Electrically charged black hole solutions in generalized gauge field theories

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Outline

Overview of the topic

- The non-linear BI action (1934): Generalization of Maxwell theory $L = X = -\frac{1}{2}F_{\mu\nu}F^{\mu\nu} = \vec{E}^2 - \vec{B}^2$ as

$$L_{BI} = \beta^2 \left(1 - \sqrt{1 - \frac{X^2}{\beta^2}}\right)$$

to obtain a finite energy for the electron field $\longrightarrow$ Cornerstone of many investigations in non-linear field theories
- Some developments in gravitating field configurations
  - Coupling to gravity of BI-like like and other non-linear electrodynamics (NED) models, leading to black hole-like solutions (*Garcia et al 84, Demianski 86, Gibbons et al 95, Hassaine et al 08...*)
  - NED models in AdS spaces, motivated by the AdS/CFT correspondence (*Fernando 04, Cai 04, Dey 04*)
  - Higher-order curvature gravity theories (Lovelock) with NED models (*Wiltshire 88, Aiello et al 04, 05*)
  - BI-like actions for non-abelian gauge fields in gravity, solutions with hair? (*Volkov 99, Dyadichev et al 00, and many others*)

Action and classification of the models

- Action:

$$S = \int d^4x\sqrt{-g}\left(\frac{R}{16\pi G} - \varphi(X,Y)\right)$$

$\varphi(X,Y)$: arbitrary function of the two standard field invariants $X = -\frac{1}{2}F_{\mu\nu}F^{\mu\nu} = \vec{E}^2 - \vec{B}^2$, $Y = -\frac{1}{2}F_{\mu\nu}F^{*\mu\nu} = 2\vec{E}\cdot\vec{B}$

- "Matter" side $\varphi(X,Y)$ restricted by some physical *"admissibility"* conditions
  1. φ must be a continuous, derivable and single-valued function on its domain of definition of the $X-Y$ plane
  2. Parity invariance $\varphi(X,Y) = \varphi(X,-Y)$
  3. Positive definite character of energy for *any* field configuration $\rho \geq \left(\sqrt{X^2+Y^2}+X\right)\varphi_X + Y\varphi_Y - \varphi(X,Y) \geq 0$
- In flat space: $\partial_\mu(\varphi_X F^{\mu\nu} + \varphi_Y F^{*\mu\nu}) = 0$ lead, for ESS solutions $(\vec{E} = E(r)\frac{\vec{r}}{r}, \vec{H} = 0)$ to a first-integral

$$r^2\varphi_X E(r) = q$$

- Models are classified according to the character (finite or divergent) of the flat-space energy

$$\varepsilon(q) = 4\pi \int_0^\infty r^2 T_0^0(r,q) = q^{3/2}\varepsilon(q=1)$$

Let us assume a field behaviour $E(r) \sim r^p$ at $r \to \infty$ and $r \sim 0$

- (I) **Finite-energy cases:**
  $r \to \infty$: $p < -1$: Asymptotically vanishing fields: slower than ($-2 < p < -1$, **B1**), equal to ($p = -2$, **B2**) or faster than Coulombian behaviour ($p < -2$, **B3**)
  $r \sim 0$: $-1 < p < 0$: Divergent central-field behaviour (**A1**)
  $p = 0 : E(r) \sim a - br^\sigma$: Finite value at the center (**A2**)
- (II) **Divergent-energy cases:**
  $r \sim 0$: $p \leq -1$ Ultraviolet divergent fields (**UVD**)
  $r \to \infty$: $-1 \leq p < 0$: Infrared divergent fields (**IRD**)

Metric structure

Back to the gravitational context...

- Source symmetry $T_0^0 = T_1^1 \rightarrow$ static spherically symmetric line element

$$ds^2 = \lambda(r)dt^2 - \lambda^{-1}(r)dr^2 - r^2(d\theta^2 + \sin^2\theta d\vartheta^2)$$

- Metric

$$\lambda(r) = 1 + \frac{C}{r} + \frac{8\pi}{r}\int r^2 T_0^0(r,q)dr$$

- Limits of integration of $\int r^2 T_0^0(r,q)dr$ and interpretation of C depend on the NED model
- In addition the field equations $\nabla_\mu(\varphi_X F^{\mu\nu} + \varphi_Y F^{*\mu\nu}) = 0$ lead, for this line element and ESS solutions, to the **same** first-integral as in flat space

(A) Asymptotically Schwarzschild-like solutions

- Combination of a **B**-case at $r \to \infty$ with **A1**, **A2** or **UVD** at the center

$$\lambda(r) = 1 - \frac{2M}{r} + \frac{2\varepsilon_{ex}(r,q)}{r}$$

where M: ADM mass, and

$$\varepsilon_{ex}(r,q) = 4\pi \int_r^\infty R^2 T_0^0(R,q)dR$$

exterior integral of energy, a **monotonically decreasing a concave function of r**

