Astrophysical Black Holes as Particle Colliders

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Based on Phys. Rev. Lett. 104, 021101 (2010) [arXiv:0911.3363 [gr-qc]]

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The problem

- A rotating black hole

  Mass M

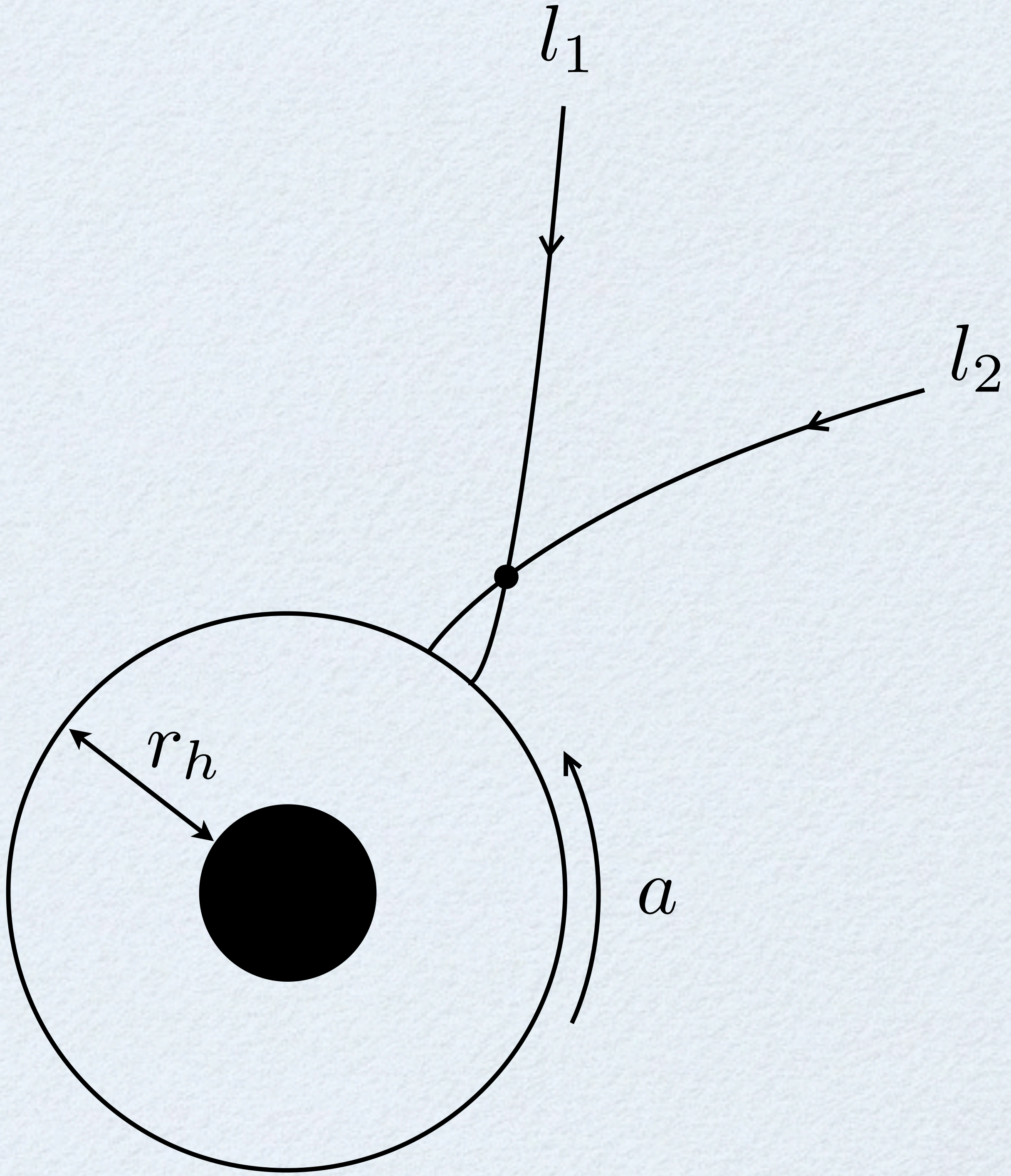
  Angular momentum J

  $a = J/M$

- Two particles falling freely from rest at infinity

  Mass m

  Angular momenta l_1, l_2



How much can the center of mass energy be?

