Dedicated to the memory of Professor Brian Edgar

On explicit thermodynamic functions and extremal limits of Myers-Perry black holes

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Outline

- Myers-Perry (MP) black holes
- Extremal limits ↔ Kerr bounds
- Thermodynamic geometry and its applications
- Conjecture on ultraspinning instabilities

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Myers-Perry (MP) black holes

MP BH = solution to Einstein equation in higher dimensions (Myers-Perry 1986)

Non-rotating solutions = Reissner-Nordström in *d* dimensions <---> simple !

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2 dS_{(d-2)}$$

NB: *(d-2)* sphere

$$f(r) = 1 - \frac{\mu}{r^{d-3}} + \frac{q^2}{r^{2(d-3)}}$$

$$\mu = \frac{16\pi M}{(d-2)\Omega_{(d-2)}}$$

$$q = \sqrt{\frac{8\pi G}{(d-2)(d-3)}Q}$$

ADM mass and charge

Rotating MP black holes

Basically Kerr-like black holes in higher dimensions ---> a lot more complicated (in Boyer-Lindquist)

$$ds^2 = -d\bar{t}^2 + (r^2 + a_i^2)(d\mu_i^2 + \mu_i^2 d\bar{\phi}_i^2) + \frac{\mu r^2}{\Pi F}(d\bar{t} + a_i \mu_i^2 d\bar{\phi}_i)^2 + \frac{\Pi F}{\Pi - m r^2} dr^2$$

$$d\bar{t} = dt - \frac{m r^2}{\Pi - m r^2} dr, \qquad d\bar{\phi}_i = d\phi_i + \frac{\Pi}{\Pi - m r^2} \frac{a_i}{r^2 + a_i^2} dr,$$

$$F = 1 - \frac{a_i^2 \mu_i^2}{r^2 + a_i^2}, \qquad \Pi = \prod_{i=1}^{(d-1)/2} (r^2 + a_i^2).$$

Areas of event horizon in odd and even dimensions

$$A = \frac{\Omega_{(d-2)}}{r_+} \prod_i (r_+^2 + a_i^2) \qquad A = \Omega_{(d-2)} \prod_i (r_+^2 + a_i^2)$$

In general it is very difficult to solve for r_+

In this work we study and derive a fundamental relation of MP black holes with all the allowed number of spins appearing in any number of dimensions.

Temperature functions help us determine extremal limits

Definition: Extremal black holes have zero temperature (vanishing surface gravity)

It is tricky to derive the mass formula in arbitrary dimensions when angular momenta are turned on. In 2006 [PRD **73**, 605 (2006)] we did it for a single spin in any *d*

$$M = \frac{d-2}{4} S^{\frac{d-3}{d-2}} \left(1 + \frac{4J^2}{S^2}\right)^{1/(d-2)}$$

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In arXiv:1001.5550 we do for multiple-spin MP black holes, namely for the black holes with $M(S, J_1, J_2, ..., J_n)$

$$M = \frac{d-2}{4} S^{\frac{d-3}{d-2}} \prod_{i}^{n} \left(1 + \frac{4J_i^2}{S^2}\right)^{\frac{n}{d-2}} .$$

n = number of angular momenta

This gives us the temperature in terms of explicit state parameters

$$T = \frac{\frac{d-3}{4} \prod_{i}^{n} \left(1 + \frac{4J_i^2}{S^2}\right) - \frac{1}{2} \sum_{k}^{n} \frac{4J_k^2}{S^2} \prod_{i \neq k}^{n} \left(1 + \frac{4J_i^2}{S^2}\right)}{S^{\frac{1}{d-2}} \prod_{i}^{n} \left(1 + \frac{4J_i^2}{S^2}\right)^{\frac{d-3}{d-2}}} .$$

We study specific cases

(I) All nonzero spins *J* equal (we are first) 

(II) All spins *J* equal (done before but no explicit formulas given)