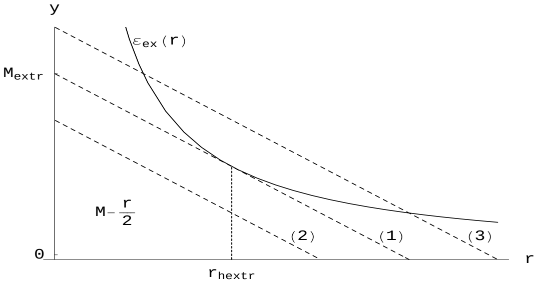
- Horizons: Zeros of $\lambda(r)$, implying

$$M - \frac{r}{2} = \varepsilon_{ex}(r,q)$$

$\rightarrow$ given by the cutting points between $\varepsilon_{ex}(r,q)$ and the beam of straight lines $M - \frac{r}{2}$

Family **UVD** ($r \sim 0$) + **B**-field ($r \to \infty$)

Example: $\varphi(X) = \alpha X$



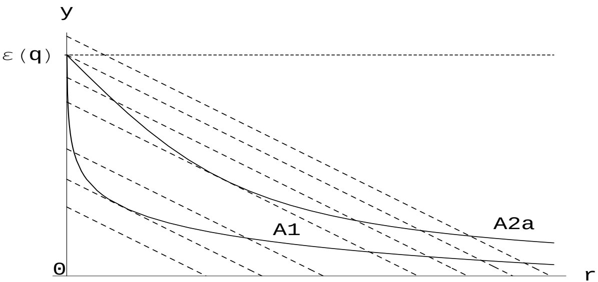
1. Family of extreme black holes (EBH) with radius $8\pi r_{hextr}^2 T_0^0(r_{hextr}, q) = 1$, and mass: $M_{hextr}(q) = \frac{r_{hextr}(q)}{3} + \frac{16\pi q}{3} A_0(r_{hextr}, q)$
2. $M < M_{extr}(q)$: No horizons: naked singularity (NS)
3. $M > M_{extr}(q)$: Two-horizon BH (Cauchy and event)

- Always a time-like singularity at the center

Families **A1** or **A2** ($16\pi qa > 1$) at $r \sim 0$ + **B**-field at $r \to \infty$

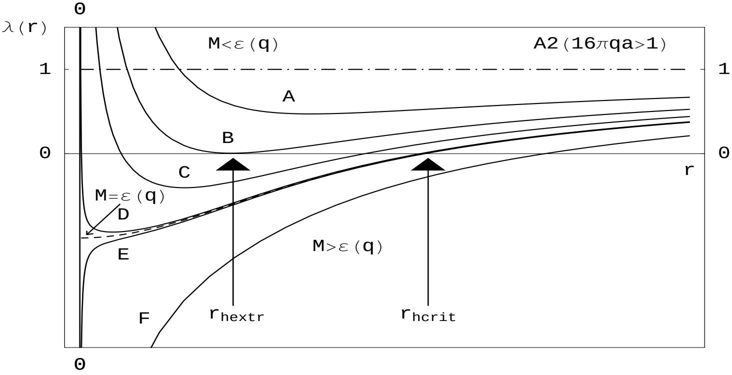
Example: A1 $\varphi(X,Y) = \frac{X}{2} + \mu(X^2 + 7/4 Y^2)$, A2: BI

- New configurations, as a consequence of the finiteness of $\varepsilon(q)$

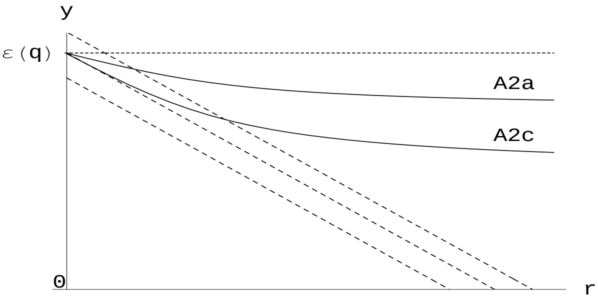


1. $M = M_{extr}(q)$: EBH
2. $M < M_{extr}(q)$: No horizons (NS)
3. $M_{extr}(q) < M < \varepsilon(q)$: BH with two horizons
4. $M > \varepsilon(q)$: Single-horizon BH
5. $M = \varepsilon(q)$ (critical): Need of analyzing the central metric behaviour

- In case A1, behaviour of the metric at the center: $\lambda(r) \rightarrow 1 - 2\frac{(M-\varepsilon(q))}{r} - O(1/r^p)$: Single-horizon BH for the critical configuration
- In case A2 ($16\pi qa > 1$): $\lambda(r) \rightarrow 1 - 16\pi qa - 2\frac{(M-\varepsilon(q))}{r}$. In the critical configuration: $\lambda(r) \rightarrow 1 - 16\pi qa < 0$ at the center



Family **A2** ($16\pi qa \leq 1$) around $r \sim 0+$ **B**-field at $r \to \infty$



1. $M < \varepsilon(q)$: No horizons (NS)
2. $M > \varepsilon(q)$: Single-horizon BH. When $2(M - \varepsilon(q)) \to 0^+$, $r_h \to 0$
3. $M = \varepsilon(q)$ (critical): $\lambda(0) = 1 - 16\pi qa \geq 0 \to$ NS or BH with a vanishing radius horizon