Subtleties and Motivation

We know that:

- Particles are infinitely blue shifted near the horizon
- Penrose process

However:

- These do not really reveal what will happen to the center of mass energy for freely falling, colliding particles
- No Penrose process has to be involved (though it could)
- There have been claims that center of mass (cm) energy can get infinite
- LHC wouldn't be needed!!

Center of mass energy

Assume that you are in the local Lorentz frame

- Special relativity in recovered
- the center of mass energy is well known in this case

$$E_{\rm cm}^2 = (mu_1 + mu_2)^2 = 2m^2(1 + u_1 \cdot u_2)$$

Simply notice that interpreting the dot product appropriately

$$u_1 \cdot u_2 = \eta_{\mu\nu} u_1^\mu u_2^\nu \to g_{\mu\nu} u_1^\mu u_2^\nu$$

this is a covariant expression and it will hold in any curved background

$$E_{\rm cm}^2 = 2m^2(1 + g_{\mu\nu} u_1^\mu u_2^\nu)$$

Rotating black hole

Assuming again equatorial motion on a Kerr spacetime

$$\left(E_{\rm cm}^{Kerr}\right)^2 = \frac{2\,m^2}{r(r^2-2r+a^2)}\times$$
$$\Big(2a^2(1+r)-2a(l_2+l_1)-l_2l_1(-2+r)+2(-1+r)r^2$$
$$-\sqrt{2(a-l_2)^2-l_2^2r+2r^2}\sqrt{2(a-l_1)^2-l_1^2r+2r^2}\,\Big)$$

Taking the collision to occur at $r=r_h=1+\sqrt{1-a^2}$

and assuming for simplicity that $a=1$

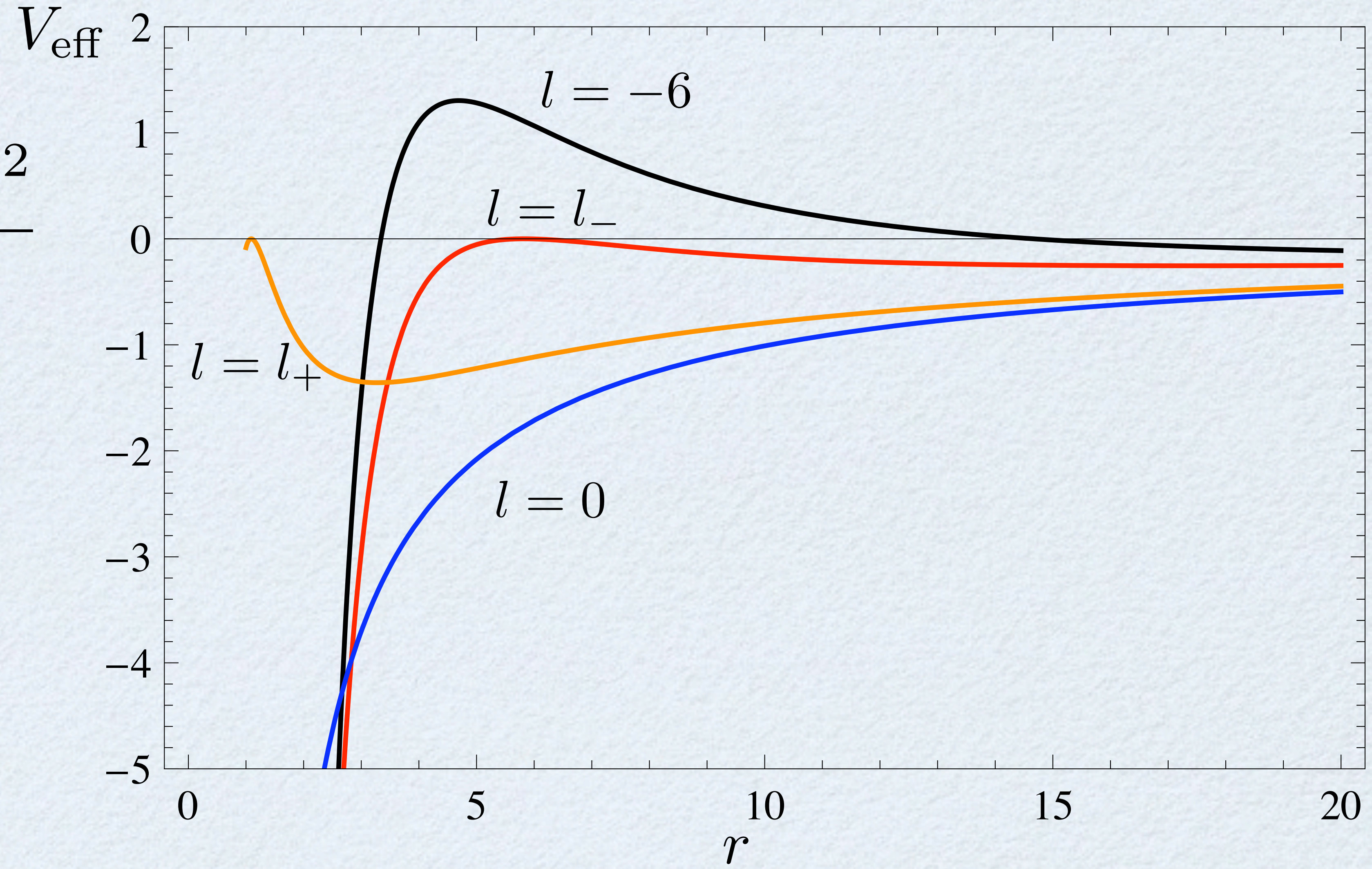
$$E_{\rm cm}^{Kerr}(r\to r_h)=\sqrt{2}m\sqrt{\frac{l_2-2}{l_1-2}+\frac{l_1-2}{l_2-2}}$$

