Entropy

- A fixed classical solution usually has no entropy :
  (What is the "entropy" of the Coulomb potential $\Phi = Q/r$?)
  … But if matter/radiation disappears into the black hole,
  what happens to its entropy?
- Maybe M_{irr}^2 (which increases classically: Christodoulou)
  is a kind of "entropy"?
  To get units of entropy need to divide Area, A by $(\text{length})^2$
  … But there is no fixed length scale in classical Gen. Rel.
- Planck length $\ell_{Pl}^2 = \hbar G / c^3$ involves $\hbar$
- So Bekenstein suggested $S_{BH} = \gamma\, k_B\, A/L_{Pl}^2$ with $\gamma \sim O(1)$
- Hawking (1974) argued black holes emit thermal radiation at

$$T_H = \frac{\hbar c^3}{8\pi G k_B M}$$

Apparently then the first law, $dE = T_H\, dS_{BH}$ fixes $\gamma = 1/4$

Great, But …

A few new problems appeared…

- Hawking Temperature requires trans-Planckian frequencies
- $S_{BH} \propto A$ is non-extensive and *HUGE* (factor of 10^{19})
- In the classical limit $T_H \rightarrow 0$ (very cold) but $S_{BH} \rightarrow \infty$ (? !)
- $E \propto T^{-1}$ implies negative heat capacity

$$\frac{dE}{dT} << 0 \quad \Rightarrow \text{highly unstable}$$

Equilibrium Thermodynamics cannot be applied

- Information Paradox: Where does the information go?

  (Pure states → Mixed States? Unitarity ?)
- What is the statistical interpretation of S_{BH} ?

  Boltzmann asks: $S = k_B \ln W$??

Horizon in Quantum Theory

- Infinite Blueshift Surface

$$\omega_{local} = \omega_\infty \, (1 - 2GM/r)^{-1/2}$$

No problem classically, but in quantum theory

$$E_{local} = \hbar\omega_{local} = \hbar\omega_\infty \, (1 - 2GM/r)^{-1/2} \rightarrow \infty$$

$\hbar \rightarrow 0$ and $r \rightarrow 2GM$ limits do not commute ($\Rightarrow$ non-analyticity)

- *Singular* coordinate/gauge transformations need not be harmless
- Energies becoming trans-Planckian should call into doubt the semi-classical fixed metric approximation
- Large local energy densities/stresses are generic near the horizon

$$\langle T^a{}_b \rangle \sim \hbar\omega^4{}_{local} \sim \hbar M^{-4} \, (1 - 2GM/r)^{-2}$$

The geometry does may not remain unchanged down to $r = 2GM$

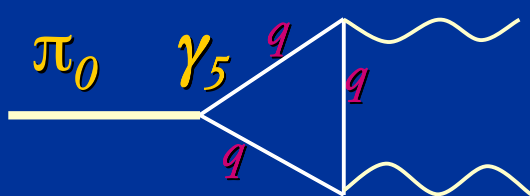
Quantum Backreaction is important

Effective Field Theory & Quantum Anomalies

- EFT = Expansion of Effective Action in *Local* Invariants
- Assumes Decoupling of Short (*UV*) from Long Distance (*IR*)
- But *Massless* Modes do not decouple
- Massless Chiral, Conformal Symmetries are *Anomalous*
- Macroscopic Effects of Short Distance physics
- Special Non-Local Terms Must be Added to Low Energy EFT
- *IR* Sensitivity to *UV* degrees of freedom
- Important on horizons because of large blueshift/redshift

Chiral Anomaly in QCD

- QCD with N_f massless quarks has an apparent $U(N_f) \otimes U_{ch}(N_f)$ Symmetry
- But $U_{ch}(1)$ Symmetry is Anomalous
- Effective Lagrangian in Chiral Limit has $N_f^2 - 1$ (*not* N_f^2) massless pions at low energies
- Low Energy $\pi_0 \rightarrow 2\gamma$ dominated by the anomaly



$$\partial_\mu j^{\mu 5} = e^2 N_c F_{\mu\nu} \tilde{F}^{\mu\nu} / 16\pi^2$$

- ***No Local*** Action in chiral limit in terms of $F_{\mu\nu}$ but ***Non-local*** *IR Relevant Operator* that violates naïve decoupling of *UV*
- ***Measured*** decay rate verifies $N_c = 3$ in QCD Anomaly Matching of *IR* ↔ *UV*

2D Gravity

$$S_{cl}[g] = \int d^2x \sqrt{g}(\gamma R - 2\lambda)$$

has no local degrees of freedom in 2D, since

$$g_{ab} = \exp(2\sigma)\bar{g}_{ab} \to \exp(2\sigma)\eta_{ab}$$

(all metrics conformally flat) and

$$\sqrt{g}R = \sqrt{\bar{g}}\bar{R} - 2\sqrt{\bar{g}}\,\bar{\Box}\,\sigma$$

gives a total derivative in S_{cl}.

