Isotropization of non-diagonal Bianchi I symmetric spacetimes with collisionless matter

(*based on arXiv 1007.0184*)

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Why study Bianchi spacetimes?

- Inhomogeneous spacetimes are quite difficult
- BKL-analysis of cosmological singularities, general solutions behave like homogeneous solutions (Bianchi IX "Mixmaster")

Why am I studying Bianchi spacetimes?

- To study the (mathematical) structure of the Einstein equations
- Study of the Einstein equations with a *kinetic description* of the matter
- Study of the asymptotics, first the “easy” direction in an “easy” Bianchi type

My spacetime: the easiest case, Bianchi I

- ${}^4g = -dt^2 + g_{ab}(t)dx^a dx^b$
- The “easiest” homogeneous spacetime which is not (necessarily) isotropic
- Translations in $\mathbb{R}^3$ as homogeneity group
- “Basis” for higher Bianchi types
- The only Bianchi type besides type VII_0 which includes Flat Friedmann, but in principle all Bianchi types are compatible with a *quasiisotropic* epoch...

My matter model: collisionless matter

- Vlasov = Boltzmann without collision term
- Kinetic description $f(t, x^a, p^a)$ often used in (astro)physics
- Perfect fluid is somehow natural in an isotropic spacetime, assuming a linear equation of state makes life simple...
- Vlasov avoids certain unphysical singularities

- For Bianchi I the Vlasov equation

$$\frac{\partial f}{\partial t} + 2k^a_b p^b \frac{\partial f}{\partial p^a} = 0.$$

  can be solved explicitly and f if expressed as a function of p_i is independent of time
- In particular

$$\rho = \int f_0(p_i)(m^2 + g^{cd} p_c p_d)^{\frac{1}{2}} (\det g)^{-\frac{1}{2}} dp_1 dp_2 dp_3$$

$$S_{ab} = \int f_0(p_i) p_a p_b (m^2 + g^{cd} p_c p_d)^{-\frac{1}{2}} (\det g)^{-\frac{1}{2}} dp_1 dp_2 dp_3$$

$$T_{0a} = 0$$

- The fact that the dependence on p_i remains makes the whole system a borderline case between dynamical systems and systems of PDE due to the "integro-differential" aspect. We will use PDE-techniques.

3+1 decomposition and initial value formulation

- Instead of $G_{\mu\nu} = 8\pi T_{\mu\nu}$ we use spatial metric g_{ab} and the second fundamental form k_{ab}.
- Then EE for Bianchi I simplify to

$$\dot{g}_{ab} = -2k_{ab}$$
$$\dot{k}_{ab} = Hk_{ab} - 2k_{ac}k_b^c - 8\pi(S_{ab} - \frac{1}{2}g_{ab}\,\mathrm{tr}\,S) - 4\pi\rho g_{ab}$$

where we have used the notations $\mathrm{tr}\,S = g^{ab}S_{ab}$, $H = \mathrm{tr}\,k$. From the constraint equations:

$$\begin{aligned} -k_{ab}k^{ab} + H^2 &= 16\pi\rho \\ T_{0a} &= 0 \end{aligned}$$

Physical motivation: Stability of Einstein-de Sitter

- Matter-dominated Era, relatively small eigenvelocities of galaxies
- Does Velocity dispersion decay due to expansion?
- Isotropization?
- "Structural stability" of the fluid model at late times?
- Yes for Bianchi I + reflection symmetry [Rendall 96]
- This model is diagonal, what happens for the non-diagonal case?

Small data assumptions

- Close to isotropic: shear parameter, $F = \frac{\sigma_{ab}\sigma^{ab}}{H^2}$ is small, where σ_{ab} is the trace-free part and H the trace of the second fundamental form
- $F \sim$ temperature fluctuations $\Delta T/T$
- Maximal Velocities P are bounded, i.e. the spacetime is close to dust, where P is

$$P(t) = \sup\{|p| = (g^{ab}p_a p_b)^{\frac{1}{2}} | f(t,p) \neq 0\}$$

Theorem

Consider any C^∞ solution of the Einstein-Vlasov system with Bianchi I-symmetry and with C^∞ initial data. Assume that $F(t_0)$ and $P(t_0)$ are sufficiently small. Then at late times one can make the following estimates:

$$\begin{aligned} H(t) &= -2t^{-1}(1+O(t^{-1})) \\ P(t) &= O(t^{-\frac{2}{3}+\epsilon}) \\ F(t) &= O(t^{-2}) \end{aligned}$$

[2nd eq. implies that asymptotically there is a dust-like behaviour $S_{ab}/\rho = O(t^{-\frac{4}{3}+\epsilon})$]

Generalized Kasner exponents

- Let λ_i be the eigenvalues of k_{ij} with respect to g_{ij}, i.e., the solutions of $\det(k^i_j - \lambda\delta^i_j) = 0$
- We define $p_i = \frac{\lambda_i}{H}$ as the *generalized Kasner exponents*. They satisfy the first but not the second Kasner relation
- Example: Einstein - de Sitter ($p_i = \frac{1}{3}$)

$$^4g = -dt^2 + t^{\frac{4}{3}}(dx^2 + dy^2 + dz^2).$$

Theorem

Consider the same assumptions as in the previous theorem. Then

$$p_i = \frac{1}{3} + O(t^{-1})$$

and

$$g_{ab} = t^{+\frac{4}{3}}[\mathcal{G}_{ab} + O(t^{-2})]$$
$$g^{ab} = t^{-\frac{4}{3}}[\mathcal{G}^{ab} + O(t^{-2})]$$

where $\mathcal{G}_{ab}$ *and* $\mathcal{G}^{ab}$ *are independent of* t*.*

Outlook

- Other Bianchi types?
- As a first step analysis of diagonal, but non-LRS spacetimes starting with Bianchi II
- There are other solutions which may act as "attractors", for instance the Collins-Stewart solution for Bianchi II
- Later: higher Bianchi types and inhomogeneous space-times as Gowdy-symmetric ones
- Is it possible to remove the small data assumption(s)?