In case (I) we have the extremal limit (only for $2n > d-3$) in (J, M) coordinates

$$\frac{J^{d-3}}{M^{d-2}}\bigg|_{extr} = \frac{2^{d-n-1}(2n-d+3)^{\frac{2n-d+3}{2}}(d-3)^{\frac{d-3}{2}}}{(d-2)^{d-2}n^n}$$

This is the so-called Kerr bound in *d* dimensions for *n* nonzero equal spins expressed in (J, M) coordinates

Homework: what is the Kerr bound in $d = 4$?

d	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$
4	$\frac{S}{J} \geqslant 2$				
5	$\frac{S}{J} \geqslant 0$	$\frac{S}{J} \geqslant 2$			
6	-	$\frac{S}{J} \geqslant \frac{2}{\sqrt{3}}$			
7	-	$\frac{S}{J} \geqslant 0$	$\frac{S}{J} \geqslant \sqrt{2}$		
8	-	-	$\frac{S}{J} \geqslant \frac{2}{\sqrt{5}}$		
9	-	-	$\frac{S}{J} \geqslant 0$	$\frac{S}{J} \geqslant \frac{2}{\sqrt{3}}$	
10	-	-	-	$\frac{S}{J} \geqslant \frac{2}{\sqrt{7}}$	
11	-	-	-	$\frac{S}{J} \geqslant 0$	$\frac{S}{J} \geqslant 1$

Extremal limits in (*S*, *J*) coordinates for MP black holes in various spectrum of dimensions, *d*, for different angular momenta, *n*. A dash means that no extremal limit exists.

Thermodynamic geometry: an information geometric method

The Hessian of the entropy function can be thought of as a metric tensor on the state space. In the context of thermodynamical fluctuation theory Ruppeiner has argued that the Riemannian geometry of this metric gives insight into the underlying statistical mechanical system.

So we have the Ruppeiner metric defined as the Hessian of the entropy on the state space of the thermodynamic system

$$g_{ij}^{R} = -\partial_i \partial_j S(M, N^a)$$

M denotes mass/internal energy, N^a denotes mechanically conserved parameters such as charge and spin

Ruppeiner geometry tells us about the underlying statistical mechanics of the system

The Ruppeiner metric is conformally related to the Weinhold metric, i.e.

$$g_{ij}^R = \frac{1}{T} g_{ij}^W$$

where *T* is the system's temperature

$$g_{ij}^W = \partial_i \partial_j M(S, N^a)$$

Since we cannot obtain entropy functions of MP black holes in analytic form, the Weinhold metric is thus the starting point. We can transform it to Ruppeiner metric whenever we wish.

In this work we study both Ruppeiner and Weinhold geometries of the MP black holes in arbitrary dimensions. Results are

(A) Ruppeiner curvature scalars
(B) State space plots

We use **CLASSI** in computing all geometrical quantities.

Ruppeiner curvature for **(d, n) – MP black holes**

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$$R_R = -\frac{1}{S}\frac{1+3\frac{2n-d+3}{d-3}\frac{4J^2}{S^2}}{(1+\frac{2n-d+3}{d-3}\frac{4J^2}{S^2})(1-\frac{2n-d+3}{d-3}\frac{4J^2}{S^2})}$$

If $2n > d-3$ the curvature diverges at the extremal limit

$$\frac{J}{S} = \frac{\sqrt{d-3}}{2\sqrt{2n-d+3}},$$

Extremal limit!

