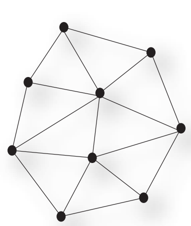
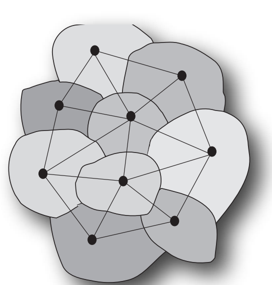
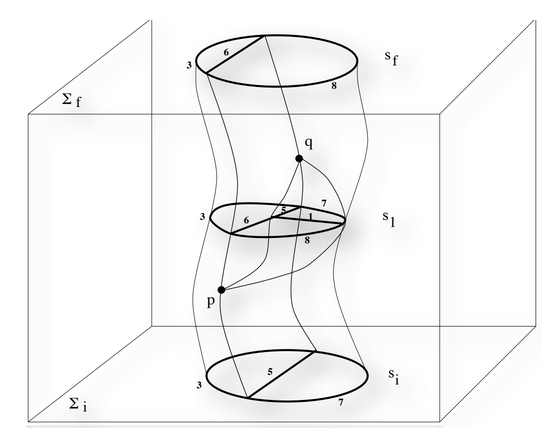
Loop Quantum Gravity

carlo rovelli



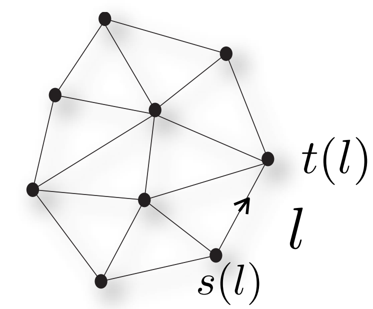




- The theory is defined by the triple $(\mathcal{H}, \mathcal{W}, \mathcal{A})$
  - $\mathcal{H}$ is Hilbert space
  - $\mathcal{W} : \mathcal{H} \longrightarrow \mathbb{C}$ is a map that defines the dynamics
  - $\mathcal{A}$ is an algebra of operators

Hilbert space:

- $\tilde{\mathcal{H}} = \bigoplus_{\Gamma} \mathcal{H}_\Gamma$ Γ: Abstract graph :



- Graph Hilbert space: $\mathcal{H}_\Gamma = L_2[SU(2)^L/SU(2)^N]$
  - Gauge transformations $\psi(U_l) \to \psi(V_{s(l)} U_l V_{t(l)}^{-1}),$ $V_n \in SU(2)^N$
- $\mathcal{H} = \tilde{\mathcal{H}}/\sim$ where $\sim$ defined identifying states on subgraph.

- The space $\mathcal{H}_\Gamma$ admits a basis $|\Gamma, j_l, i_n\rangle$ labelled by a spin for each link and an intertwiner for each node. These states are called "spin network states".

Operator algebra:

- $\vec{L}_l = \{L_l^i\}, i = 1, 2, 3$ left-invariant vector field for each link l : "gravitational field operator (tetrad)"

- U_l : "Holonomy of the Ashtekar-Barbero connection along the link".

- Composite operators:

  - Area: $A_\Sigma = \sum_{l \in \Sigma} \sqrt{L_l^i L_l^i}.$

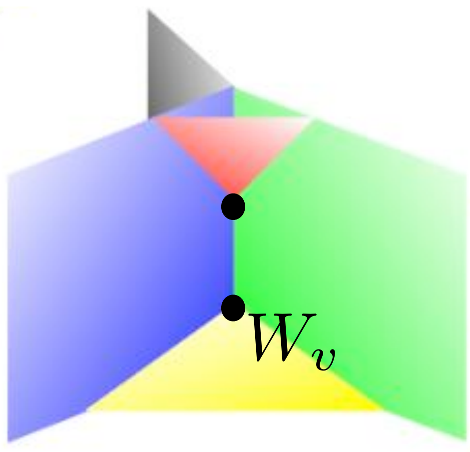
  - Volume: $V_R = \sum_{n \in R} V_n, \qquad V_n^2 = \frac{2}{9} |\epsilon_{ijk} L_l^i L_{l'}^j L_{l"}^k|.$

  - Angle: $L_l^i L_{l'}^i.$

- The spin network basis $|\Gamma, j_l, i_n\rangle$ diagonalizes the area and volume operators.

Dynamics: $|\psi\rangle = |\Gamma, j_l, i_n\rangle$

$$\mathcal{W}(\psi) = \sum_{\partial\sigma=\psi} \prod_f d_{j_f} \prod_v W_v$$



σ : spinfoam

σ: two-complex Δ with faces f and edges e colored with spins j_f and intertwiners i_e, bounded by Γ, j_l, i_n . $\sigma = (\Delta, j_f, i_e)$: "spinfoam".

