The Generalized Second Law
in Universes with quantum corrected entropy relations

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Plan of the talk

- Gravity $\leftrightarrow$ Thermodynamics
- Bekenstein Entropy-Area relation for a Black-Hole
- Clausius relation and Equipartition of energy $\leftrightarrow$ Einstein equations
- Quantum corrections to Entropy
- Induced modified Friedmann equations
- Investigation of the GSL in such contexts.

Thermodynamical properties of Black Holes

Black holes and entropy

J.D. Bekenstein *Phys. Rev. D 7, 2333-2346, 1973*

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Particle creation by black holes

S.W. Hawking *Commun. Math. Phys. 43, 199 (1975)*

$$T = \frac{\hbar c^3}{G k_B} \frac{1}{8\pi M}$$

Black holes emit thermal radiation

k_B stands for the Boltzmann constant, A the area of the horizon, $l_{pl} = \sqrt{G\hbar/c^3}$ the Planck's length and M the

Generalised Second law of Thermodynamics (GSL)

Generalized second law of thermodynamics in black-hole physics

J.D. Bekenstein *Phys. Rev. D 9, 3292 (1974)*

GLS

The sum of ordinary entropy outside black holes and the total black hole entropy never decreases

Moreover, the GSL predicted that the emergent Hawking radiation entropy shall more than compensate for the drop in black hole entropy.

Thermodynamical properties of horizons

G.W. Gibbons and S.W. Hawking *Phys. Rev. D, 15, 2738, 1977*
P.C.W. Davies*Class. Quantum Grav. 4 L225, 1987*

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Temperature for apparent horizon in a FRW Universe

They show that there is indeed a Hawking radiation with temperature $T = 1/2\pi r_A$, for locally defined apparent horizon of a Friedmann-Robertson-Walker universe with any spatial curvature

r_A is the apparent horizon radius

Cosmological horizons and the generalised second law of thermodynamics Rong-Gen Cai, Li-Ming Caob, Ya-Peng Hu *Class. Quantum Grav. 26, 155018, 2009*

Thermodynamics and Gravity

Thermodynamics of space-time: The Einstein equation of state

T. Jacobson *Phys. Rev. Lett., 75:1260, 1995*

Formulation of the Einstein's equations from the Clausius relation

$$\delta Q = T dS$$

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Surface density of spacetime degrees of freedom from equipartition law in theories of gravity

T. Padmanabhan *arXiv:1003.5665v1 [gr-qc], 2010*

Assumption of the the field equations and derivation of the equipartition law

$$E = \frac{1}{2} n k_B T$$

Quantum entropy corrections

Quantum corrections to the semi-classical entropy-law

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Quantum corrections to the semi-classical entropy-law

- Logarithmic corrections arise from loop quantum gravity due to thermal equilibrium fluctuations and quantum fluctuations

$$S \propto \left[\frac{A}{4l_{pl}^2} + \alpha \ln \frac{A}{4l_{pl}^2} \right] .$$

- power-law corrections appear in dealing with the entanglement of quantum fields in and out the horizon

$$S \propto \frac{A}{4l_{pl}^2} \left[1 - K_\alpha A^{1-\alpha/2} \right] .$$

In the expressions above, α denotes a dimensionless parameter whose value is currently under debate.

Towards modified Friedmann equations

We wish to examine thermodynamical behavior of the system consisting in the apparent horizon of a spatially flat FRW universe and the fluid within it.

The FRW metric can be written as

$$ds^2 = h_{ab} dx^a dx^b + \tilde{r}^2 d\Omega^2,$$

where $\tilde{r} = a(t)r$ and $h_{ab} =$ diag$(-1, a(t)^2)$.
The apparent horizon is

$$\tilde{r}_{AH} = \frac{c}{H},$$

where $H = \dot{a}/a$ denotes the Hubble function.

Energy

In both Clausius relation and equipartition principle, the left hand side represents the amount of energy that crosses the apparent horizon within a time interval dt in which the apparent horizon evolves from $\tilde{r}_{AH}$ to $\tilde{r}_{AH} + d\tilde{r}_{AH}$

$$dE = A_{AH} T_{\mu\nu} k^{\mu} k^{\nu} dt \, .$$

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- $T_{\mu\nu} = (\rho + P/c^2) u_{\mu} u_{\nu} + P g_{\mu\nu}/c^2$
- k^a is the generator of the horizon, $k^a = (1, -Hr, 0, 0)$.