For $2n < d-3$ the curvature will diverge at

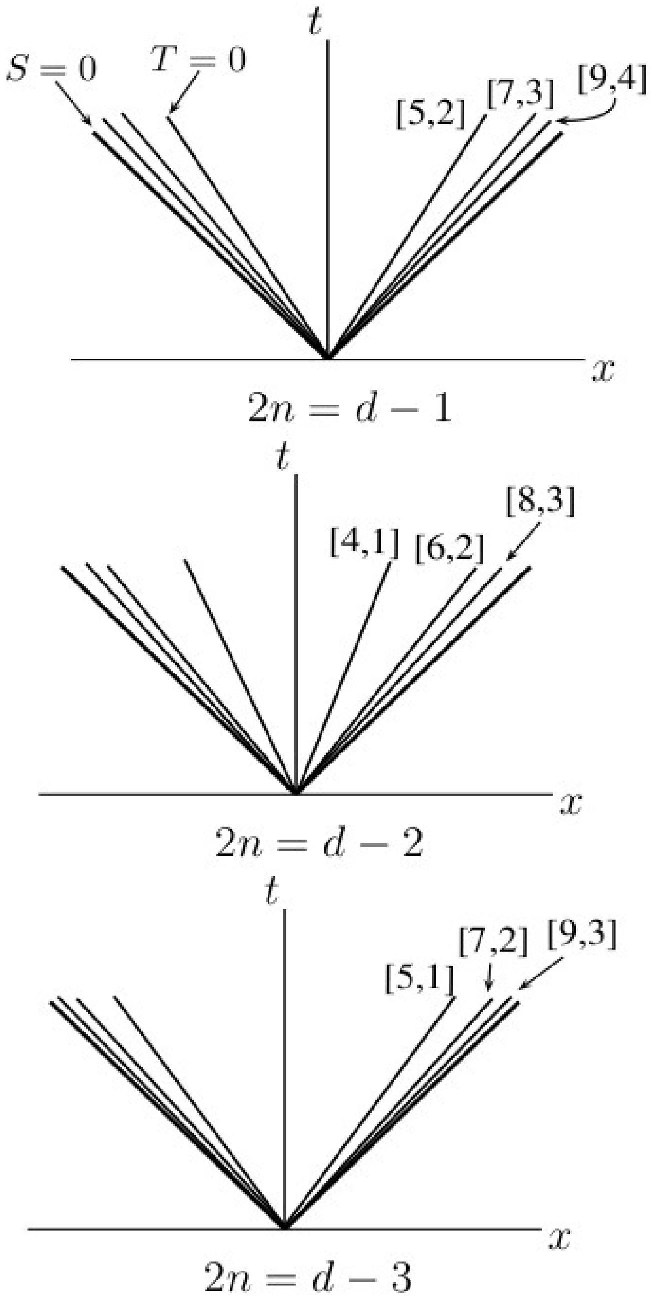
$$\frac{J}{S} = \frac{\sqrt{d-3}}{2\sqrt{d-2n-3}}$$

Not extremal limit but VERY important as we will see!

For $2n = d-3$ the curvature is reduced to

$$R_R = -\frac{1}{S}$$

without a divergence.



Lessons from geometry!

The MP black hole solutions in three series:

Series 1 when *2n = d-1*
Series 2 when *2n = d-2*
Series 3 when *2n = d-3*.

For each series we have three cases as shown in the figures. Numbers in the brackets refer to [*d, n*] with *d* the spacetime dimension, and *n* the number of nonzero spins.

Summary

MP black holes of dimension *d* with *n* equal nonzero spins and $2n >= d-3$ all have extremal limits as expected

We should classify MP black holes in three series depending on
d and *n, i.e.*

$2n - d + 3 = 0, 1 \text{ or } 2$

MP black holes with $2n < d-3$ the Ruppeiner curvature diverges but they have no extremal limits. This is comparable to the recent finding in Ref. JHEP 1008:046,2010 (Astefanesei, Rodriguez and Theisen). They establish the minimum temperature surface on which the membrane phase of ultraspinning MP black holes takes place.

Ultraspinning occurs when at least one of the possible $[\frac{d-1}{2}]$ spins must be exactly 0.

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Our conjecture

We conjecture that the membrane phase ultraspinning MP black holes is reached at the minimum temperature in the case $2n < d-3$ which is where the Ruppeiner curvature diverges.

Thank you for your attention!

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