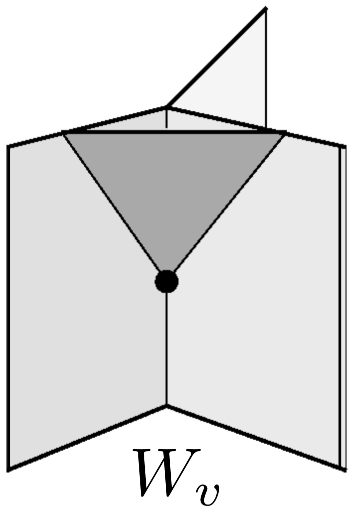
$$d_j = 2j + 1$$

$$W_v = (P_{SL(2,\mathbb{C})} \circ Y_\gamma \; \psi_v)(\mathbb{1})$$

$$Y_\gamma : \mathcal{H}_j \longmapsto \mathcal{H}_j \subset \mathcal{H}_{(p=\gamma(j+1),\, k=j)}.$$

$SU(2)$ rep $SL(2,C)$ rep

$$SU(2) \subset SL(2,\mathbb{C})$$



spinfoam vertex

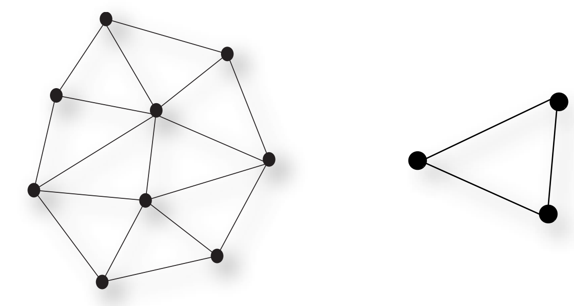
Main conjecture:

$(\mathcal{H}, \mathcal{W}, \mathcal{A})$ defines a (background independent) quantum field theory whose classical limit is general relativity

- In which sense this is a QFT ?
- In which sense it is background independent ?
- How do we compute transition amplitudes ? How do we extract physics ?
- What evidence do we have that its classical limit is General Relativity ?

1. Notice the following structure in $\mathcal{H}$: There is a natural tensor map:

$$\mathcal{T} : \mathcal{H} \otimes \mathcal{H} \to \mathcal{H}$$

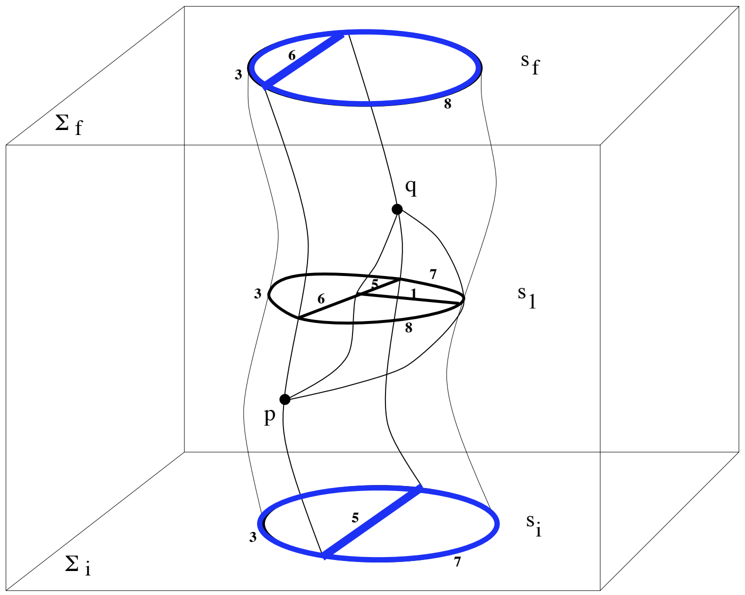


defined by $\mathcal{T}(\mathcal{H}_\Gamma \otimes \mathcal{H}_{\Gamma'}) = \mathcal{H}_{\Gamma\cup\Gamma'}$

Therefore $\mathcal{W}$ defines also maps $\mathcal{W}_2 : \mathcal{H} \to \mathcal{H}$ (cfr: $\mathcal{W}(\psi_{\rm in} \otimes \overline{\psi}_{\rm fin}) = \langle\psi_{\rm fin}|e^{-i\hat{H}t}|\psi_{\rm in}\rangle$)

➜ A two-complex is cobordism of graphs.
A spinfoam is cobordism of spin networks.

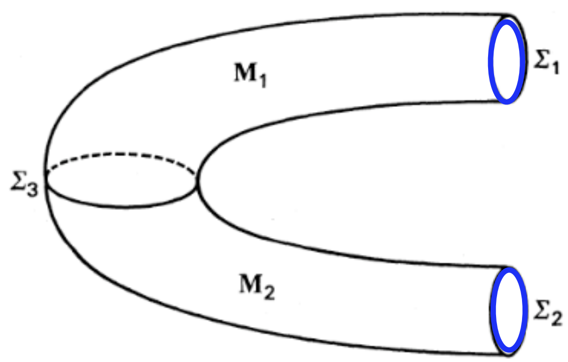
➜ A Hilbert space is associated to any connected graph and an amplitude (a state) is associated to any two-complex with the graphs as boundaries.



➜ Compare: Atiyah Topological Quantum Field Theory

Key differences from Atiyah TQFT:

