Behaviour of horizons under conformal transformations of the metric

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Outline

- Want to find a physical characterisation of the boundary of a black hole
- Trapping horizons, based on MOTS, are not invariant under conformal transformations (but the event horizon is)
- KEY IDEA: Modify the trapping horizon conditions to make them invariant under conformal transformation
- The second law can be proved for these “new” horizons

Trapping horizons

$$\theta_l = 0 = \frac{l^a \nabla_a A}{A}$$

$$\theta_n < 0$$

$$n^a \nabla_a \theta_l < 0$$

Conformal transformations

$$\tilde{g}_{ab} = W(x)\, g_{ab}$$

$$\tilde{\theta}_l = \theta_l + \frac{l^a \nabla_a W}{W}$$

Wald Entropy (and possibly others)

$$S_g = -2\pi \int \frac{\delta L}{\delta R^{abcd}} \varepsilon$$

$$\tilde{S}_g = W(x) S_g$$

Conformal trapping horizons

$$l^a \nabla_a S_g = 0$$

$$n^a \nabla_a S_g < 0$$

$$n^a \nabla_a \left(l^a \nabla_a S_g \right) < 0$$

Second law for conformal trapping horizons

$$r^a = B\, l^a + C\, n^a$$

$$r^a \nabla_a (l^a \nabla_a S) = 0$$

$$C = \frac{-B\, l^a \nabla_a (l^a \nabla_a S)}{n^a \nabla_a (l^a \nabla_a S)}$$

$$r^a \nabla_a S = B\, l^a \nabla_a S + C\, n^a \nabla_a S$$

$$r^a \nabla_a S = \frac{-B\, n^a \nabla_a S}{n^a \nabla_a (l^a \nabla_a S)} l^a \nabla_a (l^a \nabla_a S)$$

In the case of Brans-Dicke

$$r^a \nabla_a S = \frac{-B\, n^a \nabla_a S}{n^a \nabla_a (l^a \nabla_a S)} l^a \nabla_a (l^a \nabla_a S)$$

$$S = W(x) A$$

$$l^a \nabla_a (l^a \nabla_a S) = -AW \left(\frac{3}{2} \theta_l^2 + \sigma^2 + \frac{8\pi G}{W} T_{ab} l^a l^b \right) - A\,\omega\, W (l^a \nabla_a w)^2$$

END

Thank you