Cosmological solutions in F(R) Hořava-Lifshitz gravity

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E. Elizalde, S. Nojiri, S.D. Odintsov and DSG, arxiv:1006.3387

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- The theory becomes power counting renormalizable in 3+1 spacetime dimensions for $z = 3$. In such a case, $[G] = 0$ while in GR, $[G] = -2$.
- It has been pointed that, in the IR limit, the full diffeomorphisms are recovered, although the mechanism for the transition is not physically clear.

Extension of HL theory to F(R) gravity

- Hořava-Lifshitz action:

$$S = \int d^3x\, dt \sqrt{g^{(3)}}\, N\tilde{R} \quad \text{where}$$

$$\tilde{R} = K_{ij}K^{ij} - \lambda K^2 + 2\mu\nabla_\mu(n^\mu\nabla_\nu n^\nu - n^\nu\nabla_\nu n^\mu) - L^{(3)}(g_{ij}^{(3)})\,, \quad (3)$$

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- As well as in General Relativity, a natural generalization of the Hilbert-Einstein action is given by the so-called $f(R)$ gravity,

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In the case of HL gravity, we could extend the above action to,

$$S_{HL} = \int dt\, d^3x\sqrt{g^{(3)}}\, N\tilde{R} \quad \longrightarrow S = \frac{1}{2\kappa^2}\int dtd^3x\sqrt{g^{(3)}}NF(\tilde{R}) \quad (5)$$

FLRW equations

Let us assume a flat FLRW spacetime, whose metric can be written as,

$$ds^2 = -N^2 dt^2 + a^2(t) \sum_{i=1}^{3} \left(dx^i\right)^2 \ . \tag{6}$$

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Friedmann equations in Hořava-Lifshitz gravity are given by,

$$H^2 = \frac{\kappa^2}{3(3\lambda - 1)} \rho_m \, , \quad \dot{H} = -\frac{\kappa^2}{2(3\lambda - 1)} (\rho_m + p_m) \, , \tag{7}$$

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while in its extended version, it yields,

$$0 = F(\tilde{R}) - 2(1 - 3\lambda + 3\mu) \left(\dot{H} + 3H^2\right) F'(\tilde{R}) - 2(1 - 3\lambda) \dot{\tilde{R}} F''(\tilde{R}) + 2\mu \left(\dot{\tilde{R}}^2 F^{(3)}(\tilde{R}) + \ddot{\tilde{R}} F''(\tilde{R})\right) + \kappa^2 p_m \, , \tag{8}$$

$$0 = F(\tilde{R}) - 6 \left[(1 - 3\lambda + 3\mu) H^2 + \mu \dot{H}\right] F'(\tilde{R}) + 6\mu H \dot{\tilde{R}} F''(\tilde{R}) - \kappa^2 \rho_m - \frac{C}{a^3} \, , \tag{9}$$

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where $\eta = ln\frac{a}{a_0}$. (Standard $F(R)$ gravity: E.Elizalde, P.Dunsby, R.Goswani, S.Odintsov and DSG '10)

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Assuming the presence of a pressureless fluid ρ_m ($w_m = 0$). First Friedmann equation yields,

$$\begin{aligned} 0 \;=\; & (1-3\lambda+3\mu)F(\tilde{R}) - 2\left(1-3\lambda+\frac{3}{2}\mu\right)\tilde{R} + 9\mu(1-3\lambda)H_0^2\frac{dF(\tilde{R})}{d\tilde{R}} \\ & -6\mu(\tilde{R}-9\mu H_0^2)(\tilde{R}-3H_0^2(1-3\lambda+6\mu))\frac{d^2F(\tilde{R})}{d^2\tilde{R}} \end{aligned}$$

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Assuming the presence of a pressureless fluid ρ_m ($w_m = 0$). First Friedmann equation yields,

$$0 = x(1-x)\frac{d^2F}{dx^2} + (\gamma - (\alpha+\beta+1)\,x)\,\frac{dF}{dx} - \alpha\beta F - k_1 x - k_2, \tag{13}$$

where $x = \frac{\tilde{R} - 9\mu H_0^2}{3H_0^2(1+3(\mu-\lambda))}$

Late-time acceleration:ΛCDM model

Solution,

$$F(\tilde{R}) = C_1 F(\alpha,\beta,\gamma;x) + C_2 x^{1-\gamma} F(\alpha-\gamma+1,\beta-\gamma+1,2-\gamma;x) + \frac{1}{\kappa_1}\tilde{R} - 2\Lambda\,. \tag{14}$$

with,

$$\gamma = -\frac{1}{2}\,, \quad \alpha+\beta = \frac{1-3\lambda-\frac{3}{2}\mu}{3\mu}\,, \quad \alpha\beta = -\frac{1+3(\mu-\lambda)}{6\mu}\,,$$