- Boundary manifolds → graphs
- Cobordism manifolds → Two complexes



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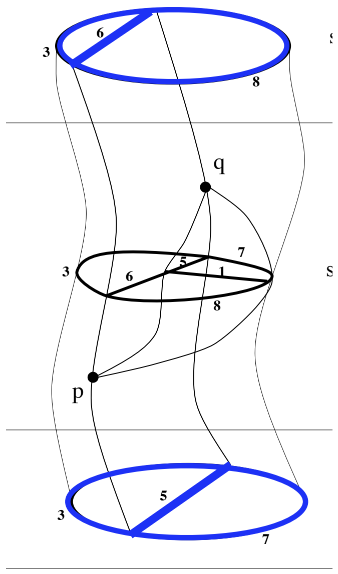
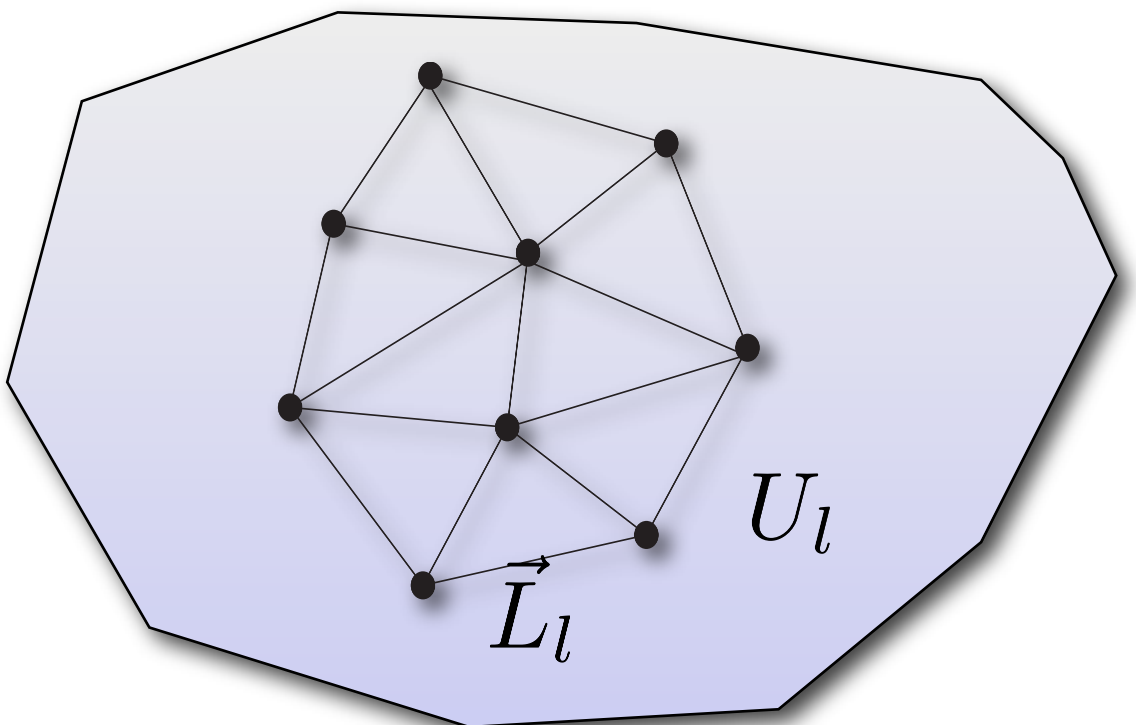


- *Finite* → *Infinite* dimensional boundary Hilbert spaces

A canonical quantization of general relativity [See Ashtekar's talk] leads to a space of states which is (up to technical details) precisely $\mathcal{H}$.



Metric space, with a triad field a Ashtekar-Barbero connection.

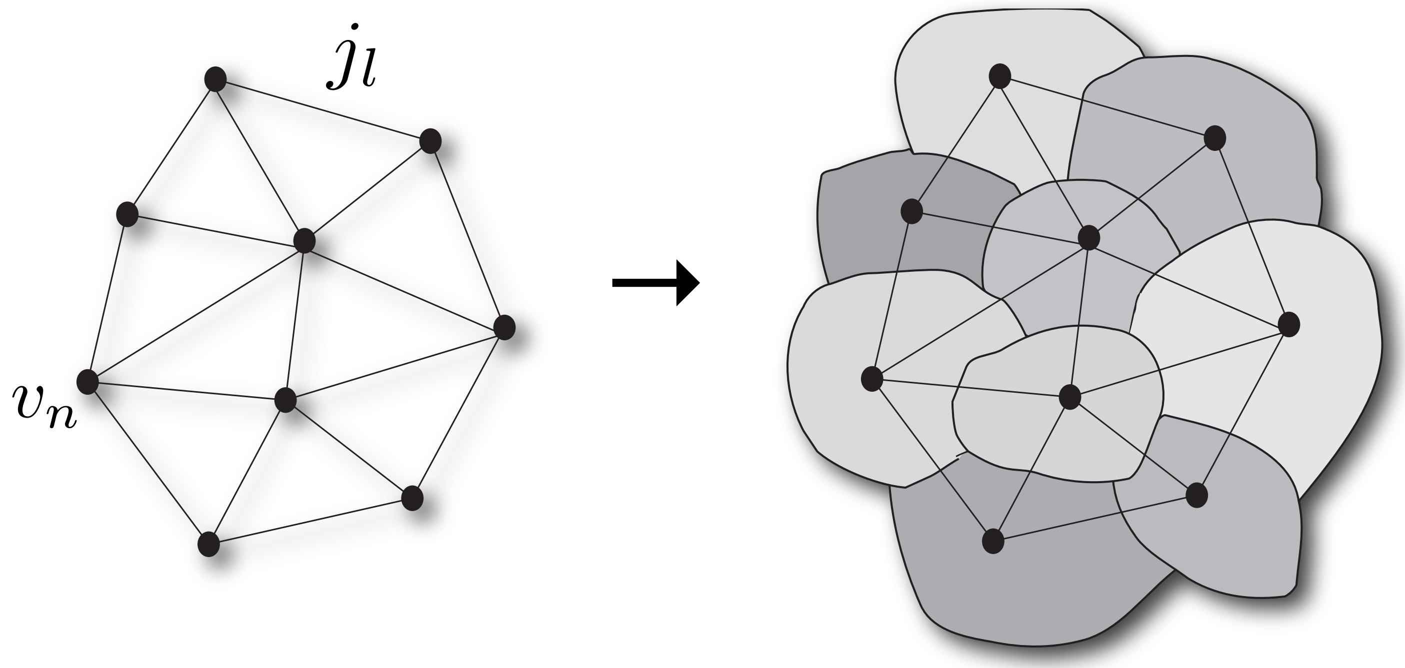
This leads to $\mathcal{H}_\Gamma$ with the operators U_l and $\vec{L}_l$.