Quantum Trace or Conformal Anomaly

$$\langle T_a{}^a \rangle = -\frac{c_m}{24\pi} R$$

$c_m = N_S + N_F$ for massless scalars or fermions.

Linearity in σ in the variational eq.

$$\frac{\delta \Gamma_{WZ}}{\delta \sigma} = \sqrt{g}\langle T_a{}^a \rangle$$

determines the Wess-Zumino Action by inspection:

2D Anomaly Action

- Integrating the anomaly linear in σ gives

$$\Gamma_{WZ} = (N/24\pi) \int d^2x \sqrt{\bar{g}}\, (-\sigma \overline{\Box} \sigma + \bar{R}\sigma)$$

- This is local but non-covariant. Note kinetic term for σ
- By solving for σ the WZ action can be also written

$$\Gamma_{WZ} = S_{anom}[g] - S_{anom}[\bar{g}]$$

- Polyakov form of the action is covariant but non-local

$$S_{anom}[g] = (-c/96\pi) \int d^2x \sqrt{g_x} \int d^2y \sqrt{g_y}\;\; R_x (\Box^{-1})_{xy} R_y$$

- A covariant and local form requires an auxiliary dynamical field φ

$$S_{anom}[g; \varphi] = (-c/96\pi) \int d^2x \sqrt{g}\, \{(\nabla\varphi)^2 - 2R\varphi\}$$

$$-\Box\varphi = R$$

Quantum Effects of 2D Anomaly Action

- Modification of Classical Theory required by Quantum Fluctuations & Covariant Conservation of $\langle T^a{}_b \rangle$
- Metric conformal factor $e^{2\sigma}$ (was constrained) becomes dynamical & itself fluctuates freely
- Gravitational 'Dressing' of critical exponents: long distance macroscopic physics
- Non-perturbative/non-classical conformal fixed point of 2D gravity: Running of Λ
- Additional non-local Infrared Relevant Operator in S_{EFT}

New Massless Scalar Degree of Freedom at low energies

Quantum Trace Anomaly in 4D Flat Space

Eg. QED in an External EM Field A_μ

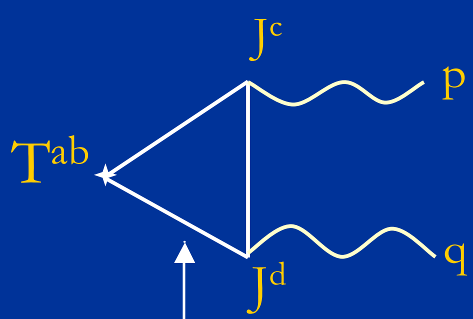
$$\langle T^{\mu}_{\mu} \rangle = \frac{e^2}{24\pi^2} F^{\mu\nu} F_{\mu\nu}$$

Triangle One-Loop Amplitude as in Chiral Case

$$\Gamma^{abcd}(p,q) = (k^2 g^{ab} - k^a k^b)(g^{cd}\, p\cdot q - q^c p^d)\, F_1(k^2) + \text{(traceless terms)}$$

In the limit of massless fermions, $F_1(k^2)$ must have a massless pole:

$k = p + q$



$$F_1(k^2) = \frac{e^2}{18\pi^2 k^2}$$

$$\rho_T(s) \to \frac{e^2}{6\pi^2}\,\delta(s)$$

Corresponding Imag. Part Spectral Fn. has a δ fn

This is a new massless scalar degree of freedom in the two-particle correlated spin-0 state

Constructing the EFT of Gravity

- Assume *Equivalence Principle* (Symmetry)
- Metric Order Parameter Field g_{ab}
- Only two strictly *relevant* operators (R, Λ)
- Einstein's General Relativity is an EFT
- But EFT = General Relativity **+** Quantum Corrections
- Semi-classical Einstein Eqs. $(k << M_{pl})$:

$$G_{ab} + \Lambda\, g_{ab} = 8\pi\, G \langle T_{ab} \rangle$$

- But there is also a quantum (trace) anomaly:

$$\langle T_a{}^a \rangle = b\, C^2 + b' \left(E - \tfrac{2}{3} \Box R\right) + b'' \Box R$$

