What can Classical Gravity tell us about Quantum Structure of Spacetime?

T. Padmanabhan

(IUCAA, Pune, INDIA)

Gravity as a Crossroad in Physics

Spanish Relativity Meeting, ERE2010, Granada, Spain

6 September 2010

THE ATTRACTION OF QUANTUM GRAVITY

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Hathaway's Einstein theories

NO fashion magazines for Hollywood actress Anne Hathaway, she spends her free time studying books on physics to enhance her knowledge on the universe.

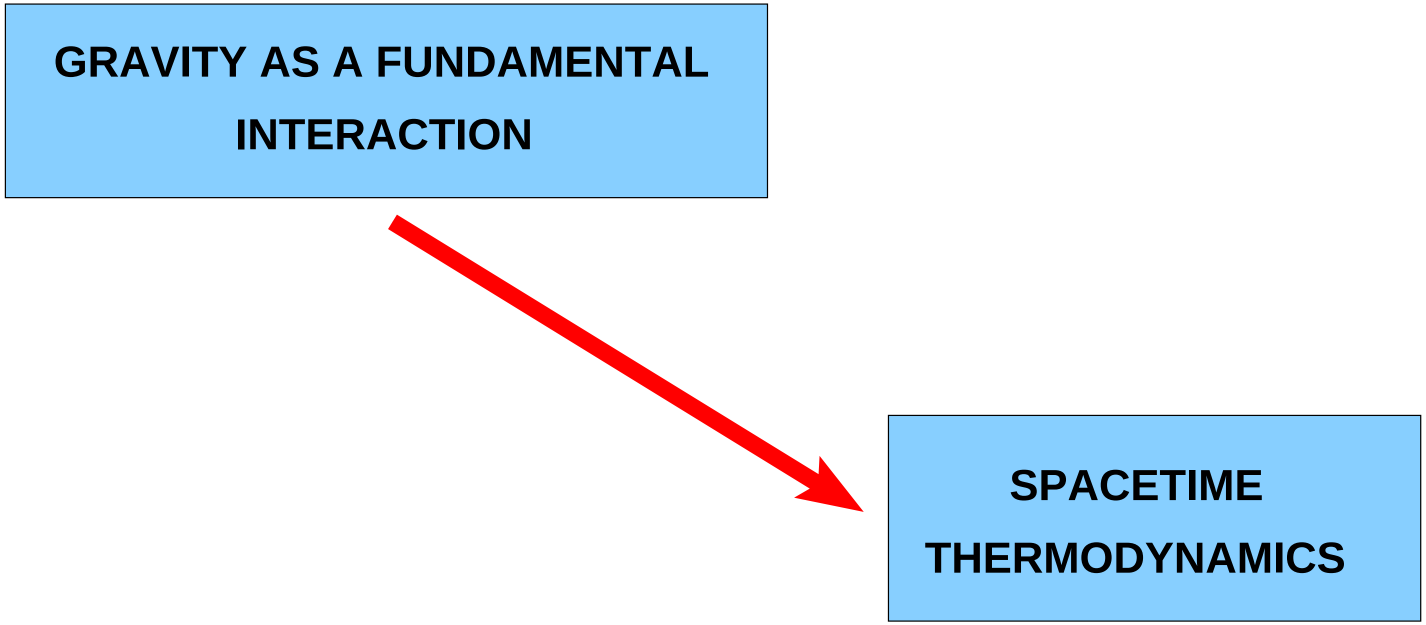
The Devil Wears Prada star admits she shuns fashion magazines and instead stocks up on books by scientist Albert Einstein and physics textbooks in a bid to better understand the universe, reported a website. "I'm interested in elementary particles. What I like thinking about is how time and space exist in the universe and how we understand it. Any spare time I have, I bury my head in a physics textbook. I'm reading a lot about Einstein. I like theories and I want to understand string theory," she said.

— IANS

CONVENTIONAL VIEW

GRAVITY AS A FUNDAMENTAL INTERACTION

CONVENTIONAL VIEW

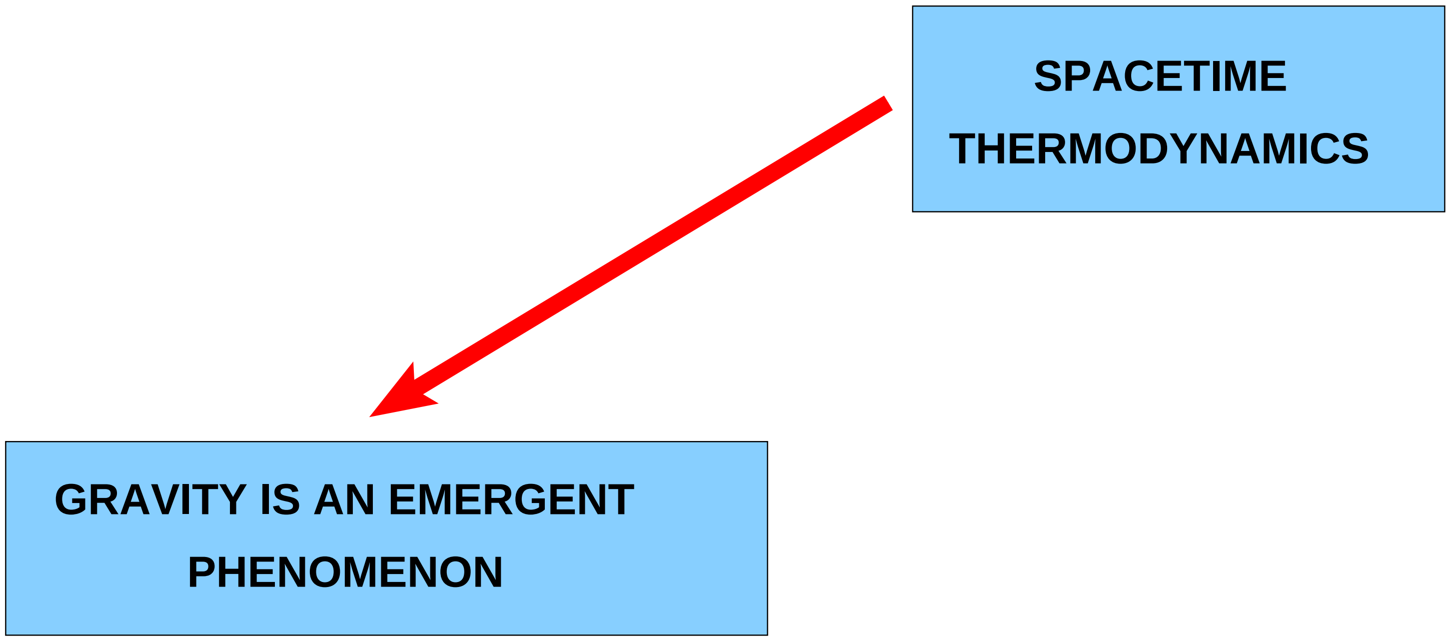


ALTERNATIVE PERSPECTIVE

SPACETIME THERMODYNAMICS

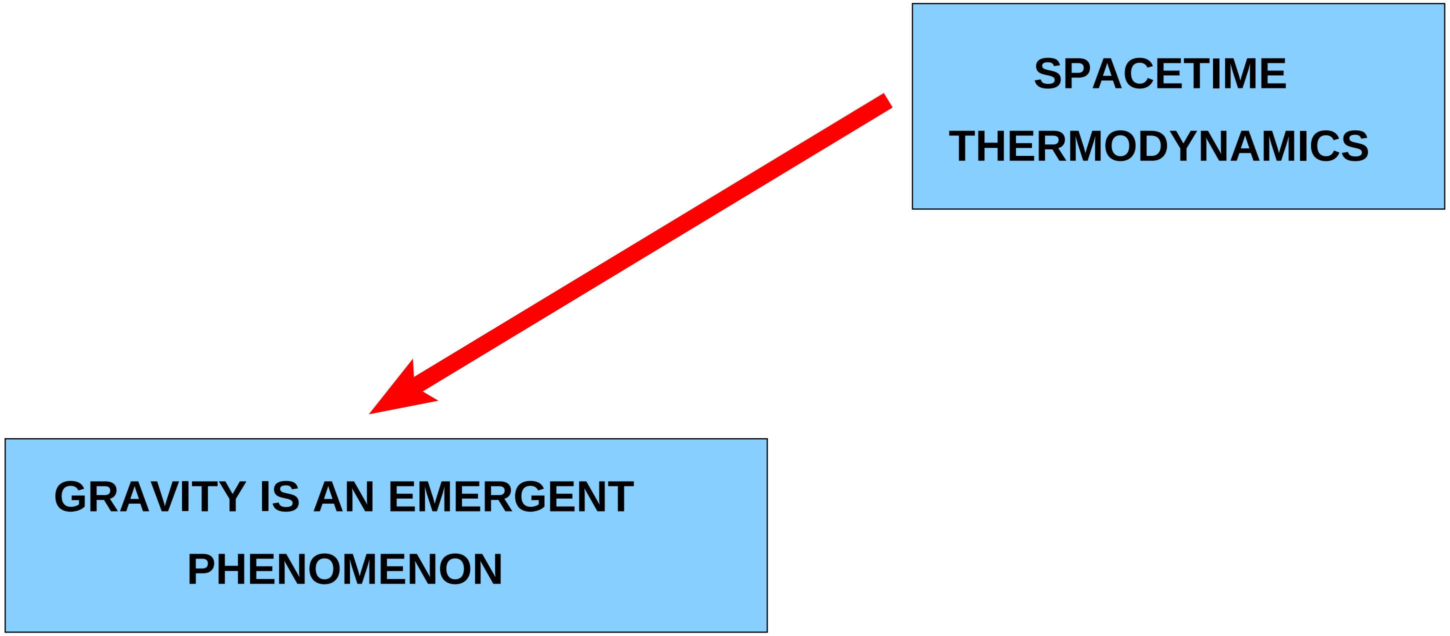
GRAVITY IS AN EMERGENT PHENOMENON

ALTERNATIVE PERSPECTIVE



GRAVITY IS THE THERMODYNAMIC LIMIT OF THE STATISTICAL MECHANICS OF 'ATOMS OF SPACETIME'

ALTERNATIVE PERSPECTIVE



GRAVITY IS THE THERMODYNAMIC LIMIT OF THE STATISTICAL MECHANICS OF ‘ATOMS OF SPACETIME’

Demonstrate existence of atoms from the fact that thermal phenomena occurs

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- EXPLORE A 'TOP-DOWN' APPROACH.

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Pick 'algebraic accidents'; look for an explanation

PLAN OF THE TALK

- THE CONVENTIONAL APPROACH TO GRAVITY AND HORIZON THERMODYNAMICS
- ‘ALGEBRAIC ACCIDENTS’ AS INTERNAL EVIDENCE FOR AN ALTERNATIVE PERSPECTIVE
- GRAVITY AS AN EMERGENT PHENOMENON
- GRAVITATIONAL DYNAMICS FROM AN ENTROPY MAXIMIZATION PRINCIPLE
- CONCLUSIONS, OPEN QUESTIONS ....