Diffeomorphism invariance: *Imbedded* graph → *Abstract* graph

Basis $|\Gamma, j_l, v_n\rangle$ in $\mathcal{H}_\Gamma$ that diagonalizes area and volume: spin network basis

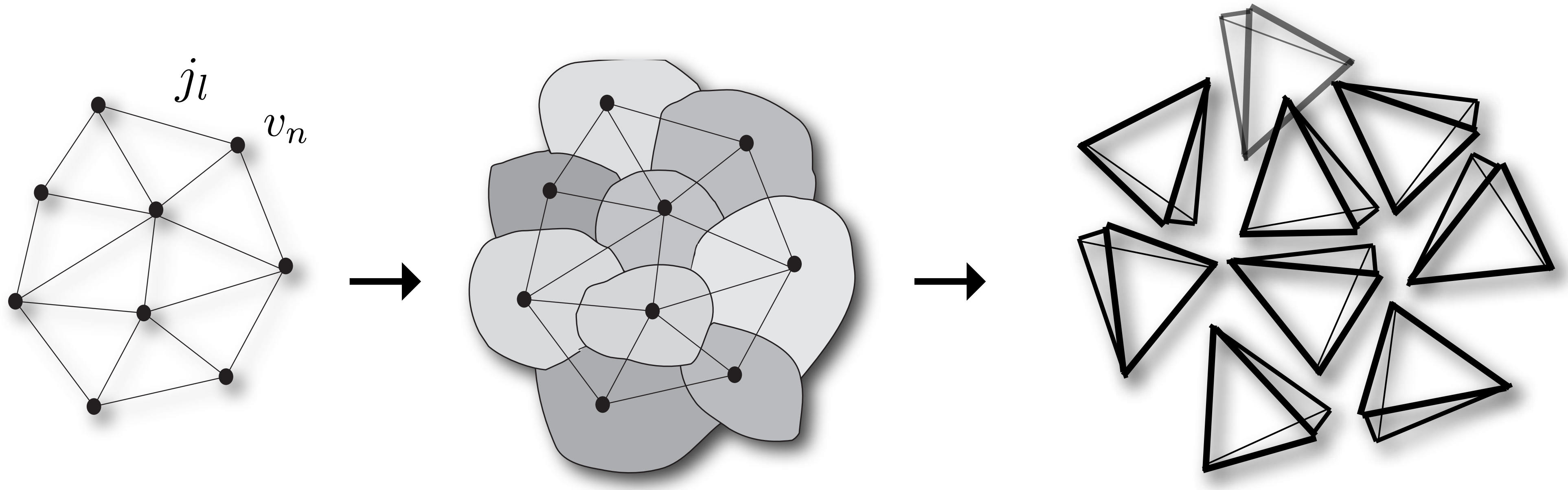
Nodes: discrete quanta of volume (chunks of space, atoms of space) with quantum number v_n .

Links: discrete quanta of area, with quantum number j_l .



A spin network state on a graph is a quantum state of geometry:
These are not states **in** space. These are states **of** space.

Physical interpretation of $\mathcal{H}_\Gamma$: the spin network states



Spin network diagonalize metric and have quantum spread extrinsic geometry

Coherent states : peaked in a given (discrete) intrinsic and extrinsic geometry
[Thiemann, Speziale CR, Livine, Bianchi Magliaro Perini]

Triangulation interpretation: Regge or "twisted" [Dittrich, Bonzom, Speziale Freidel, Livine]

Holomorphic representation: Basis of coherent states

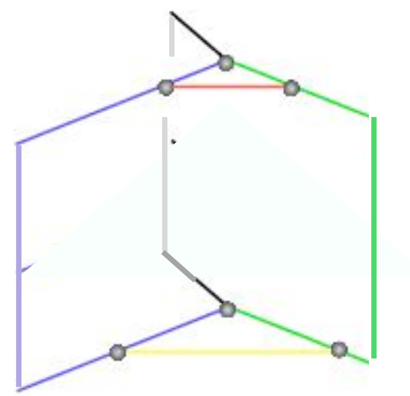
[Ashtekar Lewandowski Marolf Mourao, Bianchi Magliaro Perini]

Dynamics: $\mathcal{W}$

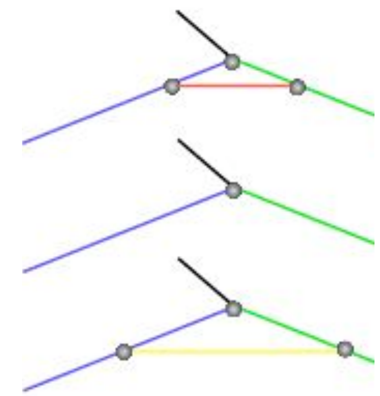
Amplitude associated to a state ψ of a boundary of a 4d region

