Principles of Einstein–Finsler Gravity and Perspectives in Modern Cosmology

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Outline

Goals

- Finsler modifications of GR derived for QG theories; Geometric models for quantum contributions and LV
- Canonical models for Einstein–Finsler gravity (EFG); principles and axioms
- Physical implications of EFG: Finsler branes, locally anisotropic cosmology & astrophysics

Reviews and new results:
S. Vacaru (in CQG, PLB, IJGMMP, JMP, JGP, IJTP)
arXiv: 1008.4912; 1004.3007; 1003.0044;
0909.3949; 0907.4278
http://www.scribd.com/people/view/1455460-sergiu

Motivation: nonlinear disps; QG & LV, cosmology

1. Deforms in Minkovski s-t: $E^2 = p^2c^2 + m_0^2c^4 + \varphi(E, p; \mu; M_P)$
$E \sim \frac{\partial}{\partial t}, p_{\widehat{i}} \sim \frac{\partial}{\partial x^{\widehat{i}}}, \omega = \frac{\partial \phi}{\partial t}\ k_{\widehat{i}} = \frac{\partial \phi}{\partial x^{\widehat{i}}}, \omega^2 = c_s^2 k^2 + c_s^2 (\frac{\hbar}{2m_0 c_s})^2 k^4 + ...$
effective c_s, $(x^1 = ct, x^2, x^3, x^4); \widehat{i}, \widehat{j}... = 2, 3, 4;$

$$\omega^2 = c^2[g_{\widehat{i}\widehat{j}} k^{\widehat{i}} k^{\widehat{j}}]^2 (1 - q_{\widehat{i}_1 \widehat{i}_2 ... \widehat{i}_{2r}} y^{\widehat{i}_1} ... y^{\widehat{i}_{2r}} / r [g_{\widehat{i}\widehat{j}} k^{\widehat{i}} k^{\widehat{j}}]^{2r})$$

light velocity in "media/ether" $c^2 = g_{\widehat{i}\widehat{j}}(x^i) y^{\widehat{i}} y^{\widehat{j}} / \tau^2 \rightarrow \check{F}^2(y^{\widehat{j}})/\tau^2$
fundamental Finsler function $F(x^i, \beta y^j) = \beta F(x^i, y^j), \beta > 0,$
$ds^2 = F^2 \approx -(cdt)^2 + g_{\widehat{i}\widehat{j}}(x^k) y^{\widehat{i}} y^{\widehat{j}} [1 + \frac{1}{r} \frac{q_{\widehat{i}_1 \widehat{i}_2 ... \widehat{i}_{2r}}(x^k) y^{\widehat{i}_1} ... y^{\widehat{i}_{2r}}}{(g_{\widehat{i}\widehat{j}}(x^k) y^{\widehat{i}} y^{\widehat{j}})^r}] + O(q^2)$
Finsler "metrics", velocities on TV, ${}^F g_{ij}(x^i, y^j) = \frac{1}{2} \frac{\partial F^2}{\partial y^i \partial y^j}$

2. Exact solutions & modified cosmology with generic off–diagonal metrics and local anisotropy.

Einstein–Finsler Gravity (EFG)

Statement I: A (pseudo) Finsler metric, ${}^{F}g_{ij}(x^k, y^a)$, DOES NOT define completely a geometric model (not Riemannian !)

Statement II: A model of Finsler geometry is defined on TV by THREE fundamental geometric objects induced by $F(x,y)$:

1. N–connection, $N_i^a(x,y)$, splitting ${}^{F}\mathbf{N} : TTV = hTV \oplus vTV$ canonically, Euler–Lagrange for $L = F^2$ are semi–sprays,
2. d–connection, N–adapted linear connect. ${}^{F}\mathbf{D} = (hD, vD)$, preferred/ canonically induced by ${}^{F}g_{ij}$ and N_i^a
3. d–metric, ${}^{F}\mathbf{g} = h\mathbf{g} \oplus v\mathbf{g}$

2 classes: a) nonmetricity, ${}^{F}\mathbf{Q} := {}^{F}\mathbf{D}\,{}^{F}\mathbf{g}$, Chern d–conn., ${}^{Ch}\mathbf{D}$
b) metricity, ${}^{F}\mathbf{Q} = 0$, Cartan d–conn., ${}^{Cart}\mathbf{D}$

Levi–Civita ${}^{F}\nabla$ is NOT adapted to nonholonomic ${}^{F}\mathbf{N}$.