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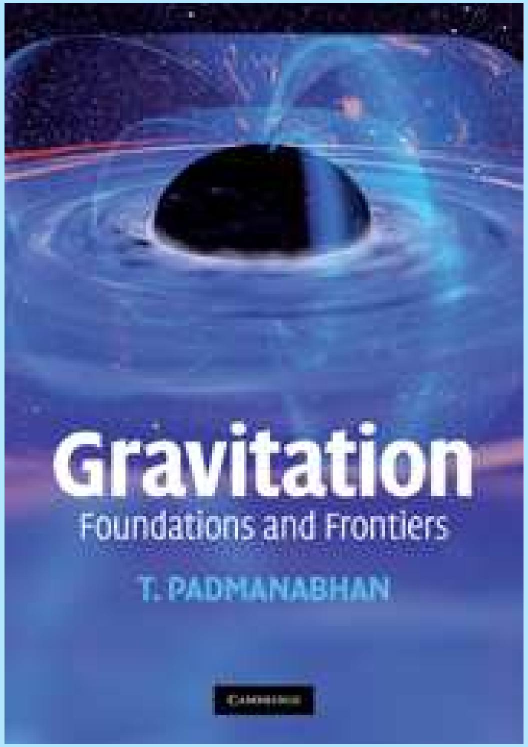

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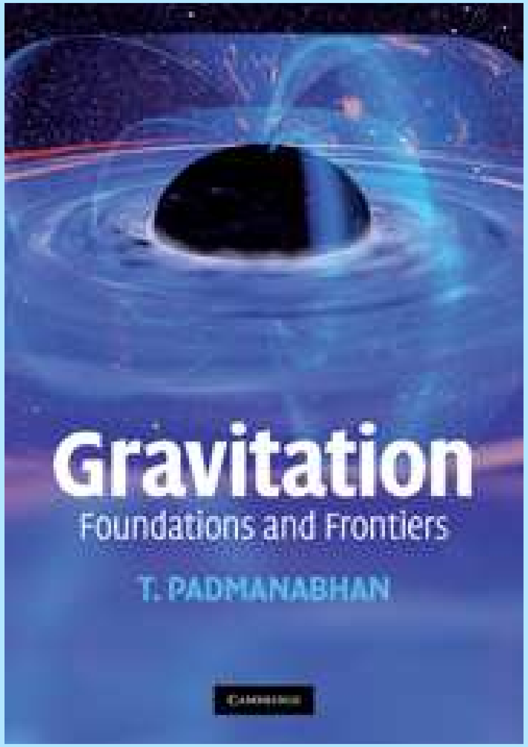
- Principle of Equivalence



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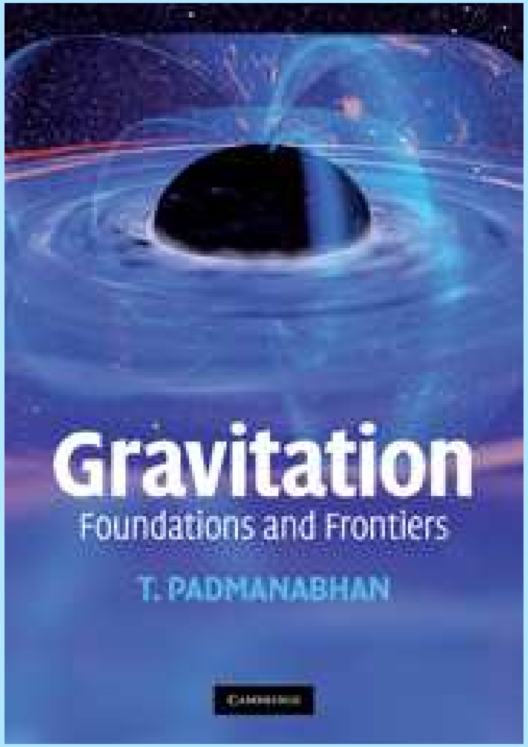

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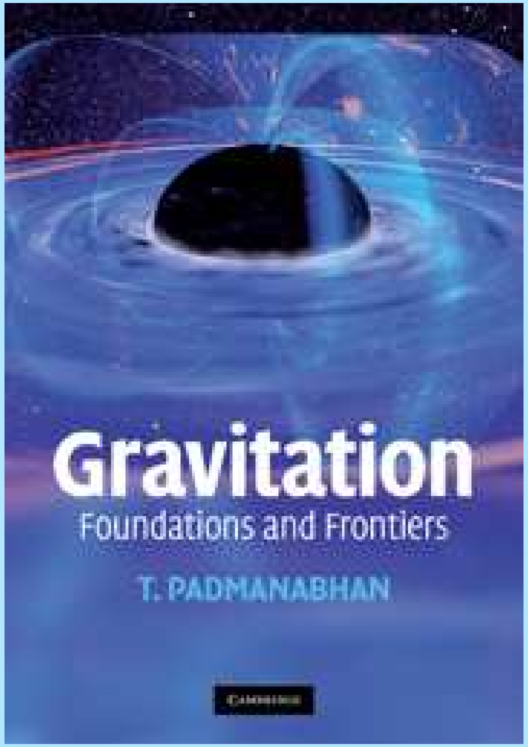
- Action functional (?)



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- Principle of Equivalence
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$$\begin{aligned} \mathcal{G}_{ab} &= P_a^{\ cde} R_{bcde} - \frac{1}{2} L g_{ab} - 2 \nabla^c \nabla^d P_{acdb} \\ &\equiv \mathcal{R}_{ab} - \frac{1}{2} L g_{ab} - 2 \nabla^c \nabla^d P_{acdb} = (1/2) T_{ab} \end{aligned}$$

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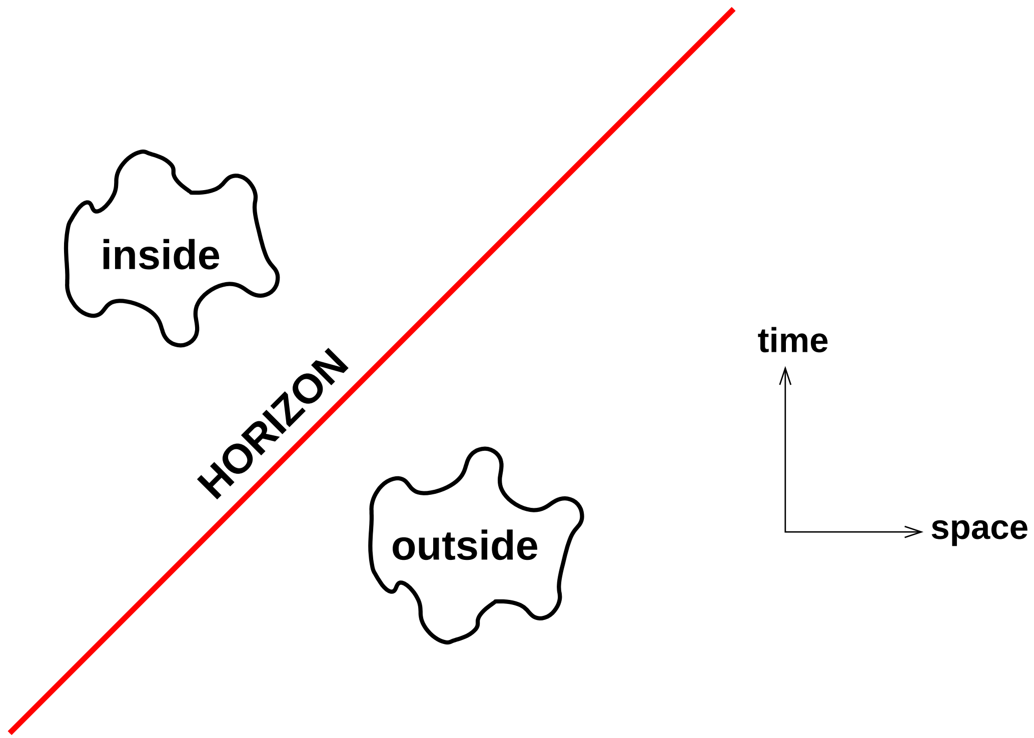
- A "nice" class of theories: $\nabla_a P^{abcd} = 0$ for which

$$\mathcal{R}_{ab} - \frac{1}{2} L g_{ab} = (1/2) T_{ab}$$

- Horizons arise inevitably in the solutions to these field equations.

ENTROPY OF HORIZONS



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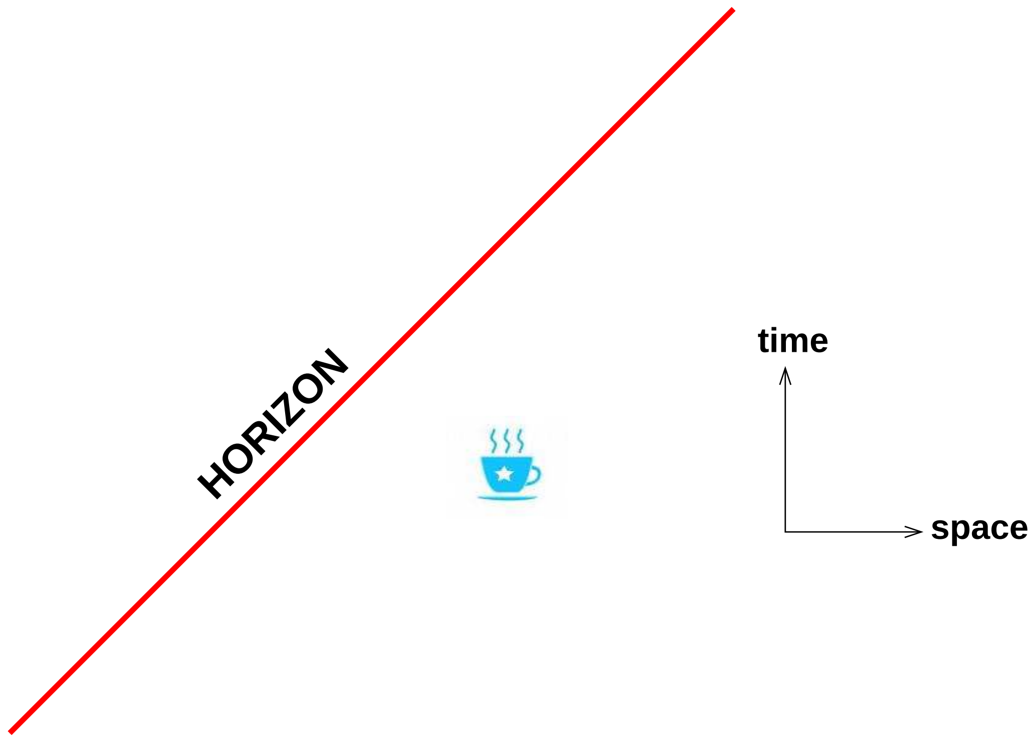

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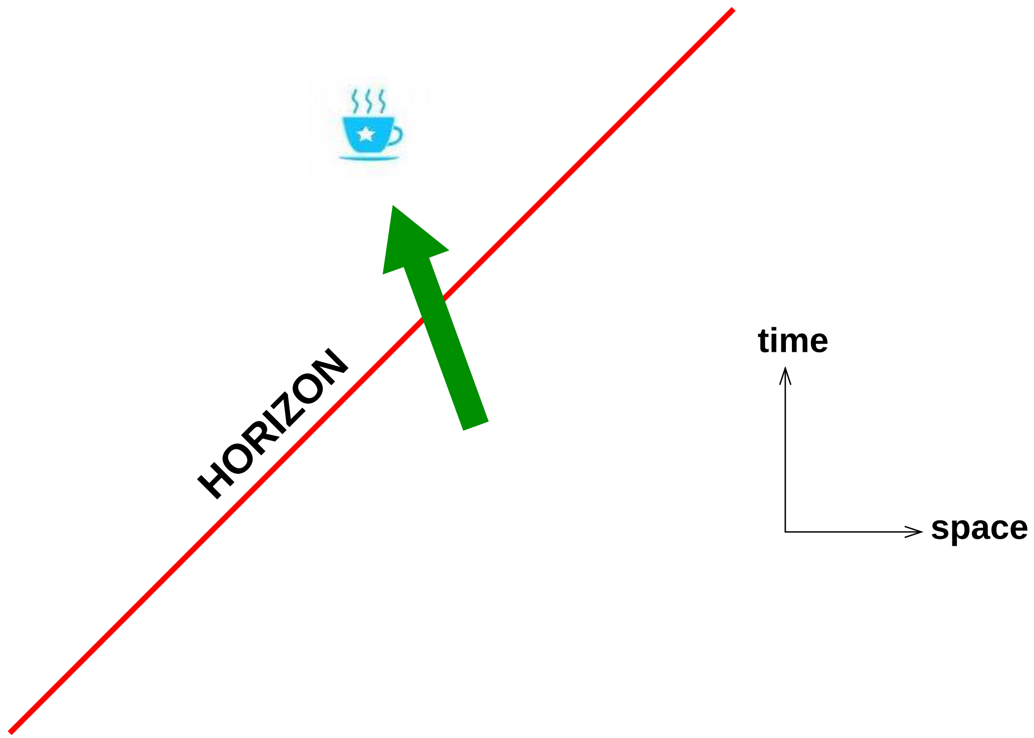

Wheeler ($\sim$ 1971): Can one violate second law of thermodynamics by hiding entropy behind a horizon ?

