FURTHER IMPROVEMENTS IN THE UNDERSTANDING OF ISOTROPIC LOOP QUANTUM COSMOLOGY

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Preamble

- One attempt to check Loop Quantum Gravity (LQG) implications is Loop Quantum Cosmology (LQC).

- Inside LQC, the simplest models are Friedmann-Robertson-Walker cosmologies coupled to a (homogeneous) massless scalar field (Ashtekar-Pawlowski-Singh).

- It has provided strong physical results related with the big bang singularity.

- Its mathematical framework is very rigorous.

Preamble

- But some important questions have to be deal with.
  - A suitable and rigorous procedure to densitize the Hamiltonian Constraint.
  - To give rise to simple superselection sectors with better physical properties than those of previous works.
  - The theory should has a unique asymptotic limit.
  - A generic (non-simplified) explicitly solvable model.

Classical model

- Flat open FRW model with spatial manifold $\sim \mathbb{R}^3 \rightarrow$ fiducial structures V_0 & $\mathcal{V}$
- The Momentum Constraint is fixed owing to the homogeneity
- Only one geometrical d.o.f.

$$\{c, p\} = 8\pi G\gamma/3$$

- The matter d.o.f is encoded by

$$\{\phi, p_\phi\} = 1$$

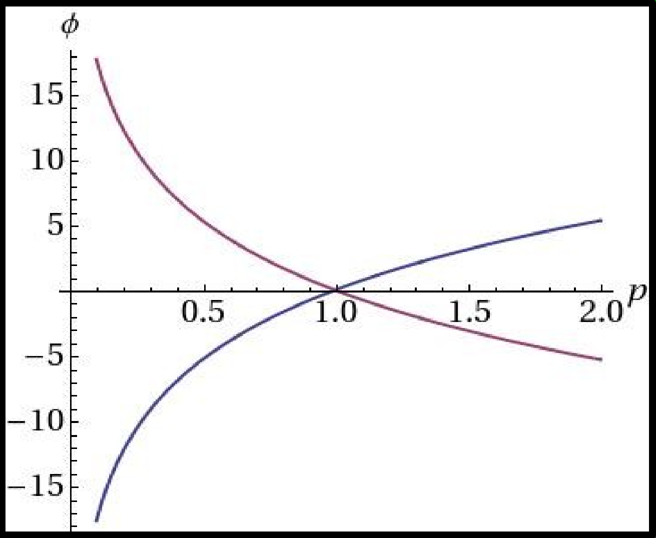
- The integrated Hamiltonian Constraint satisifies $C(N) = NC$, owing to the homogeneity, with

$$C = -6\gamma^{-2}c^2\sqrt{|p|} + 8\pi G p_\phi^2 V^{-1}, \quad V := |p|^{3/2}$$

- Friedmann equation

$$p^{-1}dp = \pm 16\pi G\, d\phi/3$$

- Classical evolution



Quantum framework

- **Matter part: standard representation** $\mathcal{H}^{kin}_{mat} = L^2(\mathbb{R}, d\phi),\ \hat{p}_\phi = -i\hbar\partial_\phi$
- **Geometrical contribution: polymeric representation**
  holonomy $h(\mu)$ → **matrix element** $\hat{\mathcal{N}}_\mu$ → **momentum repres.** $|\mu\rangle$

$$\mathcal{H}^{kin}_{grav} = \overline{span\{|\mu\rangle, \mu \in \mathbb{R}\}}, \qquad \langle\mu|\mu'\rangle = \delta_{\mu\mu'}, \qquad \hat{\mathcal{N}}_\mu|\mu'\rangle = |\mu' + \mu\rangle$$

- **Improved dynamics**
  - **Hypothesis: Minimum physical area of the system** → $\widehat{\frac{1}{\bar{\mu}}} = \frac{\widehat{\sqrt{|p|}}}{\sqrt{\Delta}}$

A new label $v(\mu)$**."Slight" modification-redefinition of the holonomy operator**

$$\hat{\mathcal{N}}_{\bar{\mu}}|v\rangle = |v+1\rangle, \qquad \hat{p}|v\rangle = sign(v)(2\pi\gamma l^2_{Pl}\sqrt{\Delta}|v|)^{2/3}|v\rangle$$

Quantum Hamiltonian Constraint

- The operator $\hat{V}^{-1}$ diverges on $|v=0\rangle$ -analog to classical singularity-. In this case one appeals to Thiemann's trick !!!