Probability amplitude $P(\psi) = |\langle W|\psi\rangle|^2$

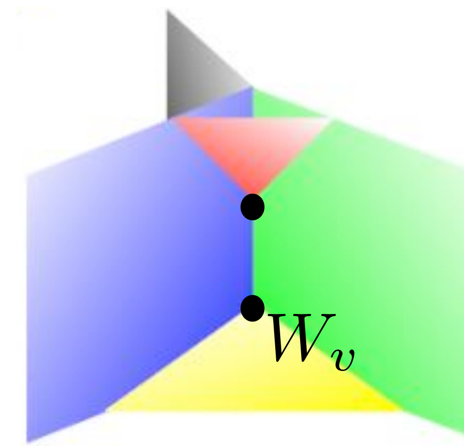
3d boundary



boundary graph

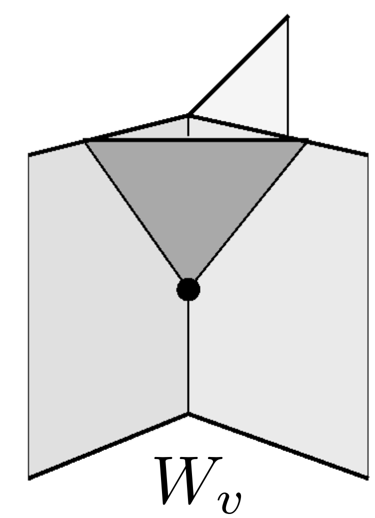


a spin network history

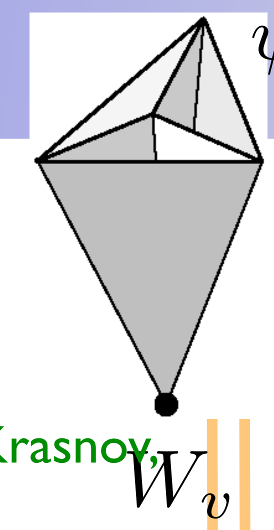


σ : spinfoam

- Superposition principle $\langle W|\psi\rangle = \sum_\sigma W(\sigma)$
- Locality: vertex amplitude $W(\sigma) \sim \prod_v W_v.$
- Lorentz invariance $W_v = (P_{SL(2,\mathbb{C})} \circ Y_\gamma \; \psi_v)(\mathbb{1})$



spinfoam vertex



Natural immersion $\mathcal{H}_\Gamma^{SU(2)} \subset \mathcal{H}_\Gamma^{SL(2,\mathbb{C})}$:

$SU(2) \subset SL(2,\mathbb{C})$

$$Y_\gamma : \mathcal{H}_j \longmapsto \mathcal{H}_j \subset \mathcal{H}_{(p=\gamma(j+1),\, k=j)}.$$

[Engle Pereira CR, Livine, Speziale, Freidel Krasnov, Lewandowski Kaminski Kisielowski, 07-10]

i) If we replace Y_γ with the identity, we obtain a TQFT which is well known: it is the Ooguri quantization of the theory $S[B,A] = \int B \wedge F$

ii) General relativity can be written as $S[e,A] = \int ((e \wedge e)^* + \frac{1}{\gamma} e \wedge e) \wedge F$

and $B = (e \wedge e)^* + \frac{1}{\gamma} e \wedge e$ iff (in a fixed gauge): $B^{ij} - \gamma\ \epsilon^{ij}{}_k\ B^{0k} = 0$

iii) Theorem [Ding CR 09]: on the image of Y_γ : $\langle \psi | B^{ij} - \gamma\ \epsilon^{ij}{}_k\ B^{0k} | \phi \rangle = 0$

Asymptotic analysis

$$W_v \sim \; e^{iS_{Regge}} \sim \; e^{iS_{Einstein-Hilbert}}$$

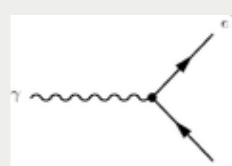
[Barrett Dowdall Fairbairn Gomes Hellmann, Pereira]

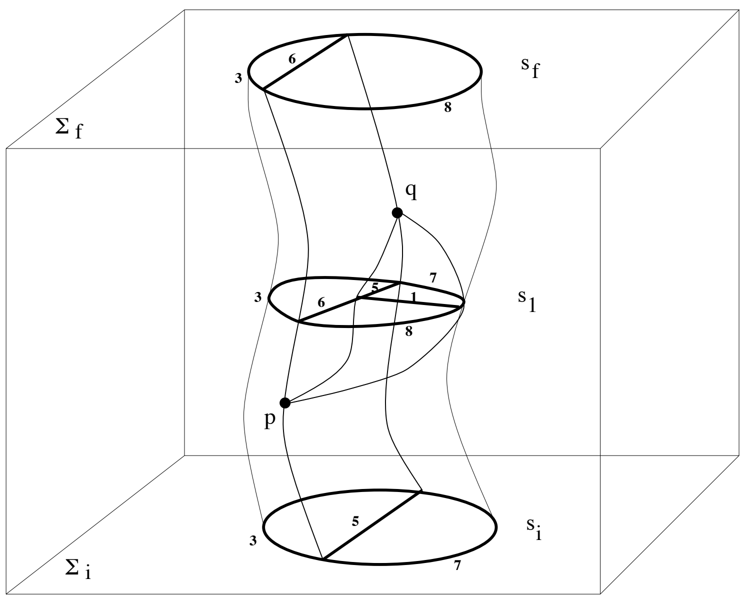
In the spin network basis W yields the cos of the action.
In the holomorphic representation, only one of the two terms of the cos survives
[Bianchi Magliaro Perini]

Dynamics: $\mathcal{W}$

$$W_v = (P_{SL(2,\mathbb{C})} \circ Y_\gamma \; \psi_v)(\mathbb{1})$$

This natural vertex amplitude appear to yield the Einstein equations in the large distance classical limit: A natural group structure based on $SU(2) \subset SL(2,\mathbb{C})$ appears to code the Einstein equations.

cfr :  $= e \; \gamma_\mu^{AB} \; \delta(p_1 + p_2 - k)$



σ : *spinfoam*

$$\langle W|\psi\rangle = \sum_{\sigma} \prod_{f} (2j_f+1) \prod_{v} W_v(\psi_v(\sigma)).$$