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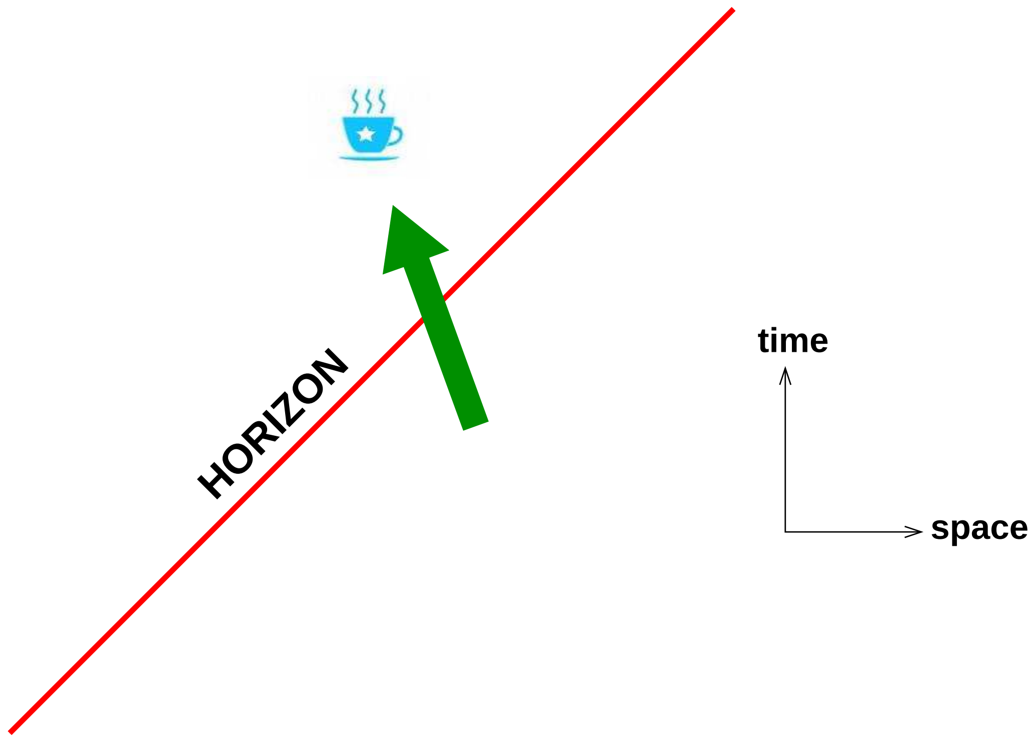

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Bekenstein (1972): No! Horizons have entropy $S \propto (Area)$ which goes up when you try this.

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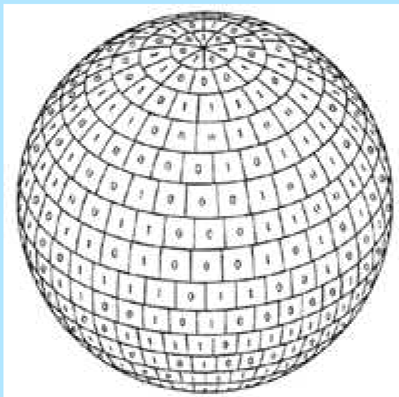
$$N = \frac{A}{A_{Planck}} = \frac{c^3 A}{G\hbar}$$



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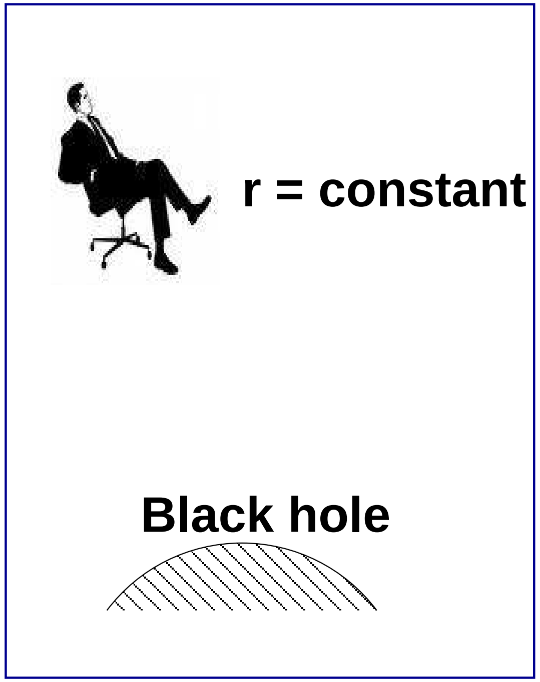
- The Boltzmann entropy is:

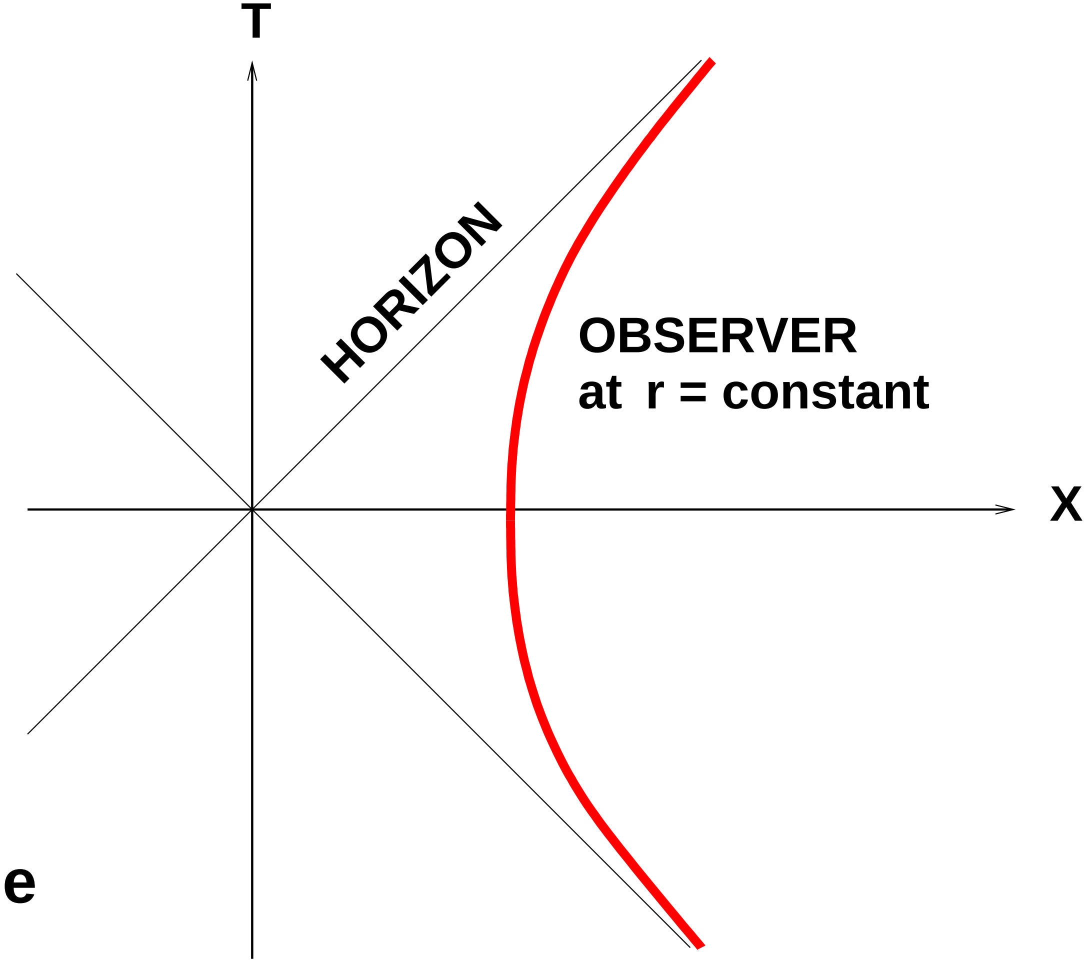
$$S = \ln \Omega \propto \frac{A}{A_{Planck}} \propto \frac{c^3 A}{G\hbar}$$

DEMOCRACY OF HORIZONS

- Hawking: Black hole horizons have a temperature (1975)



BLACK HOLE SPACETIME



Temperature
$\propto$ acceleration
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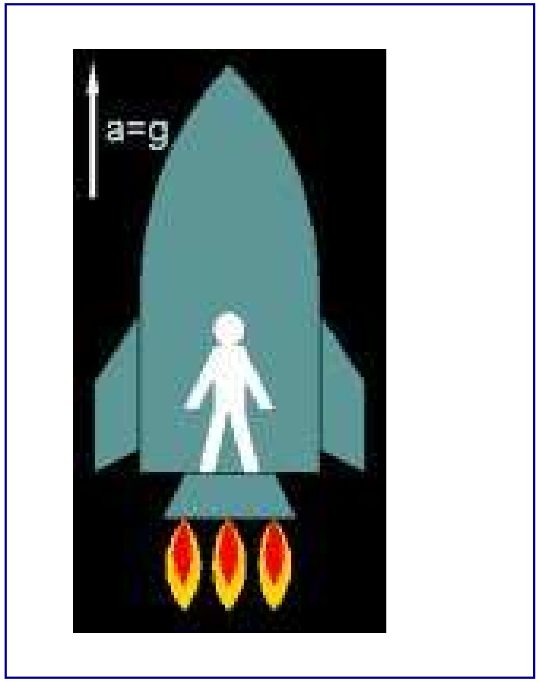
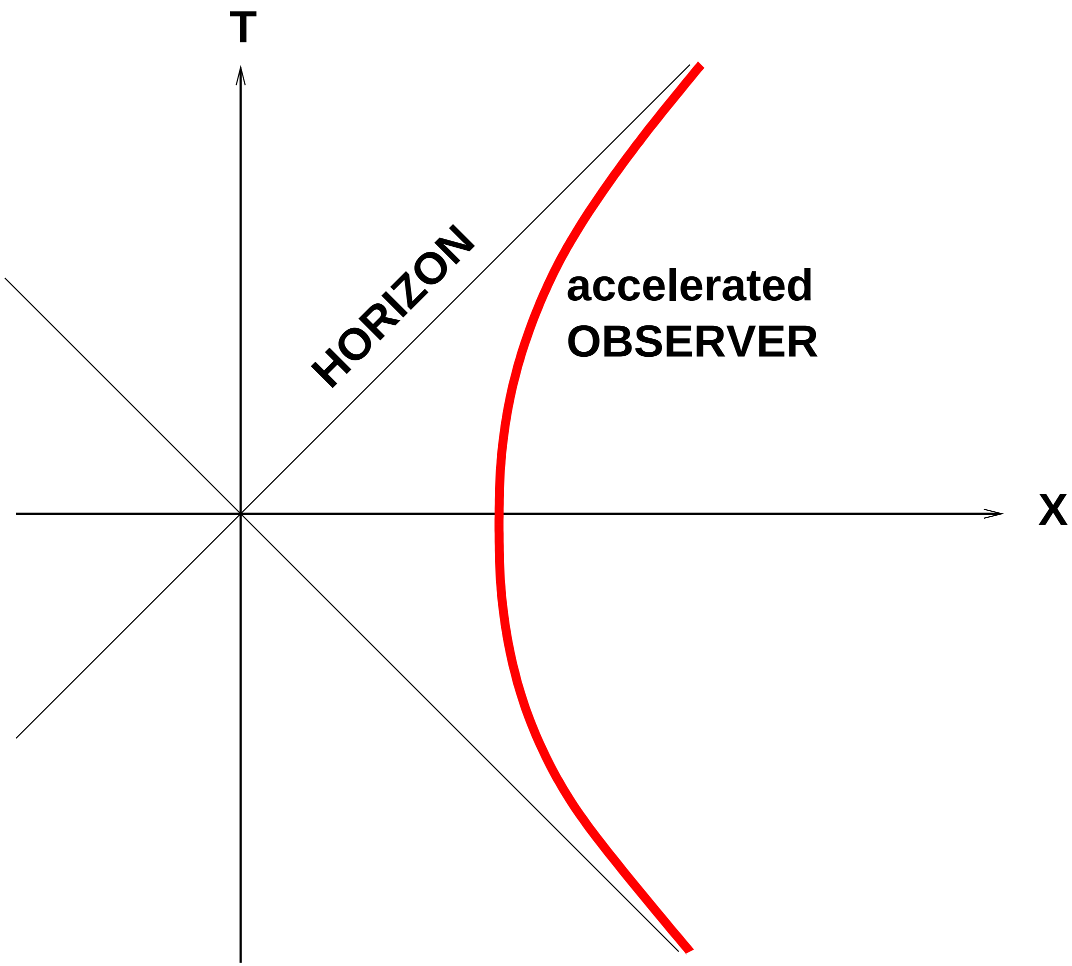
FLAT SPACETIME