$$\left[\widehat{\frac{1}{V}}\right] := \left[\widehat{\frac{1}{\sqrt{|p|}}}\right]^3 ; \quad \widehat{\frac{1}{\sqrt{|p|}}} = \frac{3}{4\pi\gamma l_{Pl}^2 \sqrt{\Delta}} \widehat{sign(p)} \widehat{\sqrt{|p|}} \left(\hat{\mathcal{N}}_{-\bar{\mu}} \widehat{\sqrt{|p|}} \hat{\mathcal{N}}_{\bar{\mu}} - \hat{\mathcal{N}}_{\bar{\mu}} \widehat{\sqrt{|p|}} \hat{\mathcal{N}}_{-\bar{\mu}} \right)$$

- The factor ordering for $\hat{C}$ comes from Bianchi I quantum models

$$\hat{C} := \left[\widehat{\frac{1}{V}}\right]^{1/2} \left(\frac{-6}{\gamma^2} \hat{\Omega}^2 + 8\pi G \hat{p}_\phi^2 \right) \left[\widehat{\frac{1}{V}}\right]^{1/2}$$

$$\hat{\Omega} := \frac{1}{4i\sqrt{\Delta}} \left[\widehat{\frac{1}{\sqrt{|p|}}}\right]^{-1/2} \widehat{\sqrt{|p|}} \left[\left(\hat{\mathcal{N}}_{2\bar{\mu}} - \hat{\mathcal{N}}_{-2\bar{\mu}} \right) \widehat{sign(p)} + \widehat{sign(p)} \left(\hat{\mathcal{N}}_{2\bar{\mu}} - \hat{\mathcal{N}}_{-2\bar{\mu}} \right) \right] \widehat{\sqrt{|p|}} \left[\widehat{\frac{1}{\sqrt{|p|}}}\right]^{-1/2}$$

no fundamental Bianchi I ordering

main feature!!!

Densitization

- The operator $\left[\widehat{1/V}\right]$ annihilates the state $|v=0\rangle$ → $\hat{C}$ does too. Its orthogonal complement remains invariant → $|v=0\rangle$ doesn't contrib.

"lives" in the dual algebraic of $span\{|v\rangle, v\in\mathbb{R}-\{0\}\}$

$$\overline{\mathcal{H}}^{kin}_{grav}=\overline{span\{|v\rangle, v\in\mathbb{R}-\{0\}\}}$$

initial singularity removed!!!

- If $(\psi|$ is a solution of $\hat{C}$, we define through $\left[\widehat{1/V}\right]$ a bijection

$$(\psi'|=(\psi|\left[\widehat{1/V}\right]^{1/2} \Rightarrow \hat{\mathcal{C}}:=\left[\widehat{1/V}\right]^{-1/2}\hat{C}\left[\widehat{1/V}\right]^{-1/2}, \quad \hat{\mathcal{C}}=-6\gamma^{-2}\hat{\Omega}^{2}+8\pi G\,\hat{p}_{\phi}^{2}$$

where $(\psi'|$ are solutions of $\hat{\mathcal{C}}$

Geometrical properties

Superselection sectors

- The action of $\widehat{\Omega}^2$ on the basis $|v\rangle$ is given by

$$\widehat{\Omega}^2|v\rangle = -f_+(v)f_+(v+2)|v+4\rangle + \left[f_+^2(v)+f_-^2(v)\right]|v\rangle - f_-(v)f_-(v-2)|v-4\rangle$$

$$f_\pm(v) = \frac{\pi\gamma\hbar G}{3}\,g(v+2)\left(sign(v\pm2)+sign(v)\right)g(v), \qquad g(v) = \frac{v^{1/6}}{\left|\,|v+1|^{1/3}-|v-1|^{1/3}\right|^{1/2}}$$

- Owing to $f_-(v)f_-(v-2)=0$ in $v\in(0,4]$, and $f_+(v)f_+(v+2)=0$ in $v\in[-4,0)$ it is possible to decouple the semiaxis $v>0$ and $v<0$

- The operator $\widehat{\Omega}^2$ relates states contained in $\mathcal{L}_\varepsilon^\pm := \{v=\pm(\varepsilon+4n),\, n\in\mathbb{N}\}$ with $\varepsilon\in(0,4]$ → superselection sectors $\widetilde{\mathcal{H}}_\varepsilon^\pm$

Geometrical properties

Spectral analysis

- **The operator $\widehat{\Omega}^2$ is essentially self-adjoint on $\widetilde{\mathcal{H}}_\varepsilon^{\pm}$ and it is positive definite with eigenvalues $\lambda \in [0, \infty)$ (continuum sprectum)**