The cm-energy diverges whenever one particle has $l=2$

M. Banados, J. Silk and S. M. West, Phys. Rev. Lett. 103, 111102 (2009)

Trajectories

$$V_{\text{eff}} = -\frac{1}{r} + \frac{l^2}{2r^2} - \frac{(l-a)^2}{r^3}$$



Types of trajectories:

- *no turning point*
- *two turning points*
- *trajectories with an unstable orbit for which*

$$V_{\text{eff}} = dV_{\text{eff}}/dr = 0 \qquad at \qquad r = r_\pm$$

$$l_\pm = \pm 2(1 + \sqrt{1 \mp a}) \qquad r_\pm = 2 \mp a + 2\sqrt{1 \mp a}$$

Extremal black hole

For an extremal black hole we have

$$a = 1 \qquad r_h = 1 \qquad r_+ = 1 \qquad l_+ = 2$$

Near the maximum

$$V_{\text{eff}} = -r_{\pm}^{-3}(r - r_{\pm})^2 + \dots$$

$$\dot{r} = \sqrt{-2V_{\text{eff}}} \qquad \rightarrow \qquad \dot{r} \propto (r - r_{\pm})^2 + \dots$$

- Proper time diverges logarithmically as critical radius is approached
- Critical radius coincides with horizon radius and collision radius

Realistic black holes

- *Accretion processes prohibit any spin factor* $a \leq 0.998$
- *MHD simulations suggest even smaller* $a \leq 0.95$

How will decreasing the spin factor affect the cm-energy?