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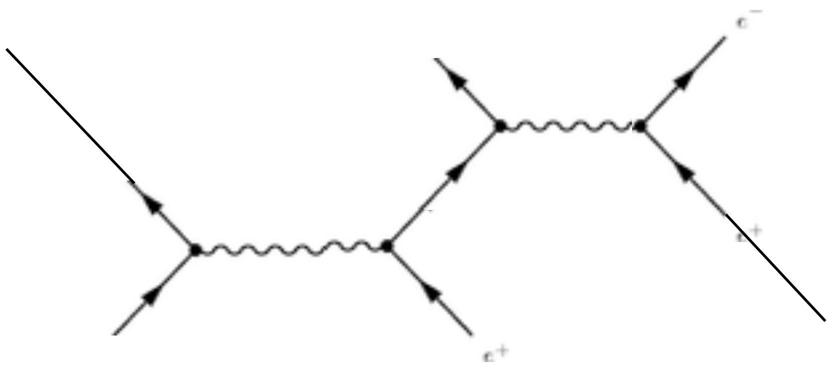
"Sum over histories" form of LQG: Dual interpretation:

I. Discrete version of:

$$W(q) = \int_{\partial g = q} Dg \ \ e^{iS_{EH}[g]}$$

II. Sum over Feynman graphs:



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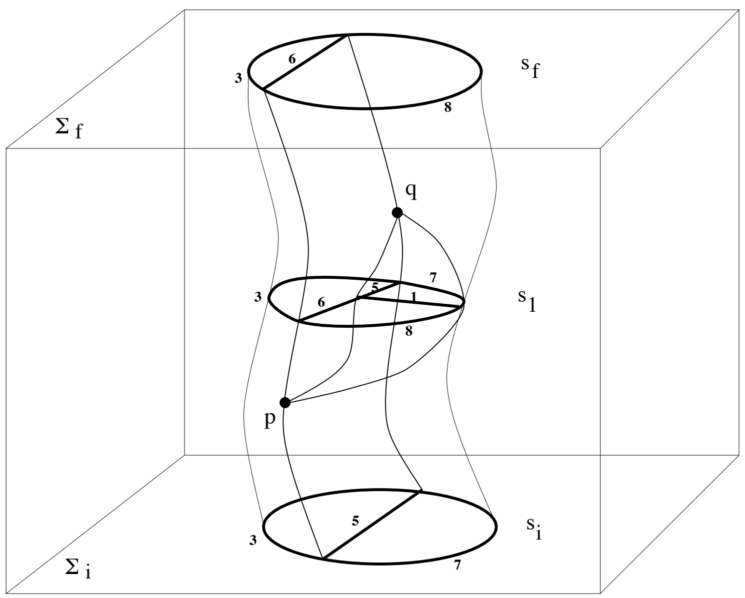

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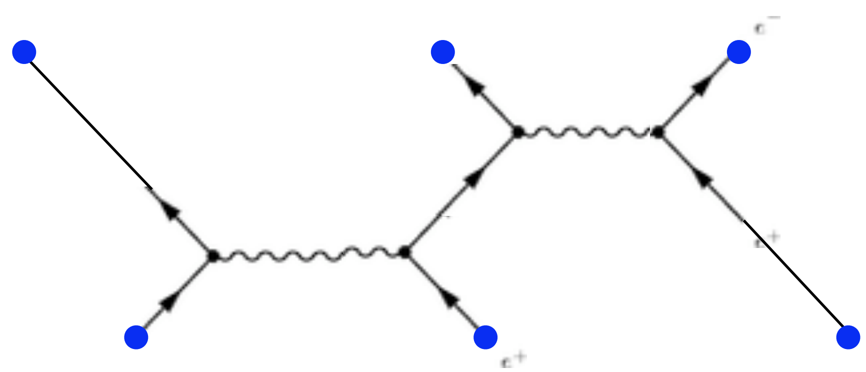

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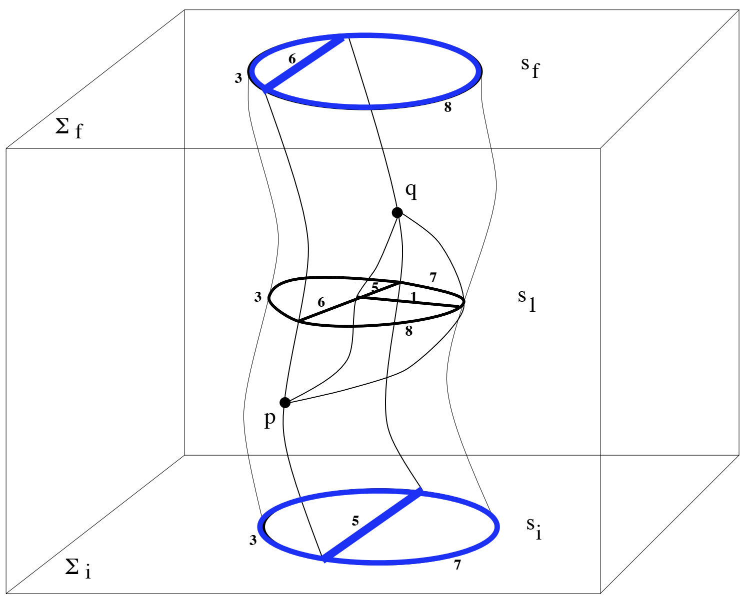

