Victoria University of Wellington

Te Whare Wānanga o te Ūpoko o te Ika a Maui

Hořava gravity

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Abstract:

- Hořava gravity is a recent (Jan 2009) idea in theoretical physics for trying to develop a quantum field theory of gravity.
- It is not a string theory, nor loop quantum gravity, but is instead a quantum field theory that breaks Lorentz invariance at ultra-high (trans-Planckian) energies, while retaining Lorentz invariance at low and medium energies.

Abstract:

- The challenge is to keep the Lorentz symmetry breaking controlled and small — small enough to be compatible with experiment.
- I will give a very general overview of what is going on in this field, paying particular attention to the disturbing role of the scalar graviton.

Key issues:

- Is Lorentz symmetry truly fundamental?
- Or is it just an "accidental" low-momentum emergent symmetry?
- Opinions on this issue have undergone a radical mutation over the last few years.
- Historically, Lorentz symmetry was considered absolutely fundamental — not to be trifled with — but for a number of independent reasons the modern viewpoint is more nuanced.

Key questions:

- What are the benefits of Lorentz symmetry breaking?
- What can we do with it?
- Why should we care?
- Where are the bodies buried?

Hořava gravity:

- Quantum Gravity at a Lifshitz Point.
  Petr Hořava. Jan 2009.
  Phys. Rev. **D79** (2009) 084008. arXiv: 0901.3775 [hep-th]
  (cited 343 times; 187 published; as of 4 Sept 2010).

- Membranes at Quantum Criticality.
  Petr Hořava. Dec 2008.
  JHEP **0903** (2009) 020. arXiv: 0812.4287 [hep-th]
  (cited 203 times; 115 published; as of 4 Sept 2010).

- Spectral Dimension of the Universe in Quantum Gravity at a Lifshitz Point.
  Petr Hořava. Feb 2009.
  Phys. Rev. Lett. **102** (2009) 161301. arXiv: 0902.3657 [hep-th]
  (cited 157 times; 86 published; as of 4 Sept 2010).

As of 4 Sept 2010 the Spires bibliographic reports that (apart from Hořava's own articles) this topic has generated:

- 14 published papers with 100 or more citations.
- 38 published papers with 50–99 citations.
- 35 published papers with 25–49 citations.
- 46 published papers with 10–24 citations.

In addition Spires reports:

- 1 as yet unpublished paper with 100 or more citations
- 3 as yet unpublished papers with 50–99 or more citations.
- 14 as yet unpublished papers with 25–49 citations.
- 28 as yet unpublished papers with 10–24 citations.

However you look at it, this topic is "highly active".

Warning:

- There has been somewhat of a tendency to charge full steam ahead with applications, (typically cosmology), without first fully understanding the foundations of the model.
- For that matter, there is still considerable disagreement as to what precise version of the model is "best".
- When reading the literature, a certain amount of caution is advisable...

My interpretation of the central idea:

- Abandon ultra-high-energy Lorentz invariance as fundamental.
- One need "merely" attempt to recover an approximate low-energy Lorentz invariance.
- Typical dispersion relation:

$$\omega = \sqrt{m^2 + k^2 + \frac{k^4}{K^2} + \dots}$$

- Nicely compatible with the "analogue spacetime" programme...

QFT:

- Condensed matter language:

  "Critical" Lifshitz point in $(d+1)$ dimensions

  $\iff$ Dispersion relation satisfies

$$\omega \to k^d \qquad \text{as} \qquad k \to \infty.$$

- To recover Lorentz invariance, at "low" momentum
  (but still allowing $k \gg m$)
  the dispersion relation should satisfy

$$\omega \to \sqrt{m^2 + k^2} \qquad \text{as} \qquad k \to 0.$$

QFT:

- Every QFT regulator known to mankind either breaks Lorentz invariance explicitly (e.g. lattice), or does something worse, something outright unphysical.
- For example:
  - Pauli–Villars violates unitarity;
  - Lorentz-invariant higher-derivatives violate unitarity;
  - dimensional regularization is at best a purely formal trick with no direct physical interpretation...
    (and which requires a Zen approach to gamma matrix algebra).

QFT:

Standard viewpoint:

- If the main goal is efficient computation in a corner of parameter space that we experimentally know to be Lorentz invariant to a high level of precision, then by all means, go ahead and develop a Lorentz-invariant perturbation theory with an unphysical regulator — hopefully the unphysical aspects of the computation can first be isolated, and then banished by renormalization.

- This is exactly what is done, (very efficiently and very effectively), in the "standard model of particle physics".

QFT:

Non-standard viewpoint:

- If however one has reason to suspect that Lorentz invariance might ultimately break down at ultra-high (trans–Planckian?) energies, then a different strategy suggests itself.
- Maybe one could use the Lorentz symmetry breaking as part of the QFT regularization procedure?
- Could we at least keep intermediate parts of the QFT calculation "physical"?
- (Note that "physical" does not necessarily mean "realistic", it just means we are not violating fundamental tenets of quantum physics at intermediate stages of the calculation.)