- *New* (marginally) relevant operator(s) *needed*

4D Anomalous Effective Action

Conformal Parametization

$\rightarrow \qquad g_{ab} = \exp(2\sigma)\,\bar{g}_{ab}$

Since $\sqrt{g}\,F = \sqrt{\bar{g}}\,\bar{F}$

is independent of σ, and

$$\sqrt{g}\,(E - \tfrac{2}{3}\Box R) = \sqrt{\bar{g}}\,(\bar{E} - \tfrac{2}{3}\bar{\Box}\bar{R}) + 4\sqrt{\bar{g}}\bar{\Delta}_4\sigma$$

is linear in σ, the variational eq.,

$$\frac{\delta\Gamma_{WZ}}{\delta\sigma} = \sqrt{g}\langle T_a{}^a\rangle = b\,\sqrt{g}F + b'\sqrt{g}\,(E - \tfrac{2}{3}\Box R)$$

determines the Wess-Zumino Action by inspection:

$$\Gamma_{WZ} = 2b'\int d^4x\sqrt{\bar{g}}\,\sigma\bar{\Delta}_4\sigma$$
$$+ \int d^4x\sqrt{\bar{g}}\left[b\bar{F} + b'\left(\bar{E} - \frac{2}{3}\bar{\Box}\bar{R}\right)\right]\sigma\ ,$$

$$\Delta_4 \equiv \Box^2 + 2R^{ab}\nabla_a\nabla_b - \tfrac{2}{3}R\Box + \tfrac{1}{3}(\nabla^a R)\nabla_a$$

$F = C_{abcd}C^{abcd}$; $\quad E = R_{abcd}R^{abcd} - 4R_{ab}R^{ab} + R^2$

Effective Action for the Trace Anomaly
Local Auxiliary Field Form

$$S_{anom} = \frac{b}{2}\int d^4x\sqrt{-g}\left[-2\varphi\Delta_4\psi + F\,\varphi + \left(E - \frac{2}{3}\Box R\right)\psi\right]$$
$$+\frac{b'}{2}\int d^4x\sqrt{-g}\left[-\varphi\Delta_4\varphi + \left(E - \frac{2}{3}\Box R\right)\varphi\right]$$

- Two New Scalar Auxiliary Degrees of Freedom
- Variation of the action with respect to φ, ψ -- the auxiliary fields -- leads to the equations of motion,

$$\Delta_4\varphi = \frac{1}{2}\left(E - \frac{2}{3}\Box R\right) \qquad \Delta_4\psi = \frac{1}{2}F$$

$$\Delta_4 = \Box^2 + 2R^{ab}\nabla_a\nabla_b - \frac{2}{3}R\Box + \frac{1}{3}(\nabla^a R)\nabla_a$$

IR Relevant Term in the Action

The effective action for the trace anomaly scales logarithmically with distance and therefore should be included in the low energy macroscopic EFT description of gravity—

<u>**Not given in powers of Local Curvature**</u>

This is a non-trivial modification of classical General Relativity required by quantum effects in the Std. Model

$$S_{Gravity}[g,\varphi,\psi] = S_{H-E}[g] + S_{Anom}[g,\varphi,\psi]$$

Fluctuations of new scalar degrees of freedom allow $\Lambda_{\rm eff}$ to vary dynamically, and can generate a Quantum Conformal Phase of 4D Gravity where $\Lambda_{\rm eff} \to 0$

Dynamical Vacuum Energy

- Conformal part of the metric, $g_{ab} = e^{2\sigma}\ \bar{g}_{ab}$
  constrained --frozen--by trace of Einstein's eq. $R=4\Lambda$
  becomes dynamical and can fluctuate due to φ, ψ
- Fluctuations of φ, ψ describe a conformally invariant phase of gravity in 4D $\Rightarrow$ non-Gaussian statistics of CMB
- In this conformal phase G^{-1} and Λ flow to zero fixed point

I. Antoniadis, P. O. Mazur, E. M., Phys. Rev. D 55 (1997) 4756, 4770; Phys. Rev. Lett. 79 (1997) 14; N. Jour. Phys. 9, 11 (2007)