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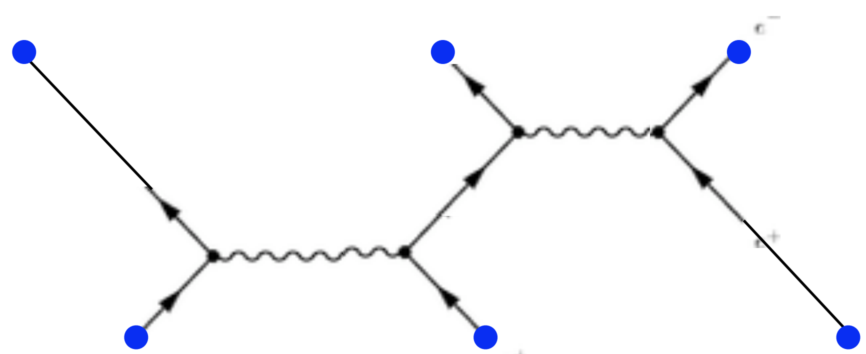


Quantization methods:

- Canonical quantization à la Dirac of the ADM constraints of General Relativity in Ashtekar-Barbero variables.
  - Holonomies as main variables
  - Diff invariance → abstract graphs
- Discretization and lattice quantization (à la QCD).
  - GR = BF+constraints → constrained are implemented weakly on the image of Y_γ
  - Lattice spacing independence
- Quantum geometry methods

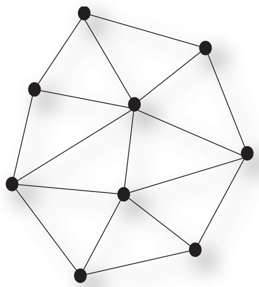
All these converge to the structure $(\mathcal{H}, \mathcal{W}, \mathcal{A})$

Physical assumptions:

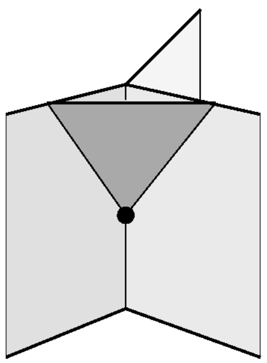
- General Relativity (with standard matter couplings, in Ashtekar formulation)
- Standard quantum mechanics (modified to be general covariant)
- Diffeomorphism invariance fully implemented

There is no physics without approximations.

- Graph expansion: Restricting the theory to $\mathcal{H}_\Gamma$ is a truncation of wavelengths short with respect to the total size of the region considered (cfr: cosmology) (cfr: lattice QCD).



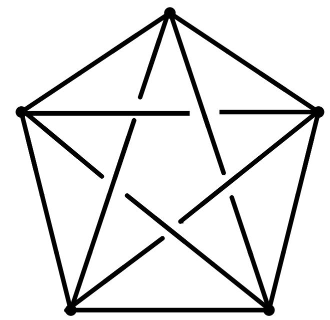
- Vertex expansion: Similar to the vertex expansion in QED: number of "elementary processes considered in the transition amplitude"

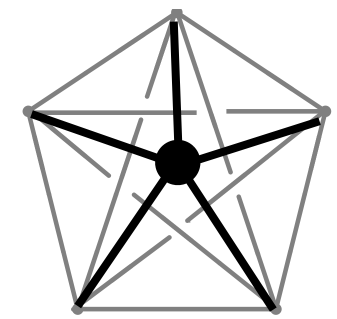


- Large distance expansion: Large with respect to the Planck length (classical limit).

$$j_l \gg 1$$

Results: 1. graviton propagator

Graph $\Gamma_5 =$  .

Vertex 

Boundary state: ψ_L coherent state determined by the (intrinsic and extrinsic) geometry of the boundary of a **flat** 4-simplex.

→ Background info input dynamically via the boundary state.

Amplitude:

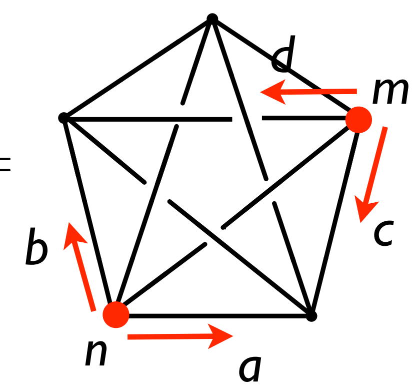
$$W^{abcd}_{mn} = \langle W | \vec{L}_{na} \cdot \vec{L}_{nb} \; \vec{L}_{mc} \cdot \vec{L}_{md} | \psi_L \rangle_c$$

corresponds to the perturbative QFT's graviton propagator

$$W^{abcd}(x_m, x_n) = \langle 0 | g^{ab}(x_n) g^{cd}(x_m) | 0 \rangle_c$$

Matches to first order ! [Bianchi Magliaro Perini]

Graph $\Gamma_5 =$



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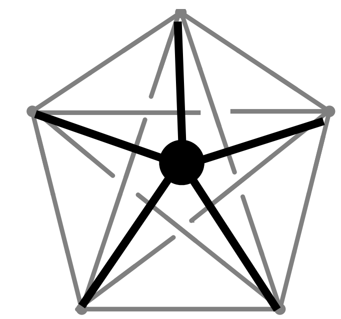
Vertex