QFT:

Consider a "physical" but Lorentz-violating regulator:

- Dispersion relation:

$$\omega^2 = m^2 + k^2 + \frac{k^4}{K_4^2} + \frac{k^6}{K_6^4}.$$

We call this a "trans–Bogoliubov" dispersion relation.

- Compare with standard condensed-matter Bogoliubov dispersion relation:

$$\omega^2 = k^2 + \frac{k^4}{K^2}.$$

QFT:

- QFT propagator [momentum-space Green function]:

$$G(\omega, k) = \frac{1}{\omega^2 - \left[m^2 + k^2 + \frac{k^4}{K_4^2} + \frac{k^6}{K_6^4}\right]}.$$

- Note rapid fall-off as spatial momentum $k \to \infty$.
- This improves the behaviour of the integrals encountered in Feynman diagram calculations (QFT perturbation theory).
- In any (3+1) dimensional scalar QFT, with arbitrary polynomial self-interaction, this is enough (after normal ordering), to keep all Feynman diagrams finite.

QFT:

For details see:

- Lorentz symmetry breaking as a quantum field theory regulator.
  Matt Visser. Feb 2009.
  Phys. Rev. **D80** (2009) 025011.
  arXiv: 0902.0590 [hep-th]

- Renormalization of Lorentz violating theories.
  Damiano Anselmi, Milenko Halat, Jul 2007.
  Phys. Rev. **D76** (2007) 125011.
  arXiv: 0707.2480 [hep-th]

- Weighted scale invariant quantum field theories.
  Damiano Anselmi, Jan 2008.
  JHEP **0802** (2008) 051.
  arXiv: 0801.1216 [hep-th]

QFT:

Language borrowed from condensed matter:

Lifshitz point of order z in $(d+1)$ dimensions:

- Dispersion relation:

$$\omega^2 = m^2 + k^2 + \sum_{n=2}^{z} g_n \, \frac{k^{2n}}{K^{2n-2}}$$

- Equivalent QFT propagator [Green function]:

$$G(\omega, k) = \frac{1}{\omega^2 - \left[m^2 + k^2 + \sum_{n=2}^{z} g_n \, \frac{k^{2n}}{K^{2n-2}} \right]}$$

QFT:

Key results:

- In a $(d+1)$ dimensional scalar QFT with $z = d$, and arbitrary polynomial self-interaction, this is enough (after normal ordering) to keep all Feynman diagrams finite.
- Gravity is a little trickier, but you can at least argue for power-counting renormalizability of the resulting QFT.
- This is unexpected, seriously unexpected...
- And yes there are still significant technical difficulties... (of which more anon)...

Implications for quantum gravity:

Standard ADM decomposition:

$$\int \sqrt{-g_3}\; N \;\left\{ \mathrm{tr}[K^2] - \mathrm{tr}[K]^2 + {}^{(3)}R \right\}\; \mathrm{d}^3x\, \mathrm{d}t$$

Split spacetime into space+time.

- ${}^{(3)}R$ is intrinsic curvature of space.
- K is extrinsic curvature of space in spacetime.

Extremely useful technique:

- Leads to "canonically quantized gravity" (with all its problems).
- Classically very useful for numerical relativity.

Implications for quantum gravity:

Develop a non-standard extension of ADM:

- Choose a "preferred foliation".
- Decompose

$$\mathcal{L} = (\text{kinetic term}) - (\text{potential term})$$

- Add extra "kinetic" and "potential" terms, beyond what you expect from Einstein–Hilbert.
- Cannot now reassemble into a simple ${}^{(3+1)}R$.
- (Implicit return of the aether...)

Kinetic term:

- Consider the quantity

$$\mathcal{T}(K) = g_K \left\{ (K^{ij} K_{ij} - K^2) + \xi K^2 \right\}.$$

- (Standard general relativity would enforce $\xi \to 0$.)
- Take the kinetic action to be

$$S_K = \int \mathcal{T}(K) \, \mathrm{d}V_{d+1} = \int \mathcal{T}(K) \sqrt{g} \, N \, \mathrm{d}^d x \, \mathrm{d}t.$$

- Only two time derivatives (hiding in K) — this is good.
- Hidden "scalar graviton" when $\xi \neq 0$ — this is bad.

Potential term:

- Now consider the most general "potential term" in $(d+1)$ dimensions:

$$S_{\mathcal{V}} = \int \mathcal{V}(g)\,\sqrt{g}\,N\,\mathrm{d}^d x\,\mathrm{d}t,$$

  where $\mathcal{V}(g)$ is some scalar built out of the spatial metric and its derivatives.

- But then $\mathcal{V}(g)$ must be built out of scalar invariants — calculable in terms of the Riemann tensor and its derivatives.