- The Quantum Phase Transition to this phase characterized by the 'melting' of the scalar condensate Λ
- Λ a dynamical state dependent condensate generated by SSB of global Conformal Invariance

Stress Tensor of the Anomaly

Variation of the Effective Action with respect to the metric gives stress-energy tensor

$$T_{\mu\nu}(g_{\mu\nu}, \varphi, \psi) = -\frac{2}{\sqrt{-g}} \frac{\delta S_{anom}}{\delta g_{\mu\nu}}$$

- Quantum Vacuum Polarization in Terms of (Semi-) Classical Auxiliary potentials
- φ, ψ Depends upon the global topology of spacetimes and its boundaries, horizons

Schwarzschild Spacetime (again)

$$ds^2 = -(1 - \tfrac{2M}{r})dt^2 + \frac{dr^2}{(1 - \tfrac{2M}{r})} + r^2 d\Omega^2$$

$$\varphi = \sigma = \ln\sqrt{f} = \frac{1}{2}\ln\left(1 - \frac{2M}{r}\right) \to \infty$$

solves homogeneous $\Delta_4\varphi = 0$

Timelike Killing field (Non-local Invariant)

$K^a = (1, 0, 0, 0)$ $\qquad$ $e^\sigma = (-K_a K^a)^{\frac{1}{2}} = \sqrt{f}$

Energy density scales like $e^{-4\sigma} = f^{-2}$

Auxiliary Scalar Potentials give Geometric (Coordinate Invariant) Meaning to Stress Tensor becoming Large on Horizon

Anomaly Scalars in Schwarzschild Space

- General solution of φ, ψ equations as functions of r are easily found in Schwarzschild case

$$\left.\frac{d\varphi}{dr}\right|_S = -\frac{1}{3M} - \frac{1}{r} + \frac{2Mc_H}{r(r-2M)} + \frac{c_\infty}{2M}\left(\frac{r}{2M} + 1 + \frac{2M}{r}\right) + \frac{q-2}{6M}\left(\frac{r}{2M} + 1 + \frac{2M}{r}\right)\ln\left(1 - \frac{2M}{r}\right) - \frac{q}{6r}\left[\frac{4M}{r-2M}\ln\left(\frac{r}{2M}\right) + \frac{r}{2M} + 3\right]$$

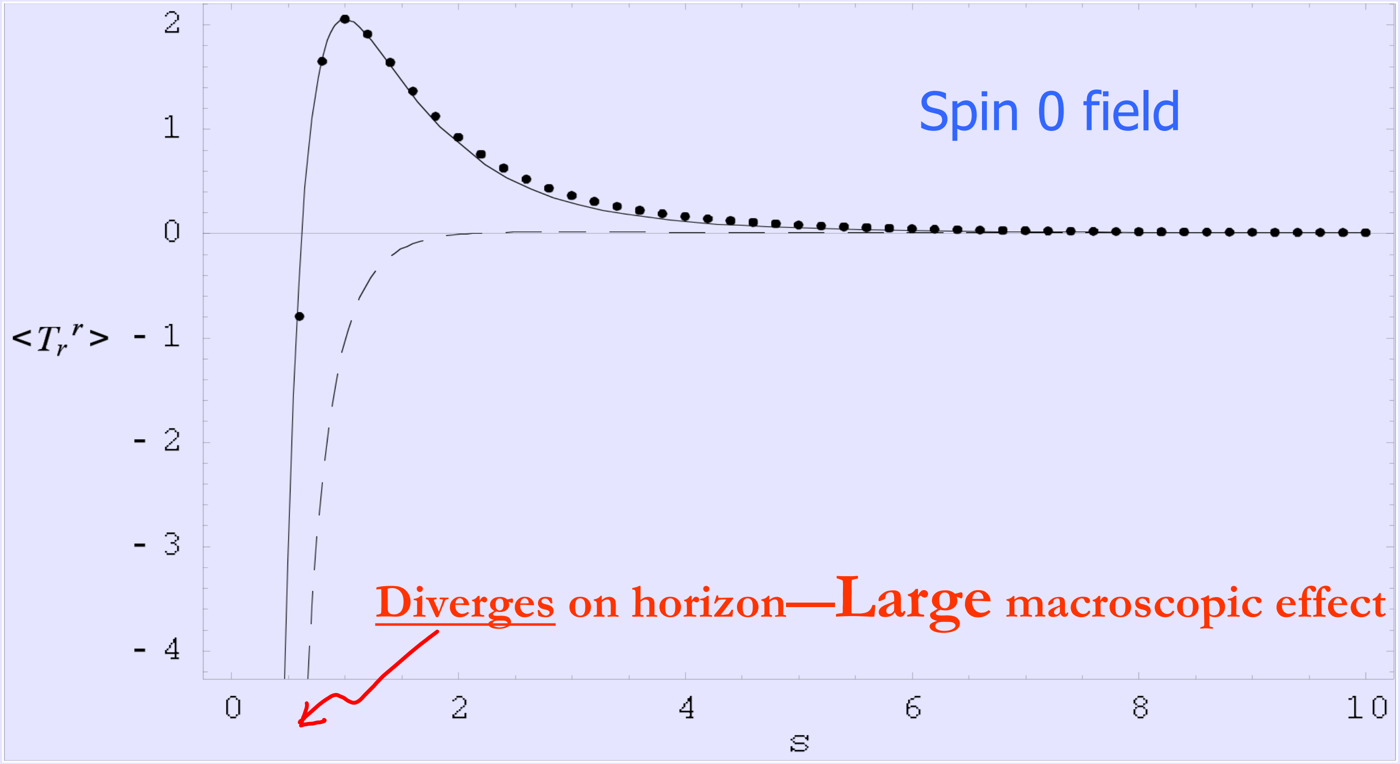
- q, c_H, c_∞ are integration constants, q topological charge
- Similar solution for ψ with q', c_H, c_∞
- Linear time dependence (p, p') can be added
- Only way to have vanishing φ as $r \to \infty$ is $c_\infty = q = 0$
- But only way to have finiteness on the horizon is