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It follows that

$$dE = 4\pi \tilde{r}^3 \left(\rho + \frac{P}{c^2} \right) \; H \; dt.$$

Modified Friedmann equations

$$H^2\left[1+g(\alpha,H)\right] = \frac{8\pi G}{3}\rho,$$
$$\dot{H}\left[1+f(\alpha,H)\right] = -4\pi G\left(\rho+\frac{P}{c^2}\right),$$

the explicit expressions of $f(\alpha,H)$ and $g(\alpha,H)$ depend on both the entropy corrections and the thermodynamical relation employed

Modified Friedmann equations

Table: Expressions for $f(\alpha, H)$ and $g(\alpha, H)$.

	Logarithmic correction
Equipartition	$f(\alpha,H) = \frac{l_p^2\alpha}{2\pi c^2}H^2\left\{1-\frac{1}{2}\ln\left(\frac{\pi c^2}{l_p^2H^2}\right)\right\}$ $g(\alpha,H) = \frac{3l_p^2\alpha}{16\pi c^2}H^2\left\{1+\frac{2}{3}\ln\left(\frac{\pi c^2}{l_p^2H^2}\right)\right\}$
Clausius	$f(\alpha,H) = \frac{l_p^2\alpha}{2\pi c^2}H^2$ $g(\alpha,H) = \frac{3l_p^2\alpha}{16\pi c^2}H^2$
	Power-law correction
Equipartition	$f(\alpha,H) = -\alpha\frac{3-\alpha}{4-\alpha}(Hr_c)^{\alpha-2}$ $g(\alpha,H) = -\frac{3-\alpha}{4-\alpha}(Hr_c)^{\alpha-2}$
Clausius	$f(\alpha,H) = -\frac{\alpha}{2}(Hr_c)^{\alpha-2}$ $g(\alpha,H) = -(Hr_c)^{\alpha-2}$

GSL

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$$\dot{S}_H \propto F(H)\dot{H}.$$

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Using Friedmann equation, it can be cast in terms of the Hubble parameter and the energy density and pressure of the fluid that fills the universe

$$\dot{S}_H = \mathcal{K}\frac{\mathcal{F}(H)}{H^3}\left(\rho + \frac{P}{c^2}\right),$$

where $\mathcal{K} = \frac{8\pi^2 c^5 k_B}{\hbar}$ and $\mathcal{F}(H)$ depends on the entropy corrections and the thermodynamic relation used to derive Friedmann equations

GSL for Logarithmic corrections (Equipartition)

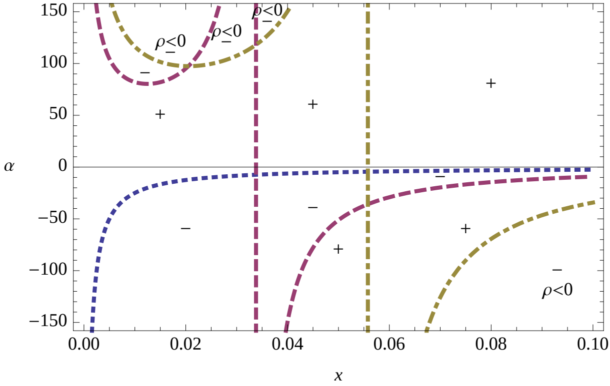


Figure: Plot of the sign of $\mathcal{F}$ depending on α along the Universe expansion, in the case of the logarithmic correction and the equipartition principle. Here, $x = l_p^2/A_{AH}$. The plot focuses on the range $0 < x < 0.1$ but in the remaining region the curves behave monotonically. Bear in mind that the smaller x, the older the universe is.

GSL for Logarithmic corrections (Clausius)

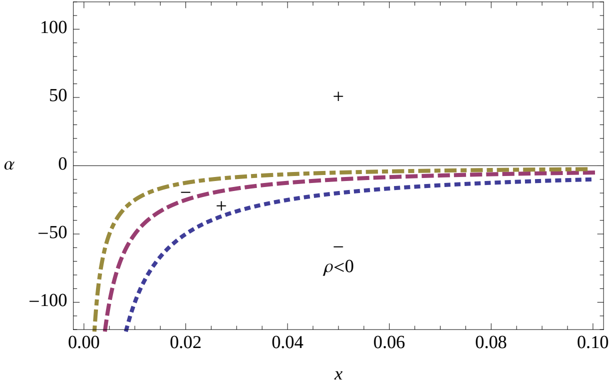


Figure: Same as before but with the equipartition principle replaced by Clausius relation

GSL for Power-Law corrections (Equipartition)