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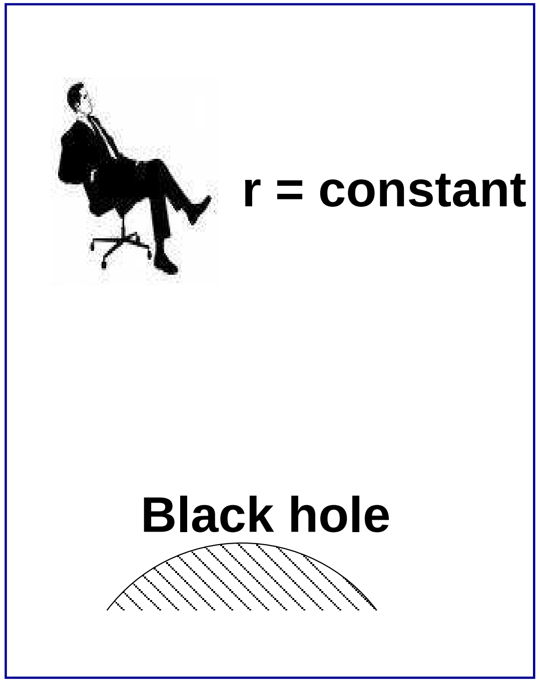

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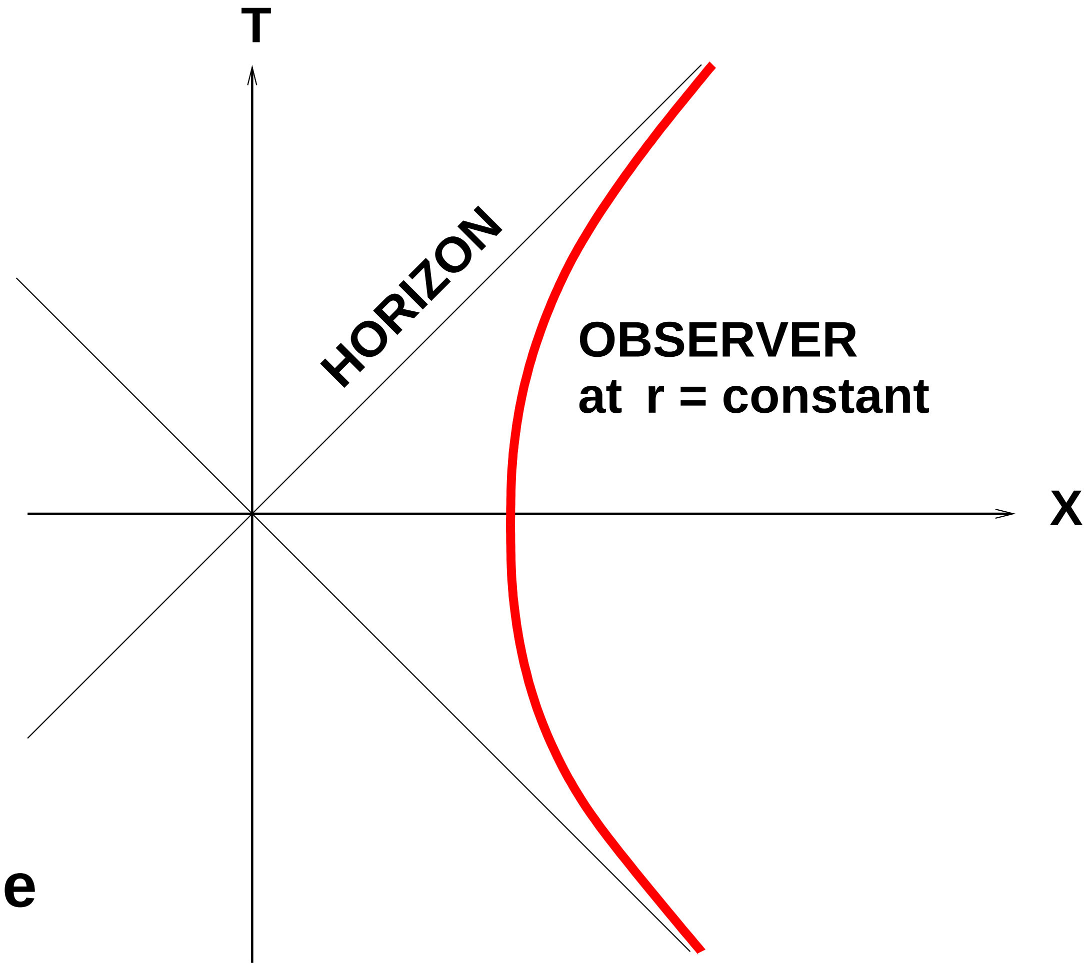

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- *The connection between horizons and temperature is quite generic.*

OBSERVERS WHO PERCEIVE A HORIZON ATTRIBUTE TO IT A TEMPERATURE

$$k_B T = \frac{\hbar}{c}\left(\frac{g}{2\pi}\right)$$

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$$\left.\begin{array}{r}\text{PERIODICITY IN}\\ \text{IMAGINARY TIME}\end{array}\right\} \Longleftrightarrow \left\{\text{FINITE TEMPERATURE}\right.$$

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SPACETIMES WITH HORIZONS EXHIBIT PERIODICITY IN IMAGINARY TIME $\Longrightarrow$ TEMPERATURE

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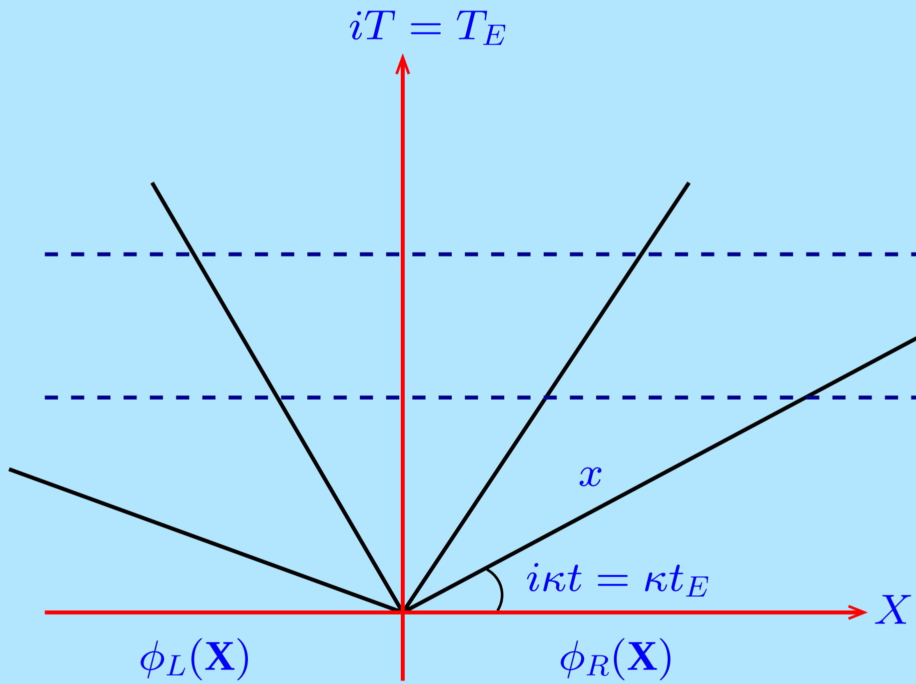

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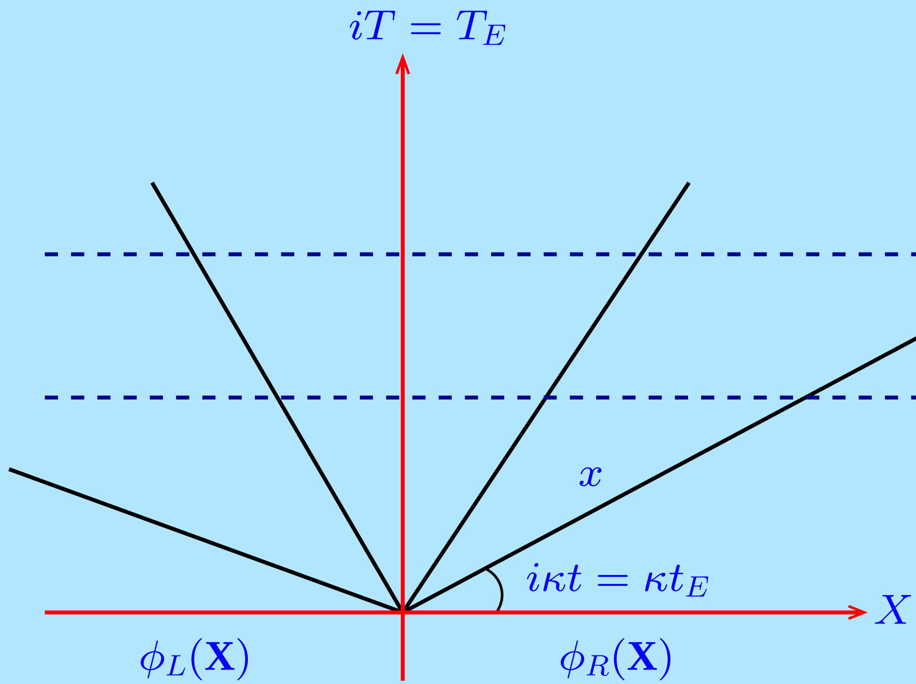

$$\langle \text{vac} | \phi_L, \phi_R \rangle \propto \int_{T_E=0;\phi=(\phi_L,\phi_R)}^{T_E=\infty;\phi=(0,0)} \mathcal{D}\phi e^{-A}$$

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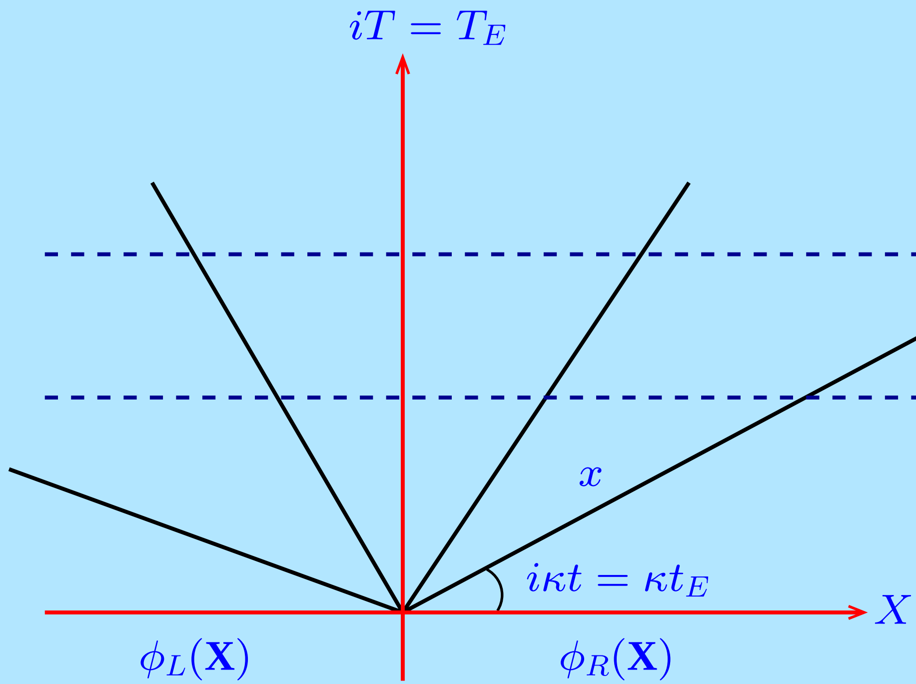