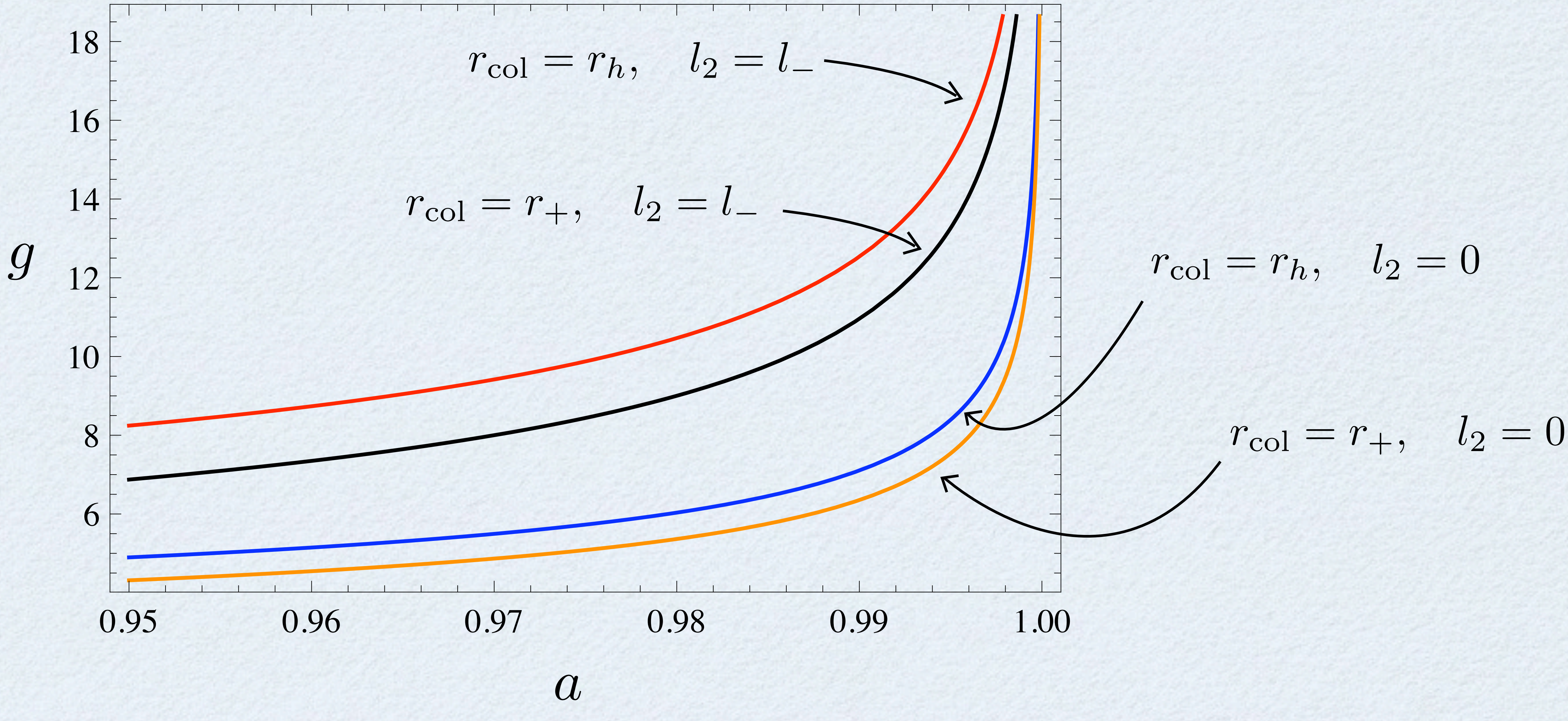
In general we have

$$\frac{E_{\rm cm}^{Kerr}}{m} = f(a, r_{\rm col}, l_1, l_2)$$

We have learned that $l_1 = l_+$ *is crucial for maximizing the energy*

$$\frac{E_{\rm cm}^{\rm max}}{m} = f(a, r_{\rm col}, l_1 = l_+, l_2) = g(a, r_{\rm col}, l_2)$$

Realistic black holes



- *In all case cm-energy grow very rapidly as* $a \to 1$
- *Hard to say more from the graph, however qualitative behaviour is the same for all cases*

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Realistic black holes

A small parameter analysis can help. Define

$$\epsilon = 1 - a$$

Then one can easily get by expanding

$$\frac{E_{\rm cm}^{max}}{m} \sim A(r_{\rm col}, l_2)\, \epsilon^{-1/4} + O(\epsilon^{1/4})$$

Typical values for $A(r_{\rm col}, l_2)$

- $r_{\rm col} = r_h, \quad l_2 = l_-, \quad A = 4.06$
- $r_{\rm col} = r_+, \quad l_2 = l_-, \quad A = 3.70$
- $r_{\rm col} = r_h, \quad l_2 = 0, \quad A = 2.20$
- $r_{\rm col} = r_+, \quad l_2 = 0, \quad A = 2.00$

Realistic black holes

Taking the best case scenario

$$\frac{E_{\rm cm}^{max}}{m} \sim 4.06\,\epsilon^{-1/4} + O(\epsilon^{1/4})$$

and we get for various values of a

- $a = 0.9, \quad \epsilon = 0.1, \quad \frac{E_{\rm cm}^{max}}{m} \sim 6.9$
- $a = 0.99, \quad \epsilon = 0.01, \quad \frac{E_{\rm cm}^{max}}{m} \sim 12.5$
- $a = 0.999, \quad \epsilon = 0.001, \quad \frac{E_{\rm cm}^{max}}{m} \sim 22.6$
- $a = 0.9999, \quad \epsilon = 0.0001, \quad \frac{E_{\rm cm}^{max}}{m} \sim 40.5$

even for unrealistic spins we don't get very high cm-energies

Back reaction effects

E. Berti, V. Cardoso and L. Gualtieri, F. Pretorius and U. Sperhake, Phys. Rev. Lett. 103, 239001 (2009)

Consider that after the collision a pair of particles in actually absorbed from the black hole