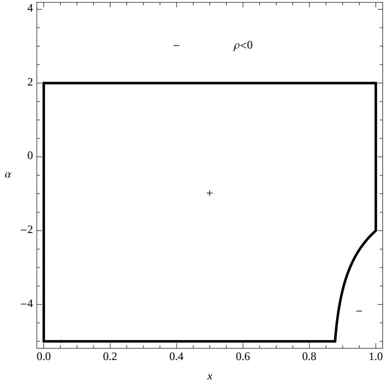
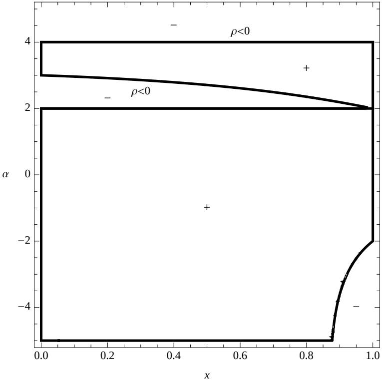


Figure: Sign of $\mathcal{F}$ depending on α along the Universe expansion in the case of power-law correction and Clausius relation. Here $x = H_0/H$.

GSL for Power-Law corrections (Clausius)



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Phantom fluid?

Could this possibility enlarge the available range for α?

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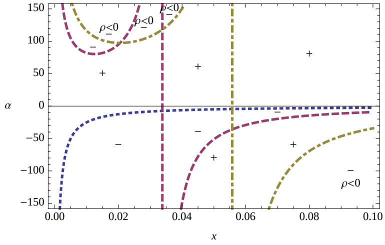

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Phantom fluid?

But an inflationary period can be obtained for $4e^3 < \alpha < 8e^{5/2}$ with

$$N = \int_{t_i}^{t_f} H dt = -\frac{1}{3}\int_{x_i}^{x_f} \frac{1}{\gamma}\frac{1+f(\alpha,x)}{1+g(\alpha,x)}\frac{dx}{x} \sim 60.$$

$$N(\alpha = 90) = 60 \rightarrow \gamma \simeq -0.002.$$

Although this is just a rough estimate it makes clear that, given an evolution for the equation of state parameter, it suffices that it slightly crosses the phantom divide-line to get a convenient amount of inflation.

Particle Production?

On a phenomenological level particle production can be described in terms of an effective bulk viscosity

$$\Pi = -3\zeta H,$$

with ζ the coefficient of bulk viscosity. Thus the total pressure is then

$$P = p + \Pi,$$

Particle production in cosmology

Ya. B Zeldovich *JETP Lett., 12:307, 1970*

Cosmology with adiabatic matter creation

W Zimdahl and D. Pavón *Phys. Lett. A, 176:57, 1993*

GSL with particle production

Entropy rate acquires a new term entirely due to the increase in the number of particles

$$\dot{S}_f = \frac{R^3 \Pi^2}{T_f \zeta}$$

where R^3 is the 3-spatial volume enclosed by the horizon and the fluid is assumed in thermal equilibrium with the horizon.

GSL

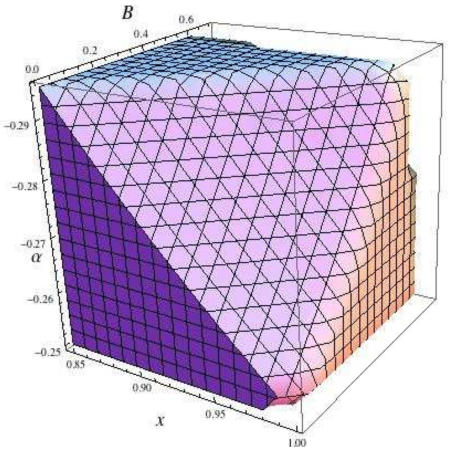
$$\dot{S} = \dot{S}_H + \dot{S}_f = \mathcal{K}\left[\frac{\Pi^2}{3\zeta c^2 H^4} + \frac{\mathcal{F}}{H^3}\left(\rho + \frac{P}{c^2}\right)\right]$$

Particle production for Logarithmic corrections

$$\mathcal{F}(\alpha, x) \geq \frac{B}{B - \gamma\sqrt{x}\,(1 + g(\alpha, x))},$$

where $B = \zeta\sqrt{16\pi}Gl_p/c^3$.
It follows that particle production allows to enlarge the α range from $-1/4 \to -1/2$ in the case of Clausius relation, and from $-1/4 \to -(2 + \ln 4)^{-1}$ in case of equipartition of energy.

Particle production for Logarithmic corrections



Conclusions

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- Our work aimed to discriminate among quantum corrections by requiring, via a classical analysis, the GSL to be fulfilled throughout the evolution of the Universe.
- Logarithmic corrections: Negative values of α are consistent with the GSL only up to $\alpha = -1/4$ or $\alpha = -1/2$ by allowing some amount of particle production.
- Power-law corrections: $3 < \alpha < 4$ corresponds to a power-law correction with an index between -1 and 0, that has been analytically or numerically obtained