$\exists$ induced by ${}^{F}\mathbf{g}$: torsion ${}^{F}\mathbf{T}$, and/or ${}^{F}\mathbf{Q}$(not Riemann-Cartan)

Einstein–Finsler spacetimes/gravity, EFG

Spacetime as a nonholonomic manifold/ bundle $\mathbf{V} := (V, \mathcal{D})$ (Vrănceanu, 1926), or TM, with a non–integrable distribution $\mathcal{D}$.

Geometric data: Finsler $(F : \mathbf{N}, \mathbf{D}, \mathbf{g})$ and Riemannian $(\nabla, \mathbf{g})$

N–anholonomic frames: $\mathbf{e}_\nu = (\mathbf{e}_i = \partial_i - N_i^a \partial_a, e_a = \partial_a)$

Sasaki d–metric: ${}^F\mathbf{g} = {}^F g_{ij}(u) dx^i \otimes dx^j + {}^F g_{ab}(u)\ {}^c\mathbf{e}^a \otimes {}^c\mathbf{e}^b$,
for ${}^c\mathbf{e}^a = dy^a + {}^c N_i^a(u)\, dx^i$.
For $\tilde{\mathbf{D}}$, standard Riemannian, Ricci, Einstein d–tensors; h-/v–splitting.

N–adapted coef.: ${}^{Cart}\mathbf{D} = \tilde{\mathbf{D}} = (h\tilde{D}, v\tilde{D}) = \{\tilde{\Gamma}^\alpha_{\gamma\tau} = (\tilde{L}^i_{jk}, \tilde{C}^a_{bc})\}$,

$$\tilde{L}^i_{jk} = \frac{1}{2}\ {}^F g^{ir}(\mathbf{e}_k\ {}^F g_{jr} + \mathbf{e}_j\ {}^F g_{kr} - \mathbf{e}_r\ {}^F g_{jk}),$$
$$\tilde{C}^a_{bc} = \frac{1}{2}\ {}^F g^{ad}(e_c\ {}^F g_{bd} + e_c\ {}^F g_{cd} - e_d\ {}^F g_{bc}).$$

Theorem: Equivalent (pseudo) Finsler & Riemannian theories if ${}^g\mathbf{D} = {}^g\nabla + {}^g\mathbf{Z}$, distortion determined by $\mathbf{g} = {}^F\mathbf{g}$.

Principles and axioms of EFG

Principles: Similarly to GR with ${}^{g}\nabla$ on V construct **EFG:** with $\mathbf{g} \sim {}^{F}\mathbf{g}, \mathbf{N} \sim {}^{F}\mathbf{N}$ and ${}^{Cart}\mathbf{D}$ on TV, or $\mathbf{V}$.

1. Generalized equivalence principle: Ideas on Free Fall and Universality of Gravitational Redshift for ${}^{Cart}\mathbf{D}$.
2. Generalized Mach principle: quantum energy/motion encoded via $(\mathbf{N}, \mathbf{g}, \mathbf{D})$ for spacetime ether with y^a.
3. Principle of general covariance extended on $\mathbf{V}$, or TV, with "mixing of Finsler parametrizations".
4. Motion eqs and conservation laws: Nonholonomc Bianchi identities for ${}^{F}\mathbf{D}$; $\nabla_i T^{ij} = 0 \rightarrow \mathbf{D}_\alpha \Upsilon^{\alpha\beta} \neq 0$.
5. Einstein–Finsler gravitational field eqs for ${}^{F}\mathbf{D}$.
6. **Axiomatics:** Constructive–axiomatic appr. (Ehlers-Pirani –Schild, EPS axioms), paradigm "Lorentzian 4–manifold" in GR; nonholon. tangent bunle on "L ..." for EFG.