(A) Asymptotically non Schwarzschild-like solutions

- Combination of **IRD**-family at $r \to \infty$ with **A1** or **A2** as $r \sim 0$ (Example: $\varphi(X, Y=0) = X^\gamma$ with $3/2 < \gamma < \infty$ (**A1**-family around the center)
- Metric:

$$\lambda(r) = 1 + \frac{C}{r} + \frac{2\varepsilon_{in}(r,q)}{r}$$

C: integration constant, no longer related to the ADM mass, and

$$\varepsilon_{in}(r,q) = 4\pi \int_0^r R^2 T_0^0(R,q) dR$$

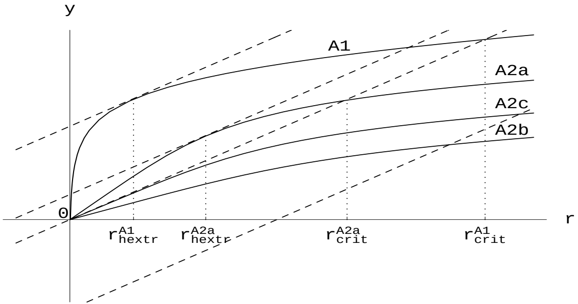
interior integral of energy, a **monotonically increasing and convex function of r**

- Horizons: Zeros of $\lambda(r)$, implying

$$\frac{C+r}{2} = \varepsilon_{in}(r,q)$$

$\rightarrow$ given by the cutting points between $\varepsilon_{in}(r,q)$ and the beam of straight lines $\frac{C+r}{2}$

- Proceeding in the same way as before...



1. Family of EBH with associated $C_{hextr}(q)$ (**A1** and **A2(**$16\pi qa > 1$**)**)
2. $C > C_{hextr}$: NS (**all cases**)
3. $0 < C < C_{hextr}$: Two-horizons BH (**A1** and **A2(**$16\pi qa > 1$**)**)
4. $C < 0$ Single-horizon BH (**A1** and all **A2** cases)
5. $C = 0$ (critical): Single-horizon BH (**A1** and **A2(**$16\pi qa \geq 1$**)**) or NS (**A2**($16\pi qa < 1$)

Notes on thermodynamics

- First BH thermo law holds (**B**-field at $r \to \infty$) for NED models (*Rasheed 97*)

$$dM = TdS + \Phi dq$$

  where $\Phi = 8\pi A_0(r_h)$
- but Smarr formula must be replaced by a generalized version

$$M = 2TS + 2\Phi q - 2\varepsilon_{ex}(r_h, q)$$

- Temperature $T = \frac{1}{4\pi}\frac{d\lambda(r)}{dr}|_{r=r_h}$ splits as $r \sim 0$ in several subcases:
  $T \to +\infty$ (**A2** $\mathbf{(}16\pi qa < 1\mathbf{)}$)
  $T \to -\infty$ (**A1**, **UVD** and **A2** $\mathbf{(}16\pi qa > 1\mathbf{)}$): similar behaviour for all r as the Reissner-Nordström one.
- The case **A2** $\mathbf{(}16\pi qa = 1\mathbf{)}$ is very special

Extension to non-abelian fields

- Taking the two standard first-order field invariants $X = -\frac{1}{2}F^a_{\mu\nu}F^{\mu\nu a}$, $Y = -\frac{1}{2}F^a_{\mu\nu}F^{*\mu\nu a}$, $a = 1\cdots N$.
- Configurations $A^a_0 \neq 0, A^a_i = 0, \forall a$ lead to N first-integrals $(X = \sum_{a=1}^{n}(E^a)^2)$

$$r^a\varphi_X E^a = q^a$$

- Combination of the first-integrals leads to the definition of $Q = \sqrt{\sum_{a=1}^{N}(q^a)^2}$ "mean-square" charge $\longrightarrow \vec{E}^a = \frac{q^a}{Q}\vec{E}(r)$
- With the identification abelian $\leftrightarrow$ mean-square non-abelian charge $(q \leftrightarrow Q)$ the metric is integrated as in the abelian case $\longrightarrow$ characterization of the BH configurations reduces also to the abelian case

Conclusions and perspectives

- Families of admissible NED models classified according to the ESS field behaviour at $r \to \infty$ and as $r \sim 0 \to$ energy finite or divergent.
  - UVD: Similar behaviour as the RN solution
  - Finite-energy: New structures arise: single-horizon black holes, finite-metrics everywhere, vanishing-horizon radius solutions...
  - *Anomalous* configurations: Solutions approaching asymptotic flatness slower than the Schwarzschild field
- Work in progress
  - Thermodynamics
  - Stability: Generalization of a flat-space (linear stability) criterion $\varphi_X - 2X\varphi_{YY} \geq 0$ to the gravitational context
  - Extension of these procedures to higher-order curvature gravity theories and AdS spaces