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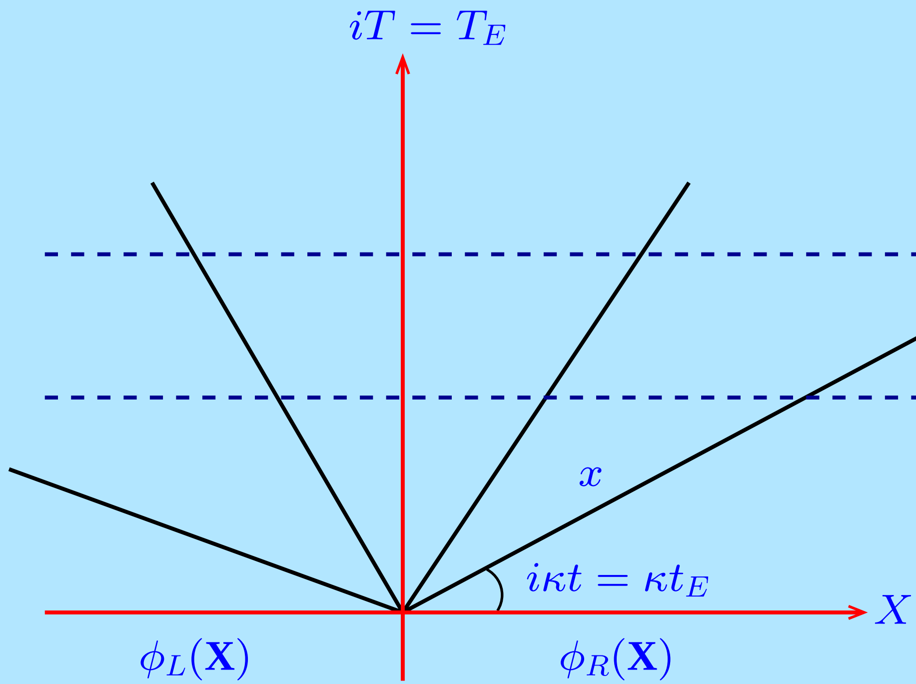

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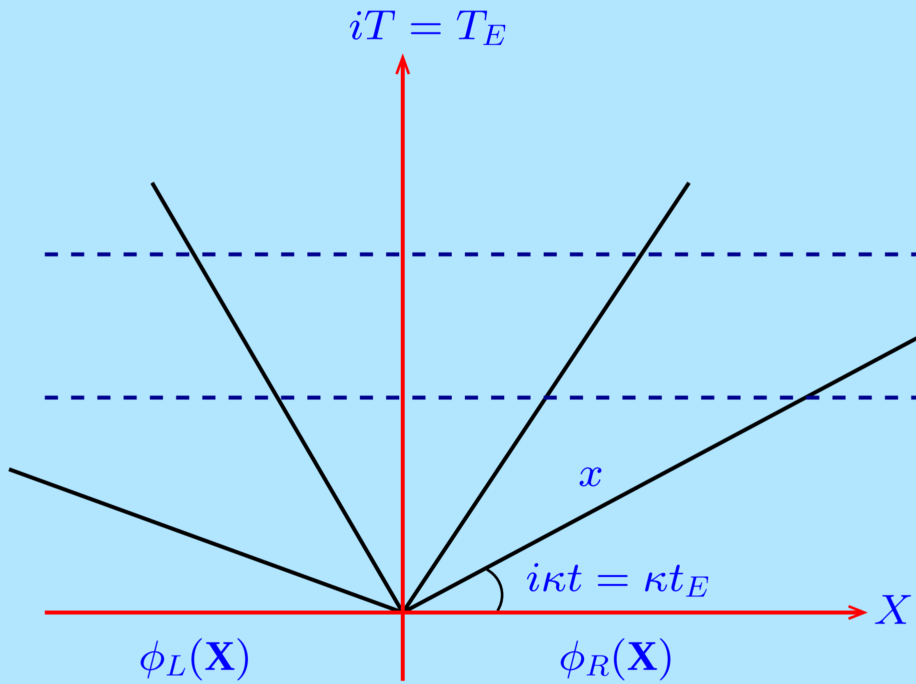

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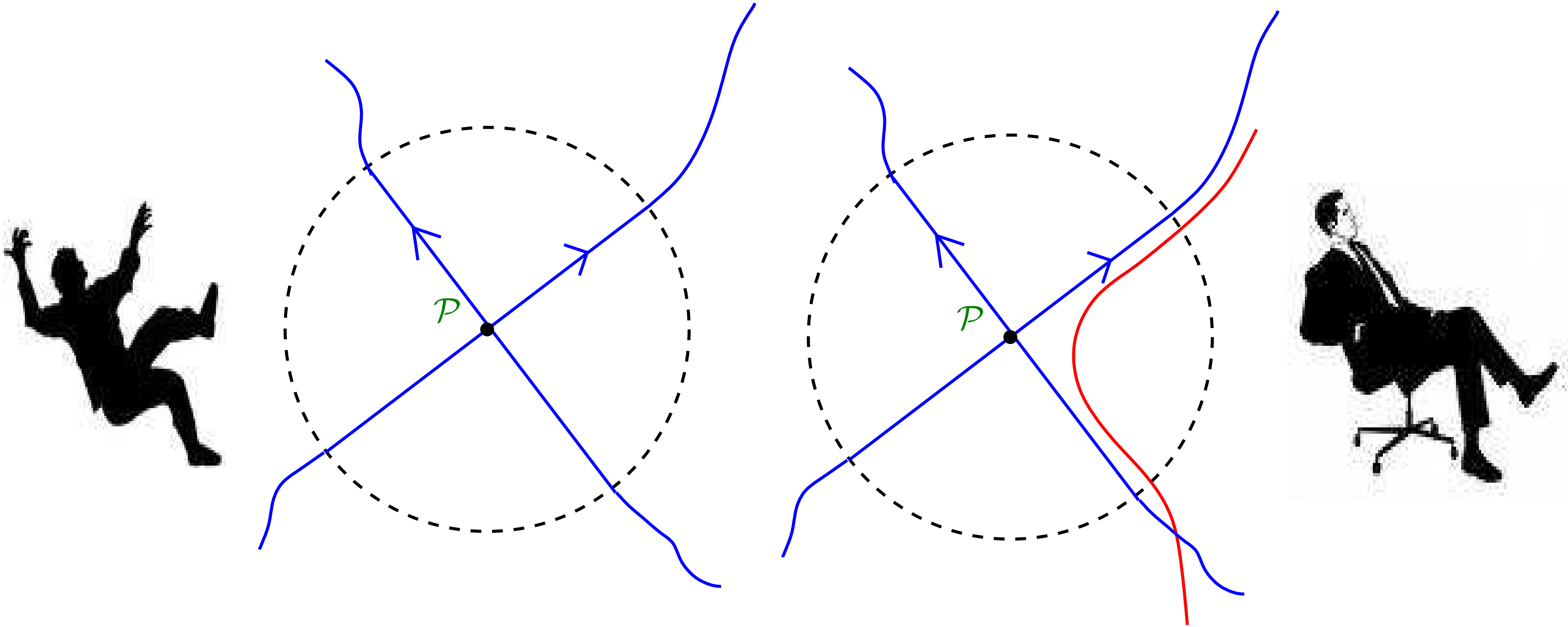
- Tracing out ϕ_L gives a density matrix:

$$\rho(\phi'_R, \phi_R) = \int \mathcal{D}\phi_L \langle \text{vac}|\phi_L, \phi'_R\rangle \langle \text{vac}|\phi_L, \phi_R\rangle \propto \langle \phi'_R|e^{-(2\pi/\kappa)H_R}|\phi_R\rangle$$



Vacuum state $\Longrightarrow$ Thermal state

(D.Kothawala,T.P, 2010)

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- NEW LEVEL OF OBSERVER DEPENDENCE IN THERMODYNAMICS (BH, dS, RINDLER ....).

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Example: $m_{inertial} = m_{grav}$ is 'internal evidence' for geometrical nature of gravity.

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- Einstein's equation at $r = a$ is (textbook result!)

$$\frac{c^4}{G}\left[\frac{ga}{c^2} - \frac{1}{2}\right] = 4\pi P a^2$$

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- Einstein's equation at $r = a$ is (textbook result!)

$$\frac{c^4}{G}\left[\frac{ga}{c^2} - \frac{1}{2}\right] = 4\pi P a^2$$

- Multiply da to write:

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- Field equations become $TdS = dE + PdV$; with :

$$S = \frac{1}{4L_P^2}(4\pi a^2) = \frac{1}{4}\frac{A_H}{L_P^2}; \qquad E = \frac{c^4}{2G} a = \frac{c^4}{G}\left(\frac{A_H}{16\pi}\right)^{1/2}$$

HOLDS TRUE FOR A LARGE CLASS OF MODELS!

- Stationary axisymmetric horizons and evolving spherically symmetric horizons in Einstein gravity, [gr-qc/0701002]
- Static spherically symmetric horizons in Lanczos-Lovelock gravity, [hep-th/0607240]
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IN ALL THESE CASES FIELD EQUATIONS REDUCE TO $TdS = dE + PdV$ ON THE HORIZON!

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IT DOES !!

In static spacetimes with horizon, Euclidean action can be interpreted as free energy of spacetime:

$$A_E = \underbrace{\beta E}_{\text{bulk term}} - \underbrace{S}_{\text{surface term}} = \beta F$$

T.P, 2004; A. Mukhopadhyay, T.P, 2006; S.Kolekar, T.P, 2010

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- Action for gravity has exactly this structure! [TP, 02, 05]

$$A_{grav} = \int d^4x\ \sqrt{-g}\,R = \int d^4x\ \sqrt{-g}\ [L_{\rm bulk} + L_{\rm sur}]$$

$$\sqrt{-g}L_{sur} = -\partial_a\left(g_{ij}\frac{\partial\sqrt{-g}L_{bulk}}{\partial(\partial_a g_{ij})}\right) \equiv \partial_a P^a$$

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- In a Riemann normal coordinates around any event $\mathcal{P}$, the action reduces to a pure surface term! One can get the field equations from $A_{total} = A_{sur} + A_{matter}$ using the horizon displacements. [TP, 2005]

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- Field equations can be interpreted as: [TP, 08, 09]

$$\left\{\begin{array}{c}\textbf{Loss of matter entropy}\\ \textbf{due to flow of energy}\\ \textbf{across the hot horizon}\end{array}\right\} = \left\{\begin{array}{c}\textbf{Gain of gravitational entropy}\\ \textbf{for infinitesimal virtual}\\ \textbf{displacements of the horizon}\end{array}\right\}$$

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GRAVITY IS AN EMERGENT PHENOMENON INVOLVING THERMODYNAMIC DESCRIPTION OF MICROSCOPIC SPACETIME DEGREES OF FREEDOM

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SOLIDS	SPACETIME
Mechanics; Elasticity ($\rho, \mathbf{v}$ ...)	Einstein's Theory (g_{ab} ...)
Statistical Mechanics of atoms/molecules	Statistical mechanics of "atoms of spacetime"

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You never took a course in 'quantum thermodynamics'!