$$c_H = 0,\ q = 2$$

- Topological obstruction to finiteness vs. falloff of stress tensor
- Five conditions on 8 integration constants for horizon finiteness

Stress-Energy Tensor in Boulware Vacuum – Radial Component

Dots – Direct Numerical Evaluation of $<T_a{}^b>$ (Jensen et. al. 1992)
Solid – Stress Tensor from the Auxiliary Fields of the Anomaly (E.M. & R. V. 2006)
Dashed – Page, Brown and Ottewill approximation (1982-1986)



Quantum Effects Near $r = R_S$

- Huge Vacuum Stresses for generic b. c. at horizon:

$$\langle T_t^t \rangle \sim \langle T_r^r \rangle \sim \left(1 - \frac{2GM}{r}\right)^{-n}, \; n = \begin{cases} 1 & \text{Unruh} \\ 2 & \text{Boulware} \end{cases}$$

- Gravitational effects of quantum matter become strong near $r = R_S$ and affect the geometry.
- Strong attractive self-interactions $\Rightarrow$ Condensation.
- If Quantum Correlations $\langle T_a^b(x)\, T_c^d(y) \ldots \rangle$ also grow when $x, y, \ldots$ approach the horizon $\Rightarrow$ Highly Entangled Quantum State.
- Possibility of Quantum Phase Transition to BEC-like phase near $r = R_S$.
- Critical region where Sound Speed = Light Speed:

$$c_S^2 = \frac{dp}{d\rho} = c^2$$

  Any Additional Increase in Pressure would violate Causality: Onset of Superluminal Modes is the Signature of a Relativistic Phase Transition.

- A Critical Surface Layer with $p = \rho$ is Necessary for Joining $p = -\rho$ Interior with Vacuum Exterior.

Gravitational Vacuum Condensates

- Gravity is a theory of spin-2 bosons
- Its interactions are attractive
- The interactions become strong near $r = R_s$
- Energy of any scalar order parameter must couple to gravity with the vacuum eq. of state,
  $$p_V = -\rho_V = -V(\phi)$$
- Relativistic Entropy Density s is (for $\mu = 0$),
  $$Ts = p + \rho = 0 \text{ if } p = -\rho$$
- Zero entropy density for a single macroscopic quantum state, $k_B \ln \Omega = 0$ for $\Omega = 1$
- This eq. of state violates the energy condition, $\rho + 3p \geq 0$ (if $\rho_V > 0$) needed to prove the classical singularity theorems
- Dark Energy acts as a repulsive core

A GBEC phase transition can stabilize a high density, compact cold stellar remnant to further gravitational collapse

A New Soln. to Einstein Eqs.