- The black hole spin is reduced by $\frac{m}{M}$
- The cm-energy is reduced as it will scales like $(1-a)^{-1/4}$

Also energy will be lost in gravitational wave radiation

$$E_{\rm tot} \sim -\log\left[1-\frac{l}{l_+}\right] \quad when \quad l \sim l_+$$

This invalidates the test particle approach

Conclusions

Using black holes as particle colliders is a fascinating idea!

Unfortunately several reasons prohibit it:

- *Astrophysical black holes are not exactly extremal*
- *Even for extremal black holes only one special trajectory allows infinite energy collisions*
- *It would still take infinite time*
- *Back reaction effects would invalidate the approach*

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Non-rotating black hole

Assuming equatorial motion on a Schwarzschild spacetime

$$\dot{r}^2/2 + V_{\rm eff}(r,l) = 0$$

$$V_{\rm eff}(r,l) = -\frac{1}{r} + \frac{l^2}{2r^2} - \frac{l^2}{r^3}$$

For trajectories with no turning points $\qquad |l| < 4$

Even for collisions on the horizon cm-energy stays finite

$$E_{\rm cm}^{Schw}(r\to 2) = \frac{m}{2}\sqrt{(l_2-l_1)^2+16}$$

so at best we can get

$$l_1 = -l_2 = 4 \qquad \to \qquad E_{\rm cm}^{Schw}(r\to 2) = 2\sqrt{5}m$$

A. N. Baushev, arXiv:0805.012 [astro-ph]

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Energy of the ejecta

- In general it is not trivial to calculate the energy of the collision products
- Conjecture: collision at the horizon will give upper bound
- Largest cm-energy for such collisions anyway

Advantage: Simple geometrical arguments give the answer

- For one of the collision products to escape its 4-momentum should be at best tangent to the horizon generator
- The 4-momentum of one of the particles will be tangent to the horizon generator as well

Energy of the ejecta

We then have

- colliding particles 4-momenta: $k,\, p$
- ejecta particles 4-momenta: $\lambda k,\, p'$

4-momentum conservation implies

$$p + k = p' + \lambda k \qquad \rightarrow \qquad p' = p + (1-\lambda)k$$

All momenta are future pointing so

$$p' \cdot p > 0 \qquad k \cdot p > 0$$

which can be used to show that

$$\lambda - 1 < \frac{p \cdot p}{k \cdot p} = \frac{m^2}{(E_{\rm cm}^2/2 - m^2)}$$

Energy of the ejecta

However, we know that $E_{\rm cm} > 2m$

$$E_{\rm cm} \to 2m \quad \to \quad \lambda \to 2$$
$$E_{\rm cm} \to \infty \quad \to \quad \lambda \to 1$$

which can be turned into the bound

$$1 < \lambda < 2$$

- Thus, the ejecta particle's Killing energy can be at most $2m$
- The result does not seem to allow the Penrose process!
- Caveat: conjecture about collision on the horizon!
- Could it be different if the collision takes place outside?

A. A. Grib and YU. V. Pavlov, arXiv:1001.0756 [gr-qc]

A curious observation...

K. Lake, arXiv:1001.5463 [gr-qc]

What is instead the collision takes place on the inner horizon?

$$r_{\rm inner} = 1 - \sqrt{1-a^2}$$

It can be shown that particles with angular momenta

$$2(1-\sqrt{1-a}) < \quad l \quad < 2(1+\sqrt{1-a})$$
$$-2(1+\sqrt{1+a}) < \quad l \quad < -2(1-\sqrt{1+a})$$

reach the inner horizon for $0 < a < 1$

The center of mass energy appears to diverge there

$$E_{\rm cm}^{Kerr}(r \to r_{\rm inner}) \to \infty$$

Planck-scale physics before reaching the Planck length?