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- In the description of gas dynamics, heat engines etc. we use two categories of variables: mechanical and thermodynamical.
- Boltzmann postulated microscopic degrees of freedom and connected the thermodynamical variables to mechanical variables of these d.o.f.
- Key new ingredient: Boltzmann postulate related thermodynamics to mechanics of microstructure.

The equipartition law

$$E = \frac{1}{2} n k_B T \to \frac{1}{2} \int dV \; \frac{dn}{dV} \; k_B T = \frac{1}{2} k_B \int dn T$$

demands the 'granularity' with finite n; degrees of freedom scales as volume.

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- Elastic constants, gas density, pressure etc are useful variables in the thermodynamic limit. Metric, curvature etc. have a similar status in the description of spacetime.
- Entropy of a gas is related to the degrees of freedom which are ignored. Entropy of spacetime is related to unobservable degrees of freedom for a given observer.

A TEST OF THE IDEA:

THE AVOGADRO NUMBER OF SPACETIME

IF SPACETIME HAS MICROSTRUCTURE AND IT CAN BE HEATED UP, IS THERE AN EQUIPARTITION LAW "$E = (1/2)nk_BT$" FOR THE MICROSCOPIC SPACETIME DEGREES OF FREEDOM ?

IF SO, CAN WE DETERMINE n?

EQUIPARTITION OF MICROSCOPIC DEGREES OF FREEDOM

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- In hot spacetimes, Einstein's equations imply the Equipartition Law for microscopic d.o.f!

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- 'Gravity is holographic'!:

$$E \equiv \frac{1}{2}k_B \int_{\partial\mathcal{V}} dn\, T_{\rm loc} \equiv \frac{1}{2}k_B \int_{\partial\mathcal{V}} dA \frac{dn}{dA}\, T_{\rm loc}$$

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- Result generalizes to any Lanczos-Lovelock model:

$$E = \frac{1}{2}k_B \int_{\partial\mathcal{V}} dn T_{loc}; \qquad \frac{dn}{dA} = \frac{dn}{\sqrt{\sigma} d^{D-2}x} = 32\pi P^{ab}_{cd}\epsilon_{ab}\epsilon^{cd}$$

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How does one close the loop on dynamics?	Use the entropy extremisation to obtain thermodynamical equations	Use the entropy extremisation to obtain gravitational field equations

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- NEW FEATURE: Thermal behaviour of spacetime is strongly observer dependent.

- The nature of independent variables q_A and the form of entropy $S[q_A]$ depend on the class of observers and the model for gravity.

- We need a (phenomenological) entropy function for spacetime maximizing which for all class of observers should give the field equations.

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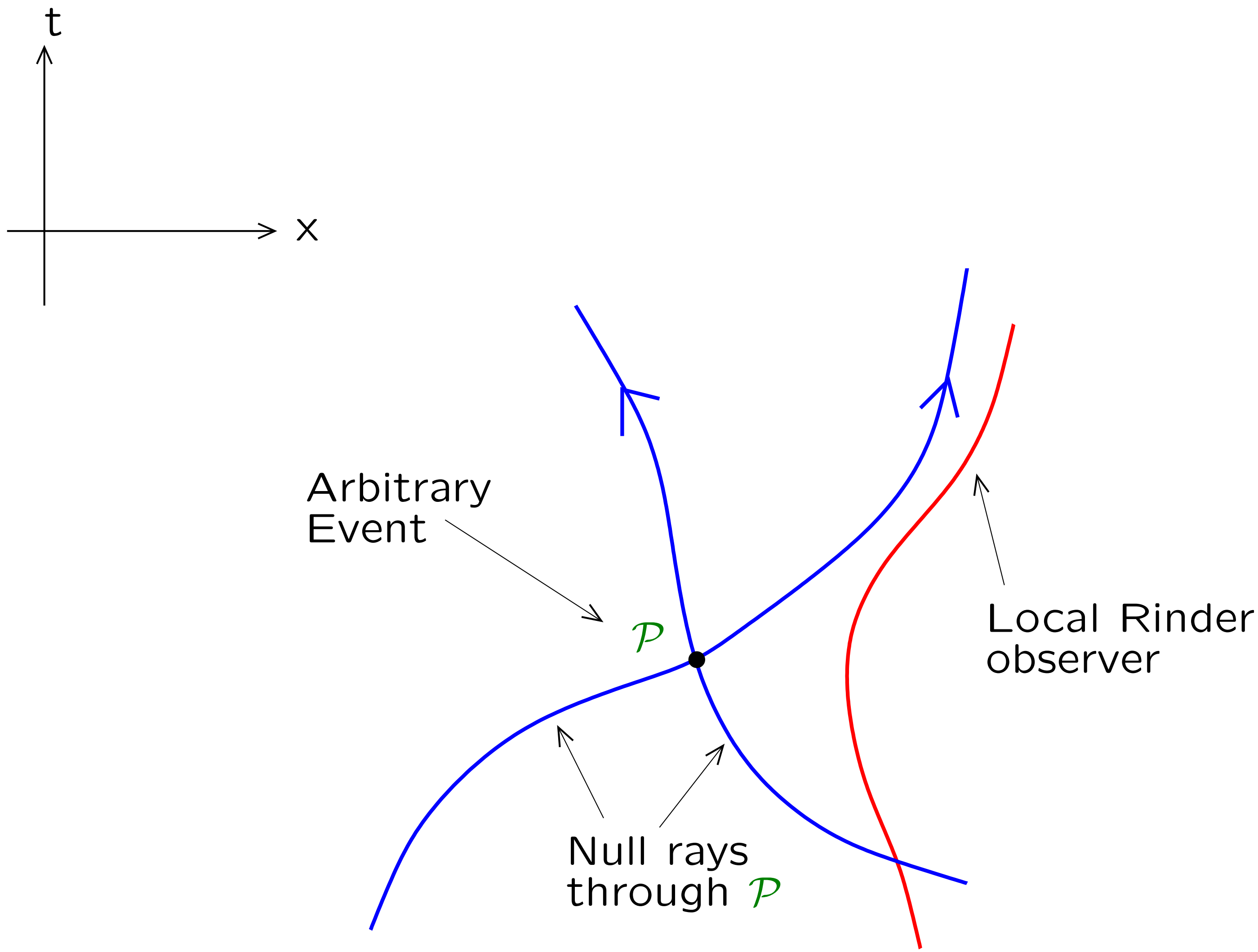
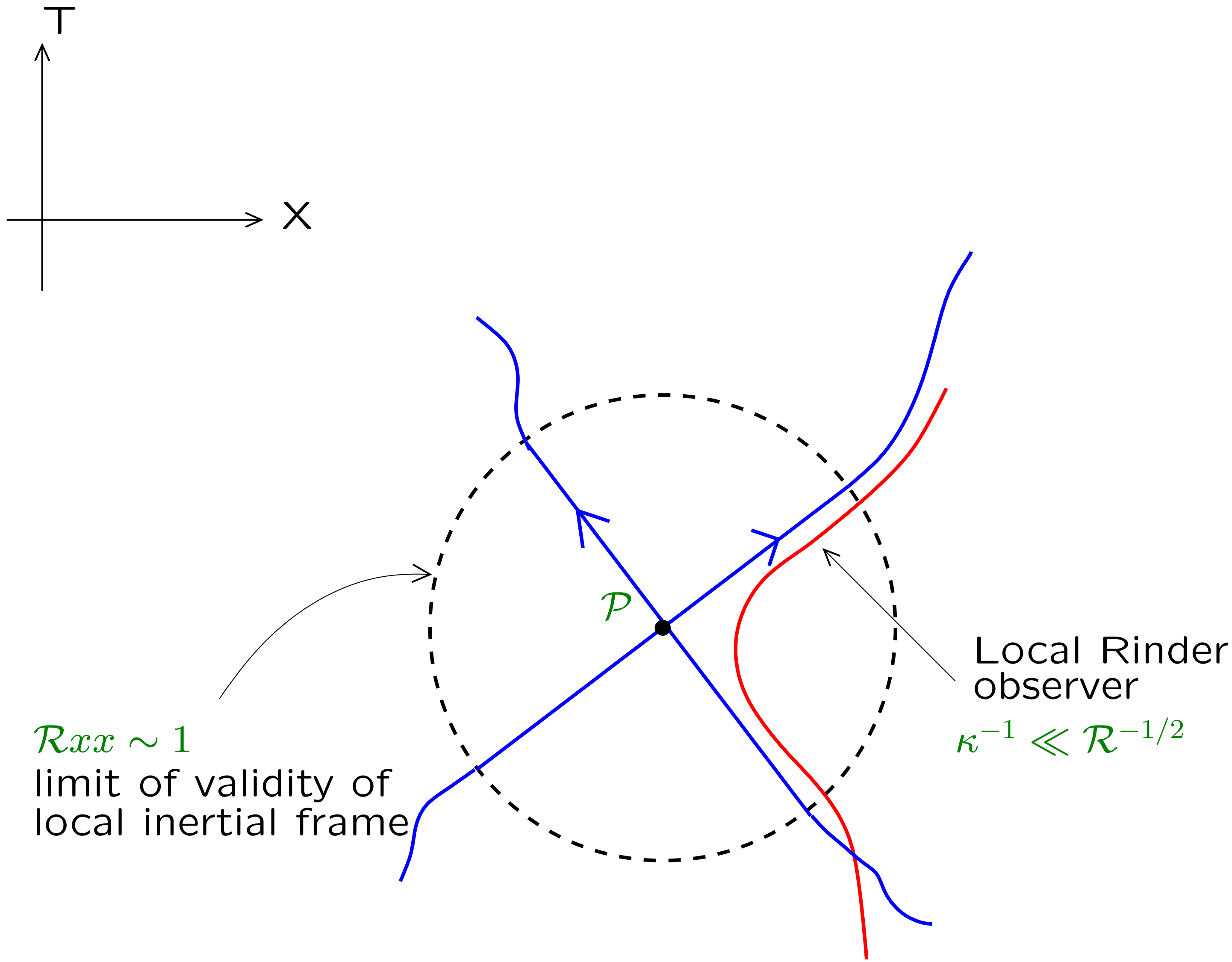