Generalized eigenfunctions

- **The eigenfunctions of $\widehat{\Omega}^2$ have the form**

$$|e_\lambda^\varepsilon\rangle = \sum_{v \in \mathcal{L}} e_\lambda^\varepsilon(v)|v\rangle, \qquad e_\lambda^\varepsilon(\varepsilon + 4\mathrm{n}) = e_\lambda^\varepsilon(\varepsilon) F(\lambda, \varepsilon, n)$$

- **They are completely determined if $\langle e_\lambda^\varepsilon | e_{\lambda'}^\varepsilon \rangle = \delta(\lambda - \lambda')$ and $e_\lambda^\varepsilon(\varepsilon) > 0$**
- **The spectral resolution of the identity $I = \int_0^\infty d\lambda |e_\lambda^\varepsilon\rangle\langle e_\lambda^\varepsilon|$ (1-fold!!!)**

Asymptotic limit

- For $v \gg 1$, the operator $\widehat{\Omega}^2$ is a differential operator

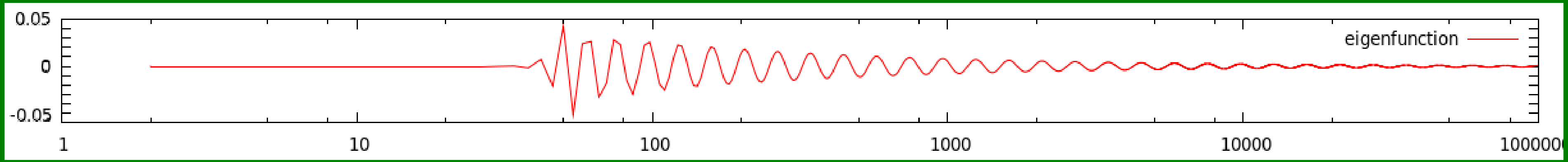
$$\underline{\widehat{\Omega}}^2 = -\beta^2 (1 + 4 v \partial + 4 (v \partial)^2) \qquad (\beta := 4 \pi \gamma G \hbar)$$

- The eigenfunctions are 2-fold $\underline{e}_\sigma$ (in) and $\underline{e}^*_\sigma$ (out)

$$\underline{e}_\sigma(v) = (2\pi \beta v)^{-1/2} e^{-i\sigma \ln|v|/\beta}, \qquad \langle \underline{e}_\sigma | \underline{e}_{\sigma'} \rangle = \delta(\sigma - \sigma'), \qquad I = \int_{-\infty}^{\infty} d\sigma |\underline{e}_\sigma\rangle\langle \underline{e}_\sigma|$$

- LQC eigenfunctions have the asymptotic limit

$$e^\varepsilon_\lambda(v) \to r \{ \exp[i\phi_\varepsilon(\sigma)] \underline{e}_\sigma(v) + \exp[-i\phi_\varepsilon(\sigma)] \underline{e}_{-\sigma}(v) \}, \quad (\sigma = \pm\sqrt{\lambda})$$



EXACT STANDING WAVE!!!

Quantum bounce mechanism

• Ingredient 1: The physical states have support on one semilattice. They "live", for example, on $v>0$. One initial data on $v=\varepsilon$ determines all the eigenfunctions. No boundary condition and no cross over $v=0$

• Ingredient 2: All the eigenfunctions behave like a standing wave for $v\gg 1$. They have the same contribution of expanding and contracting universes

I1+I2 = QUANTUM BOUNCE!!!

• With APS factor ordering, the same analysis is available for $\varepsilon=2$, and also for $\varepsilon=0$ with a boundary condition $(\Psi(v)=\Psi(-v))$
• For $\varepsilon\neq 0,\, 2$ the bounce is reached for semiclassical states, once a specific eigenf. basis (with discontinuous asympt. limit) is chosen

Comments and conclusions

- **Motivated by some drawbacks and the results in Bianchi I models**
  - **Quantum mechanics principles allow us to resolve the big bang singularity**
  - **Owing to that, it is possible establish a method to formally densitize in an intuitive and natural way (although is not the only one) the Hamiltonian Constraint**
  - **Taking into account the orientation of the triad, one can decouple their contributions contained in different semiaxis**
  - **Our theory has a well defined asymptotic limit, whose behavior let us (together with the last property) demonstrate the bouncing nature of all the trajectories**
- **We have resolved a non-simplified model, whose results are valid for each superselection sector and for all physical states**