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Nevertheless, if we impose $\mu = \lambda - \frac{1}{3}$, the solution yields,

$$F(\tilde{R}) = \frac{1}{\kappa_1}\tilde{R} - 2\Lambda\,,\quad \text{with}\quad \Lambda = \frac{3}{2}(3\lambda-1)H_0^2\,. \tag{16}$$

Late-time acceleration: Phantom dark energy

Hubble parameter in a phantom phase:

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Action in HL $F(\tilde{R})$ gravity,

$$F(R) = C_1 R^{m_+} + C_2 R^{m_-}\,, \quad \text{where} \quad m_\pm = \frac{1 - k_1 \pm \sqrt{(k_1 - 1)^2 - 4k_0}}{2}\,. \tag{18}$$

Unifying inflation and late-time acceleration

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$$F(\tilde{R}) = \tilde{R} + f(\tilde{R}), \tag{19}$$

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We consider,

$$f(\tilde{R}) = \frac{\tilde{R}^n(\alpha\tilde{R}^n - \beta)}{1 + \gamma\tilde{R}^n}\,, \tag{20}$$

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- Inflation: It is assumed that the curvature goes to infinity $\rightarrow \lim_{\tilde{R}\rightarrow\infty} F(\tilde{R}) = \alpha\tilde{R}^n$. It is found,

$$H(t) = \frac{h_1}{t}, \quad \text{where} \quad h_1 = \frac{2\mu(n-1)(2n-1)}{1 - 3\lambda + 6\mu - 2n(1 - 3\lambda + 3\mu)}. \tag{21}$$

Inflation occurs for $h_1 > 1$.

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Then, the FLRW equations reduce to

$$H^2 = \frac{\kappa^2}{3(3\lambda - 1)}\rho_m + \frac{2\Lambda}{3(3\lambda - 1)} \quad \dot{H} = -\kappa^2 \frac{\rho_m + p_m}{3\lambda - 1} \, , \tag{23}$$

which look very similar to the standard FLRW equations in GR, except for the parameter λ.
As it has been pointed out, at the current epoch the scalar $\tilde{R}$ is small, so the theory is in the IR limit, where the parameter $\lambda \sim 1$, and the equations approach the standard ones.
Then, inflation and dark energy epochs are unified under the same mechanism, due to extra terms in the gravitational action.

Newton law corrections in $F(\tilde{R})$ gravity

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$\phi = \frac{2}{3} \ln(1 + f'(A))$,

$$S = \int dt\, d^3x \sqrt{g^{(3)}}\, N \left[\tilde{K}_{ij}\tilde{K}^{ij} - \lambda \tilde{K}^2 + \left(-\frac{1}{2} + \frac{3}{2}\lambda - \frac{3}{2}\mu \right) \dot{\tilde{g}}^{ij(3)} \tilde{g}_{ij}^{(3)} \dot{\phi} \right.$$
$$\left. + \left(\frac{3}{4} - \frac{9}{4}\lambda + \frac{9}{2}\mu \right) \dot{\phi}^2 - V(\phi) + \tilde{L}(\tilde{g}^{(3)}, \phi) \right] , \tag{25}$$

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Comparing with standard $F(R)$ gravity, there is a new coupling between the scalar field ϕ and the spatial metric $\tilde{g}^{(3)ij}$, which can be dropped if the parameters are chosen to be,

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By the Chameleon mechanism, we know that the corrections on the Newton law can be restricted if the mass of the scalar field is large enough compared with the curvature,

$$m_\phi^2 = \frac{1}{2}\frac{d^2 V(\phi)}{d\phi^2} = \frac{1+f'(A)}{f''(A)} - \frac{A+f(A)}{1+f'(A)} \,. \tag{27}$$

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On the Earth $\tilde{R} \sim 10^{-50}\mathrm{eV}^2$. And for our model: $m_\phi^2 \sim \frac{\gamma \tilde{R}^{2-n}}{n(n-1)\alpha} \sim 10^{50n-100}\mathrm{eV}^2$,. Then, the Newton law corrections coming from the scalar mode of $f(\tilde{R})$ can be avoided for $n > 2$.

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- In the Einstein frame, it appears a new coupling term, absence in standard gravity, which can be dropped by fixing the parameters.
- Newtonian law corrections coming from the scalar mode of $F(\tilde{R})$ are negligible for the so-called viable standard models.