t
x
Arbitrary
Event
$\mathcal{P}$
Local Rinder
observer
Null rays
through $\mathcal{P}$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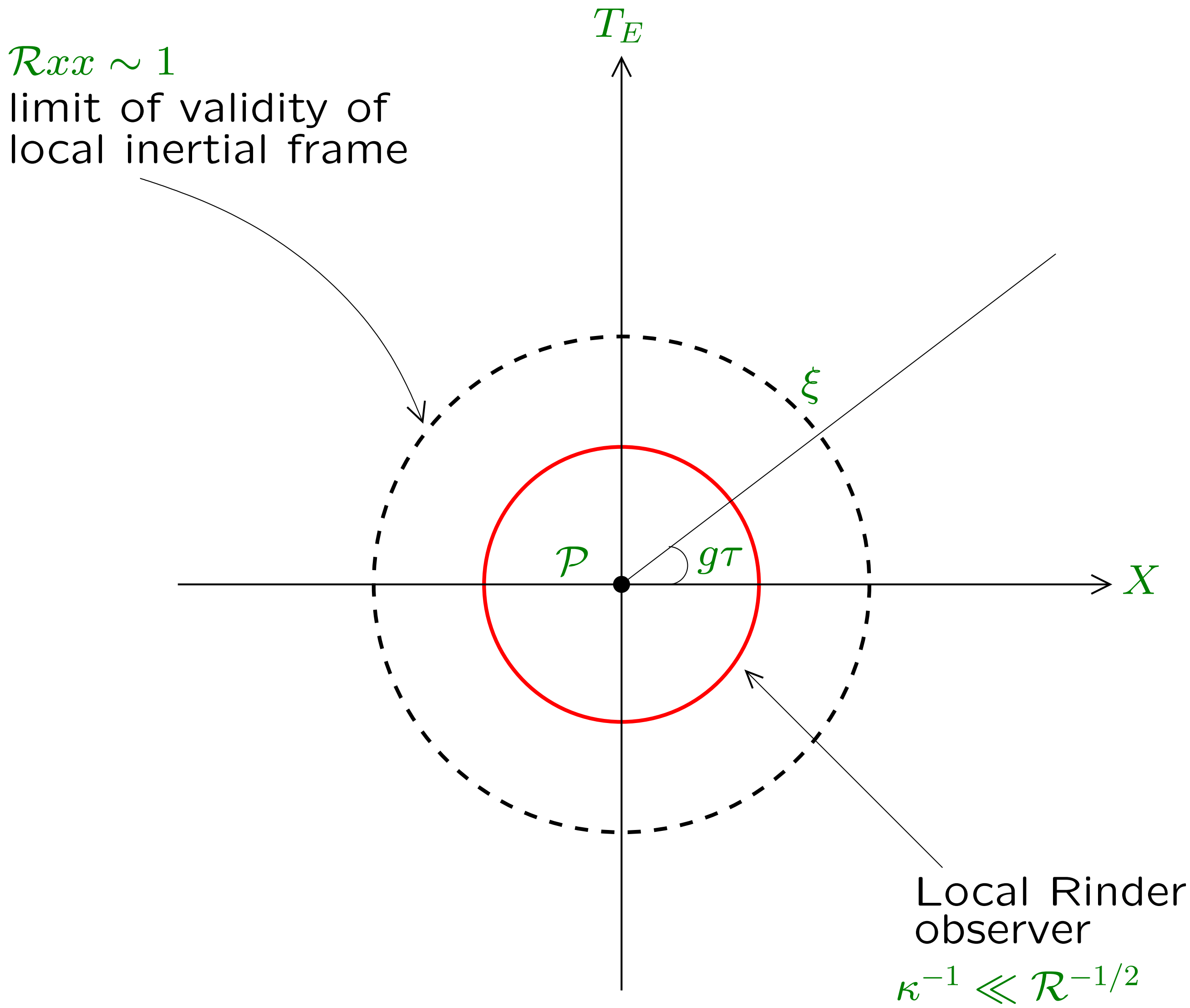

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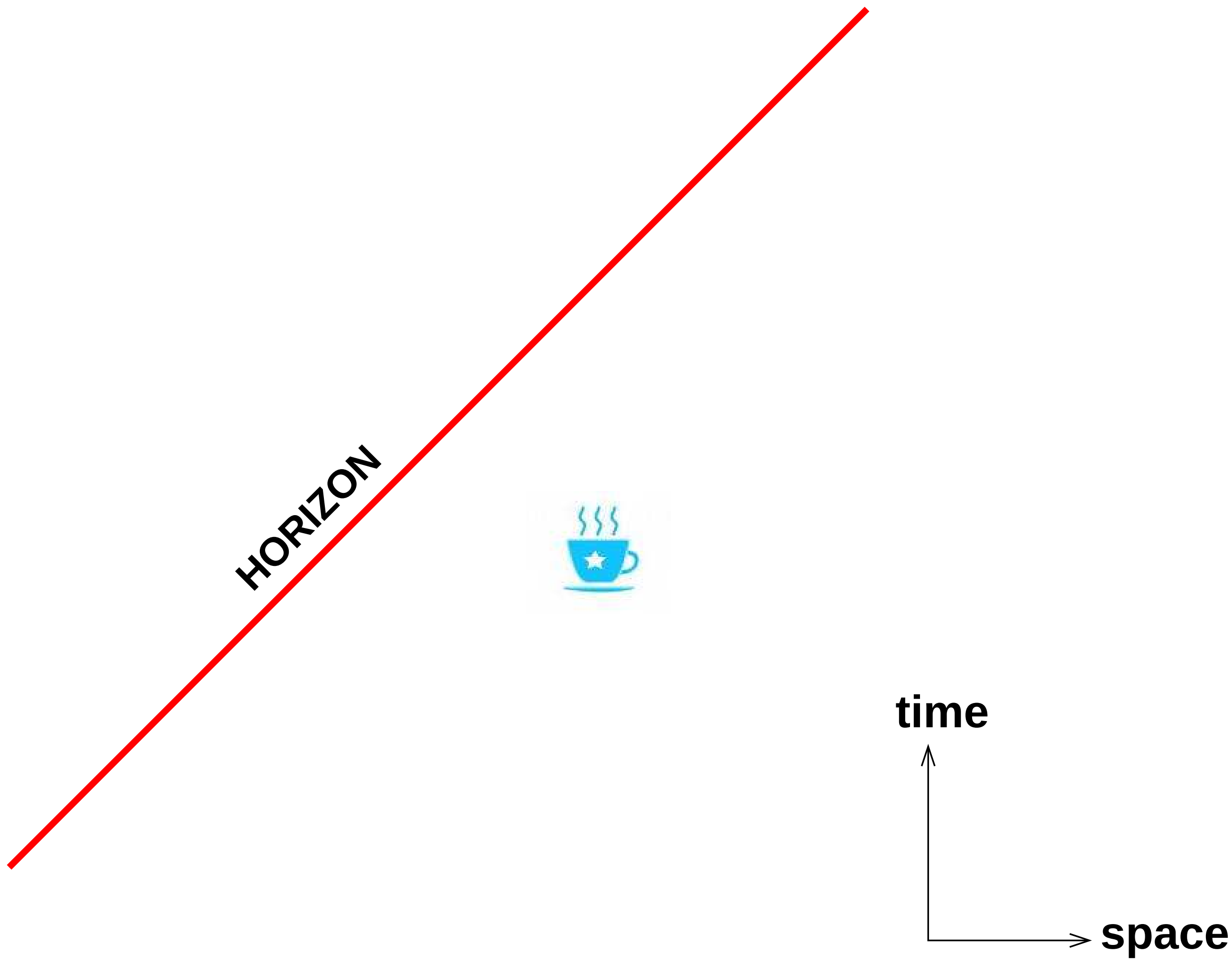
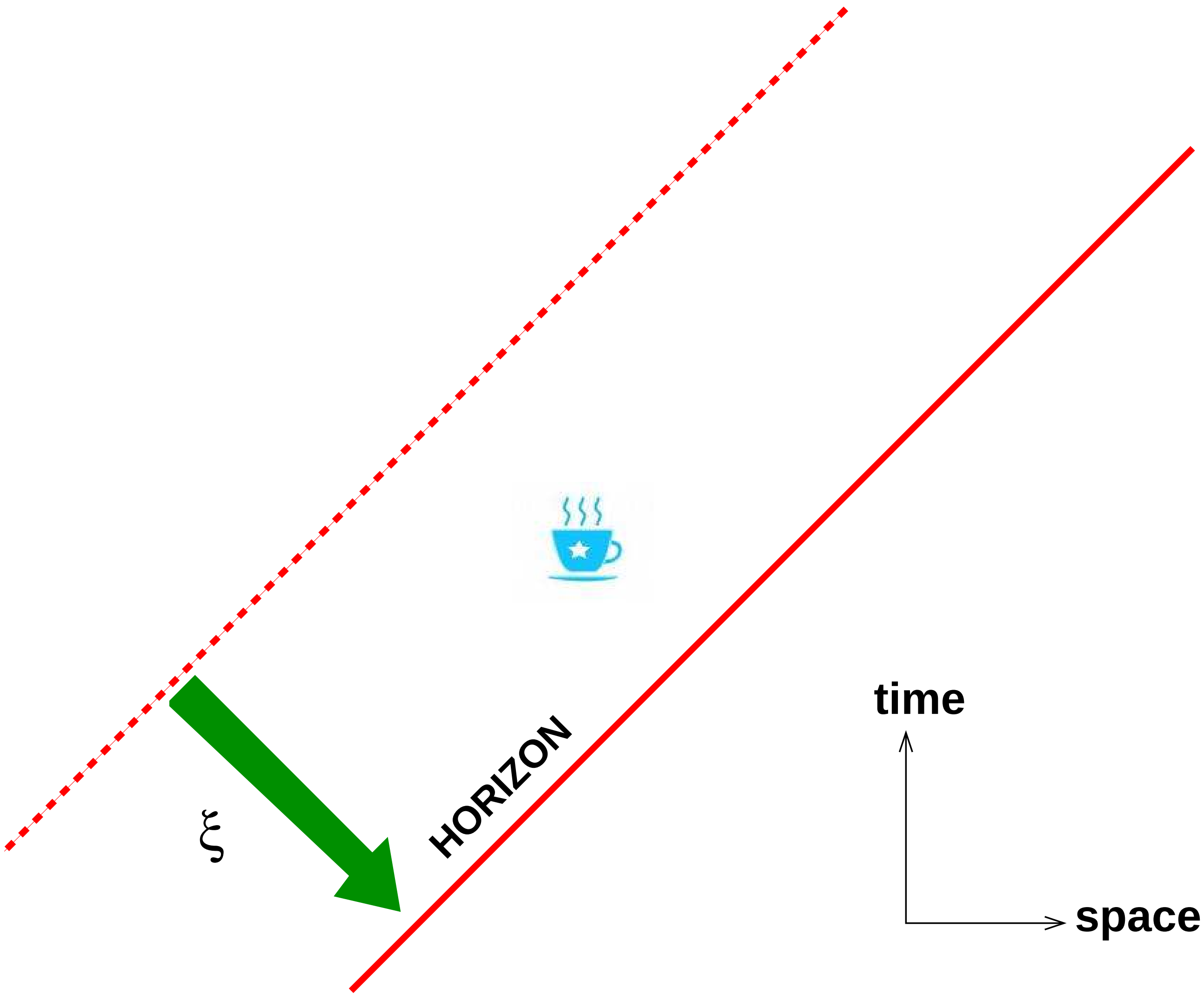