Gravitational field eqs in EFG

$\forall$ **D**, Einstein eqs: $\mathbf{E}_{\alpha\beta} = \Upsilon_{\alpha\beta}$,
h–/v–components, for $R_{ai} = R^{b}{}_{aib}$ and $R_{ia} = R^{k}{}_{ikb}$:

$$\begin{aligned} R_{ij} - \frac{1}{2}(R+S)g_{ij} &= \Upsilon_{ij}, \\ R_{ab} - \frac{1}{2}(R+S)h_{ab} &= \Upsilon_{ab}, \\ R_{ai} = \Upsilon_{ai},\ R_{ia} &= -\Upsilon_{ia}, \end{aligned}$$

Remark: For ${}^{Cart}\mathbf{D}$, general off–diagonal solutions for EFG, restrictions to GR, $\mathbf{g} = \underline{g}_{\alpha\beta}(u)\, du^{\alpha} \otimes du^{\beta}$,

$$\underline{g}_{\alpha\beta} = \begin{bmatrix} g_{ij} + N_i^a N_j^b h_{ab} & N_j^e h_{ae} \\ N_i^e\ h_{be} & h_{ab} \end{bmatrix}, \text{where } N_i^a \neq A^a_{bi}(x) y^b$$

Claim: Compactification/trapping/warping mechanism on velocity/momenta for a "new" QG and LV phenomenology.

Finsler–branes & cosmological solutions

Nonholon. trapping solutions (cosmology, with $h_3(x^i, y^3 = t)$) :

$$\mathbf{g} = g_1 dx^1 \otimes dx^1 + g_2 dx^2 \otimes dx^2 + h_3 \mathbf{e}^3 \otimes \mathbf{e}^3 + h_4 \mathbf{e}^4 \otimes \mathbf{e}^4 +$$

$$(l_P)^2 \frac{\overline{h}}{\phi^2} [\ {}^q h_5 \mathbf{e}^5 \otimes \ \mathbf{e}^5 + \ {}^q h_6 \mathbf{e}^6 \otimes \ \mathbf{e}^6 + \ {}^q h_7 \mathbf{e}^7 \otimes \ \mathbf{e}^7 + \ {}^q h_8 \mathbf{e}^8 \otimes \mathbf{e}^8]$$

$$\mathbf{e}^3 = dy^3 + w_i dx^i, \mathbf{e}^4 = dy^4 + n_i dx^i,\ \mathbf{e}^5 = dy^5 + \ {}^1 w_i dx^i,$$

$$\mathbf{e}^6 = dy^6 + \ {}^1 n_i dx^i,\ \mathbf{e}^7 = dy^7 + \ {}^2 w_i dx^i,\ \mathbf{e}^8 = dy^8 + \ {}^2 n_i dx^i.$$

$$\phi^2(y^5) = \frac{3\epsilon^2 + a(y^5)^2}{3\epsilon^2 + (y^5)^2} \text{ and } l_P \sqrt{|\overline{h}(y^5)|} = \frac{9\epsilon^4}{\left[3\epsilon^2 + (y^5)^2\right]^2},$$

N–connection coefficients determined by sources

$$\begin{aligned} {}^h\Lambda(x^i) &= \widetilde{\Upsilon}_4 + \widetilde{\Upsilon}_6 + \widetilde{\Upsilon}_8,\ {}^v\Lambda(x^i, v) = \widetilde{\Upsilon}_2 + \widetilde{\Upsilon}_6 + \widetilde{\Upsilon}_8, \\ {}^5\Lambda(x^i, y^5) &= \widetilde{\Upsilon}_2 + \widetilde{\Upsilon}_4 + \widetilde{\Upsilon}_8,\ \ {}^7\Lambda(x^i, y^5, y^7) = \widetilde{\Upsilon}_2 + \widetilde{\Upsilon}_4 + \widetilde{\Upsilon}_6. \end{aligned}$$

Conclusions

- Almost all models of QG with nonlinear dispersions can be geometrized as certain Finsler spacetimes.
- Natural/ Canonical Principles for metric compatible EFG generalizing the GR on $TV, \nabla \rightarrow {}^{Cart}\mathbf{D}$.
- Finsler branes, trapping: "new" QG/ LV phenomenology.
- **Outlook** (recently developed, under elaboration):
  - EFG is almost completely integrable, can be quantized as almost Kähler–Fedosov/ A–brane geometries, and renormalizable for bi–connection/gauge gravity models.
  - Finsler for black holes (ellipsoids, toruses, holes, wormholes, solitons); anisotropic cosmological models (off–diagonal inflation, dark energy/matter etc).
  - Noncommutative/ Ricci–Finsler flows, emergent (non) commutative Lagrange–Finsler analogous gravity and quantization, Clifford–Finsler algebroids etc.