$$R_a{}^b - \frac{1}{2} R\, \delta_a{}^b = 8\pi G\, T_a{}^b$$

- $1 - \frac{d(r\,h)}{dr} = 8\pi G\, \rho\, r^2$
- $\frac{r\,h}{f} \frac{df}{dr} + h - 1 = 8\pi G\, p\, r^2$
- $\frac{dp}{dr} + \frac{p+\rho}{2f} \frac{df}{dr} = 0 \qquad (\nabla_b T_r{}^b = 0)$

Other components follow by differentiating these

Define $h \equiv 1 - \frac{2Gm(r)}{r}$

Then $\frac{dm}{dr} = 4\pi\, \rho\, r^2$ and

$$\frac{dp}{dr} = -\frac{G(\rho+p)(m+4\pi p r^3)}{r\,(r-2Gm)} \qquad \text{(TOV eq.)}$$

Eqs. become closed when eq. of state is given:

$$p = \kappa\, \rho$$

with
$$\kappa = \begin{cases} -1, & r < r_1 \\ +1, & r_1 < r < r_2 \end{cases}$$
$$p = \rho = 0\,, \quad r_2 < r$$

A Simple Model

Proc. Natl. Acad. Sci., 101, 9545 (2004)

Main Features of New Soln.

- Vacuum Schwarzschild Exterior
- de Sitter (GBEC) Interior, No Singularity
- $\Lambda > 0$ Casimir Energy due to b.c.
- GBEC similar to Gluon Condensate in Bag Model of Hadrons
- Thin Shell of $p = \rho$, No Event Horizon
- Global Time, Unitarity, No Hawking Radiation
- Modest Entropy, No Information Paradox
- Maximizes Entropy, Completely Stable
- No Planckian Pressures or Densities
- Hydrodynamic Einstein Eqs. Valid Everywhere except at r_1, r_2 Stationary Shock Fronts
- Interior de Sitter also a Cosmological Soln.

Analog to BEC quantum transition near the classical horizon

Gravitational Vacuum Condensate Stars

Gravastars as Astrophysical Objects

- Cold, Dark, Compact, Arbitrary M, J
- Accrete Matter just like a black hole
- But matter does not disappear down a 'hole'
- Relativistic Surface Layer can re-emit radiation
- Can support Electric Currents, Large Magnetic Fields
- Possibly more efficient central engine for Gamma Ray Bursters, Jets, UHE Cosmic Rays
- Formation should be a violent phase transition converting gravitational energy and baryons into HE leptons and entropy
- Gravitational Wave Signatures
- Dark Energy as Condensate Core -- Finite Size Casimir effect of boundary conditions at the horizon

Cosmological Horizon Modes

w. P. R. Anderson & C. Molina-Paris, *Phys. Rev. D* 80, 084005 (2009)

- Variation of $\langle T^a{}_b \rangle$ in de Sitter space contains contributions from S_{anom} of scalar auxiliary fields φ, ψ
- Additional massless scalar degrees of freedom in cosmology
- The relevant scalar modes satisfy second order wave eqs. ("Inflaton without inflaton")
- Couple weakly to the metric with strength $G_N H^2 << 1$
- But grow significant close to the de Sitter horizon $r_H = H^{-1}$

$$G_N \delta \langle T^t_t \rangle \sim \frac{G_N H^4}{(1 - H^2 r^2)^2}$$

- Becomes of order of classical $R^t{}_t = 3H^2$ at a proper distance from r_H

$$\ell \sim \sqrt{r_H L_{Pl}}$$ "Healing Length"

- Same as proper distance outside the Schwarzschild horizon

New Horizons in Gravity

- Einstein's classical theory receives Quantum Corrections relevant at macroscopic Distances & near Event Horizons
- These arise from new scalar degrees of freedom in the EFT of Gravity required by the Conformal/Trace Anomaly
- At horizons these massless scalar degrees of freedom can have macroscopically large effects on spacetime
- Their Fluctuations can induce a *Quantum Phase Transition* at the horizon of a 'black hole'
- Λ_{eff} is a dynamical condensate which can change in the phase transition & remove 'black hole' interior singularity

- Gravitational Condensate Stars resolve all ‘black hole’ paradoxes ⇒ Astrophysics of gravastars testable
- The cosmological dark energy of our Universe may be a macroscopic finite size effect whose value depends not on microphysics but on the cosmological horizon scale