HORIZON
time
space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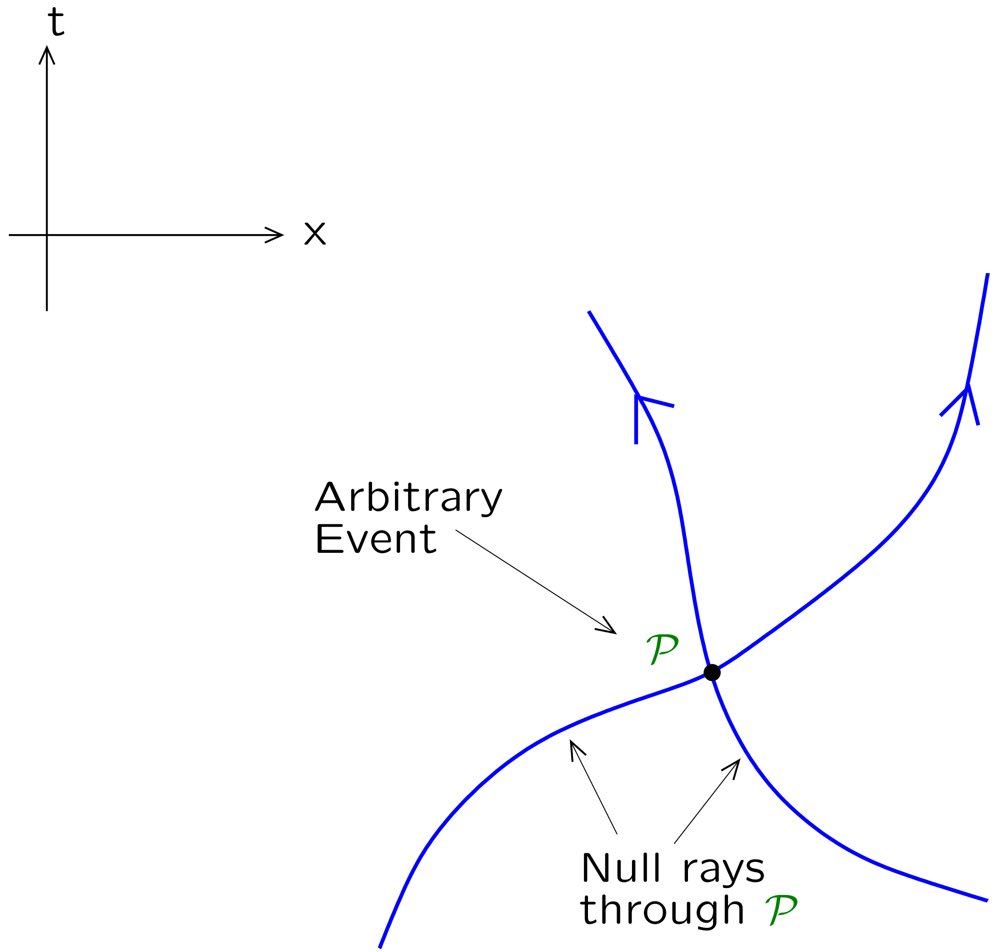
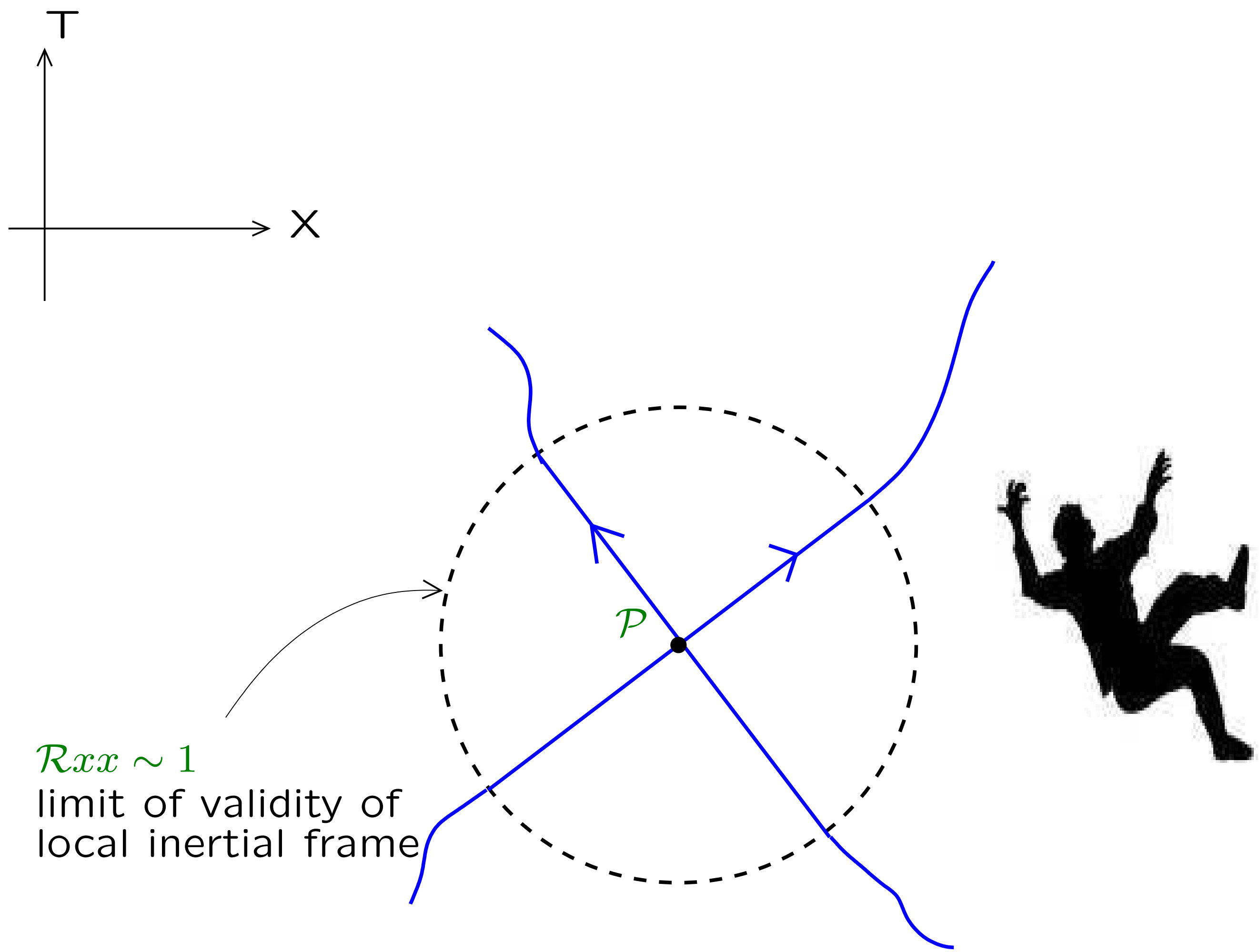
  Local Rindler frames $\Rightarrow$ Dynamics of Gravity

SPACETIME IN ARBITRARY COORDINATES

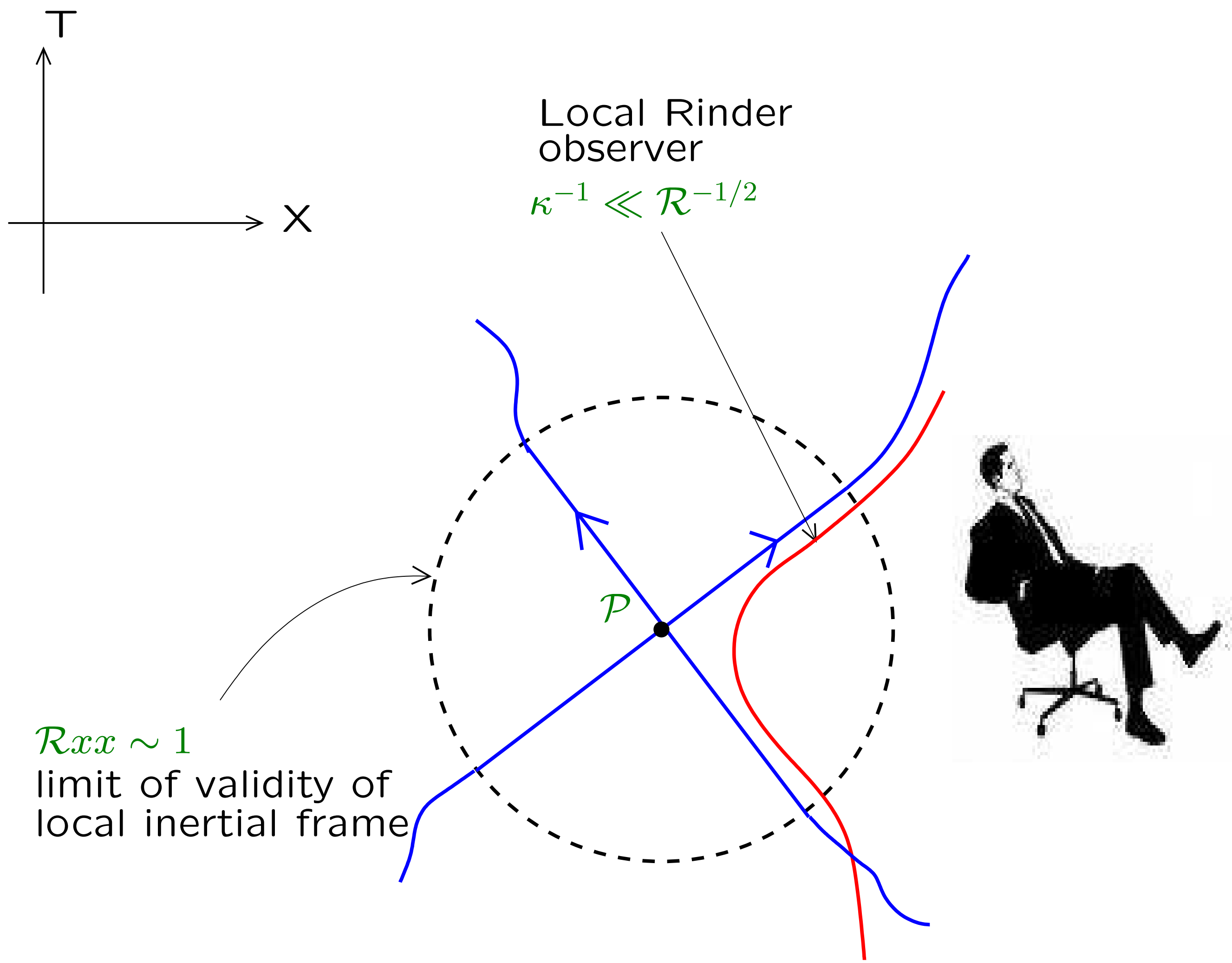


FREE – FALL OBSERVERS



Validity of laws of SR $\Rightarrow$ kinematics of gravity

LOCAL RINDLER OBSERVERS



Validity of entropy extremisation $\Rightarrow$ dynamics of gravity

A NEW VARIATIONAL PRINCIPLE

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- Associate with the virtual displacements of null surfaces an entropy which is quadratic in deformation field: [T.P, 08; T.P., A.Paranjape, 07]

$$S[\xi] = S_{grav} + S_{matt} = -\int_{\mathcal{V}} d^D x \sqrt{-g} \left[4P^{abcd} \nabla_c \xi_a \nabla_d \xi_b - T^{ab} \xi_a \xi_b \right]$$

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- Demand that $\delta S = 0$ for all null vectors should (i) constrain the background and (ii) lead to second order field equations.

- Uniquely fixes the form of P^{abcd} as

$$P^{abcd} = \left(\frac{\partial L}{\partial R_{abcd}}\right); \quad \nabla_a P^{abcd} = 0$$

where L is the Lanczos-Lovelock Lagrangian.

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- The field equations now have a new symmetry. The action and field equations are invariant under $T_{ab} \rightarrow T_{ab} + \rho_0 g_{ab}$. Gravity does *not* couple to bulk vacuum energy (cosmological constant).

ASIDE: MORE GENERAL CLASS OF MODELS

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- If we allow for higher order field equations, a more general class of models are possible with (T.P., 09; S.F.Wu, 09)

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- That is, given an $L(R^{ab}_{cd}, g_{ab})$ that leads to a field equation on varying g_{ab}, one can write down explicitly an $S[\xi^a]$ which gives the same field equations on varying ξ^a.

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- Connects with the equipartition idea.

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- Maximizing the entropy $S[n_a]$ associated with *all* null vectors gives field equations of the theory. Different forms of $S[n_a]$ lead to different theories.
- The deep connection between gravity and thermodynamics *goes well beyond Einstein's theory.* Closely related to the holographic structure action functional.

...JUST IN CASE YOU DON'T BELIEVE ME...

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Your Homework Assignment!

- Why do Einstein's equations reduce to a thermodynamic identity for virtual displacements of horizons ?
- Why is Einstein-Hilbert action holographic (and has other peculiar features) ?
- Why does the surface term in the action give the horizon entropy ? And on-shell action reduces to the free energy ?
- Why does the microscopic degrees of freedom obey thermodynamic equipartition ?
- Why is gravity immune to bulk vacuum energy ?
- Why do all these work for a much wider class of theories than just GR ?

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- Can one do better than a host of other 'QG candidate models' ? e.g., cosmological constant problem, singularity problem ...

REFERENCES

- **T.P,** *A Dialogue on the Nature of Gravity*, Proceedings of the meeting '*The Foundations of Space and Time*', Cape Town, Aug, 2009 (CUP, in press), [arXiv:0910.0839]
- **T.P,** *Thermodynamical Aspects of gravity: New Insights*, *Rep.Prog.Physics*, **73**, 046901 (2010) [arXiv:0911.5004].
- **T.P,** *Surface Density of Spacetime Degrees of Freedom from Equipartition Law in theories of Gravity*, *Phys.Rev.*, **D 81**,, 124040 (2010) [arXiv:1003.5665].

THANK YOU FOR YOUR TIME!

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- The variation (ignoring the surface term) is same as varying $(2E_{ab} - T_{ab})n^a n^b$ with respect to n_a and demanding that it holds for all n_a. This is why we get $(2E_{ab} = T_{ab})$ except for a cosmological constant.