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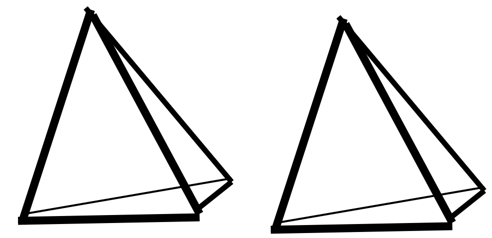
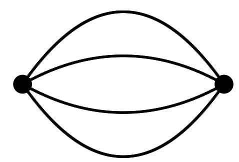
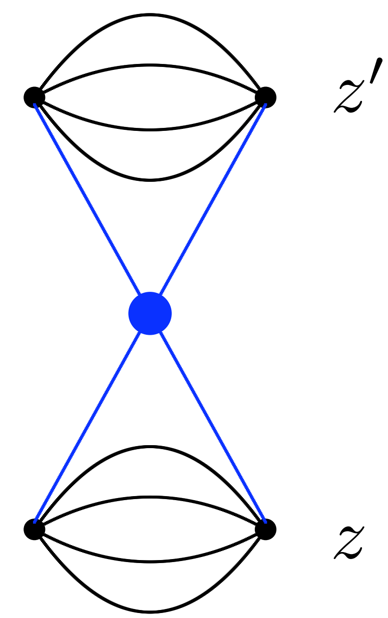
Triangulate a 3-sphere with two tetrahedra : these capture the first d.o.f.'s in a mode expansion of a cosmological metric



Dual graph: $\Delta_2^* =$  .

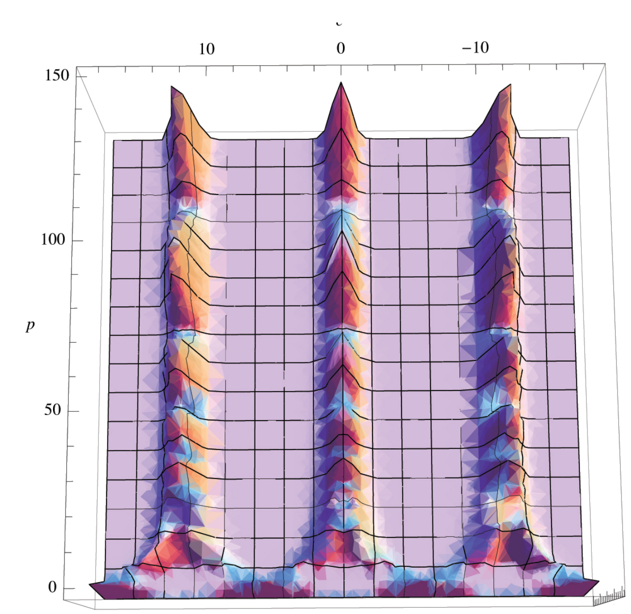


Boundary state on $\Delta_2^* \cup \Delta_2^*$ coherent state ψ_z peaked on homogeneous isotropic geometries on the 3-sphere.

Amplitude $W(z, z') = \langle W | \psi_z \otimes \psi_{z'} \rangle$

$$W(z, z') \sim z z' e^{-\frac{z^2 + z'^2}{2t\hbar}}$$

[See Vidotto's talk]



Reproduces the Friedmann dynamics:

- Is peaked on the classical solutions
- Satisfies a quantum constraint which reduces to the (gravitational part of the) Friedmann hamiltonian for $\hbar \to 0$

Open issues

- No UV divergences!   Infrared divergences?
  [Speziale Perini CR, Bonzom, Smerlak, Rivasseau Gurau Oriti]

- Scaling by radiative corrections?

- Matter in spinfoam?

- Cosmological constant?

- Relation covariant canonical formalism's dynamics?
  [Ashtekar Campigia Henderson, Wilson-Edwin Nelson, Vidotto CR]

Observations are possible. Several suggestions, but:

- No empirical support yet

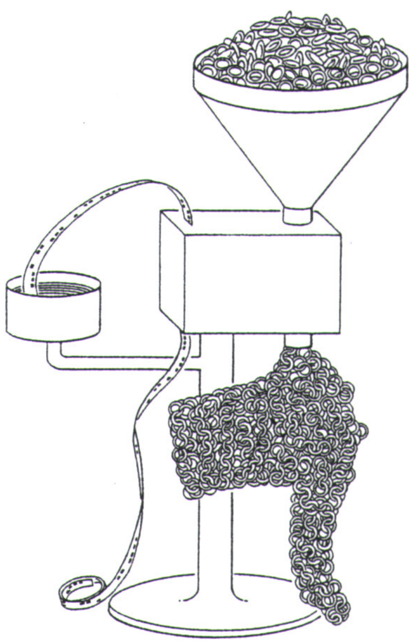
- No solid verifiable prediction yet

Summary

- The theory is defined by the triple $(\mathcal{H}, \mathcal{W}, \mathcal{A})$.
- It is a generalization of a topological QFT in the sense of Atiyah
- Kinematics: quanta of space with quantized volume and area [see Ashtekar and Livine's talks]
  Dynamics: transition amplitudes computed in expansions
- Indications supporting the conjecture that it is quantum GR:
  - derivation from canonical quantization of GR
  - derivation from discretization of GR and GR=BF+constraints.
  - asymptotic of the vertex
  - results on the low-energy limit : n-points functions, cosmology [see Vidotto's talk]
- Main physical applications
  - Loop Cosmology → Big bounce [see G. Mena-Marugan, Martin-Benito, Tanaka, Olmedo's talks]
  - Black hole entropy for real black holes [see Barbero, Diaz-Polo, Borja, talks]
- Loop Quantum Gravity provides a still incomplete, but clean and full-scale *tentative* quantum theory of space time and gravitation.

Good news

Main open problems 15 years ago:



- To construct a mathematically well defined background independent quantum field theory
- The "problem of time"
- Fate of GR singularities (Cosmological and Black Holes)
- Deriving a finite black hole entropy from first principle
- Curing ultraviolet divergences of standard field theories
- Computing quantum gravitational transition amplitudes.

Most of the open problems in quantum gravity of 15 years ago have a solution today in the context of loop gravity