- This tells us it must be constructible from objects of the form

$$\left\{(\text{Riemann})^d, [(\nabla\text{Riemann})]^2(\text{Riemann})^{d-3}, \text{etc...}\right\}.$$

Potential term:

- In general, in $d+1$ dimensions this is a long but finite list.
- All of these theories should be well-behaved as QFTs.
- All of these theories should have (in condensed matter jargon) "$z = d$ Lifshitz points".
- In the specific case $d = 3$ we have the short and rather specific list:

$$\Big\{ (\text{Riemann})^3, [\nabla(\text{Riemann})]^2, \\ (\text{Riemann})\nabla^2(\text{Riemann}), \nabla^4(\text{Riemann}) \Big\}.$$

Potential term:

- But in 3 dimensions the Weyl tensor automatically vanishes, so we can always decompose the Riemann tensor into the Ricci tensor, Ricci scalar, plus the metric.

- Thus we need only consider the much simplified list:

$$\left\{ (\text{Ricci})^3, [\nabla(\text{Ricci})]^2, (\text{Ricci})\nabla^2(\text{Ricci}), \nabla^4(\text{Ricci}) \right\}.$$

- Once you look at all the different ways the indices can be wired up this is still relatively messy.

Hořava gravity:

- It is roughly at this stage that Hořava makes his two great simplifications:
  - "projectability";
  - "detailed balance".

- Even after almost 2 years — it is still somewhat unclear whether these are just "simplifying ansatze" or whether they are fundamental to Hořava's model.

- In particular, Silke Weinfurtner, Thomas Sotiriou, and I have argued that "detailed balance" is not fundamental, and we have been carefully thinking about the issue of "projectability".

Eliminating detailed balance:

- Phenomenologically viable Lorentz-violating quantum gravity.
  Thomas Sotiriou, Matt Visser, Silke Weinfurtner. Apr 2009.
  Phys. Rev. Lett. **102** (2009) 251601.
  arXiv: 0904.4464 [hep-th]

- Quantum gravity without Lorentz invariance.
  Thomas Sotiriou, Matt Visser, Silke Weinfurtner. May 2009.
  JHEP **0910** (2009) 033.
  arXiv: 0905.2798 [hep-th]

Projectability:

What is Hořava's projectability condition?

$$N(x,t) \to N(t) \quad (\to 1)$$

This is the assertion that the lapse is always trivial (or trivializable).

- In standard general relativity the "projectability condition" can always be enforced locally as a gauge choice;
- Furthermore for "physically interesting" solutions of general relativity it seems that this can always be done (more or less) globally.

Projectability:

For instance:

- For the Schwarzschild spacetime this "projectability condition" holds globally in Painlevé–Gullstrand coordinates;
- For the Reissner–Nordström spacetime this "projectability condition" holds for $r \geq Q^2/2m$ in Painlevé–Gullstrand coordinates, (everything OK up to some point deep inside the inner horizon);
- For the Kerr spacetime this condition holds globally (for the physically interesting $r > 0$ region) in Doran coordinates;
- The FLRW cosmologies also automatically satisfy this "projectability condition".

For this purely pragmatic reason we decided to put "projectability" off to one side for a while, and first deal with "detailed balance".

What is Hořava's detailed balance condition?

$\mathcal{V}(g)$ is a perfect square.

That is, there is a "pre-potential" $W(g)$ such that:

$$\mathcal{V}(g) = \left(g^{ij} \, \frac{\delta W}{\delta g_{jk}} \, g^{kl} \, \frac{\delta W}{\delta g_{li}} \right).$$

This simplifies some steps of Hořava's algebra, it makes other features much worse.

Detailed balance:

In particular, if you assume Hořava's detailed balance, and try to recover Einstein-Hilbert in the low energy regime, then:

- You are forced to accept a non-zero cosmological constant of the "wrong sign".
- You are forced to accept intrinsic parity violation in the purely gravitational sector.

(The second I could live with, the first will require some mutilation of detailed balance, so we might as well go the whole way and discard detailed balance entirely.)

Discarding detailed balance:

There are only five independent terms of appropriate dimension:

$$R^3, \quad R R^i{}_j R^j{}_i, \quad R^i{}_j R^j{}_k R^k{}_i; \quad R\, \nabla^2 R, \quad \nabla_i R_{jk}\, \nabla^i R^{jk}.$$

These terms are all marginal (renormalizable) by power counting.

Add all possible lower-dimension terms (relevant operators, super-renormalizable by power-counting):

$$1; \qquad R; \qquad R^2; \;\; R^{ij} R_{ij}.$$

This now results in a potential $\mathcal{V}(g)$ with nine terms and nine independent coupling constants.

Physical units:

The Einstein–Hilbert piece of the action is now

$$S_{\mathrm{EH}} = \zeta^2 \int \left\{ (K^{ij} K_{ij} - K^2) + R - g_0\, \zeta^2 \right\} \sqrt{g}\; N\; \mathrm{d}^3 x\; \mathrm{d}t.$$

The "extra" Lorentz-violating terms become:

$$\begin{aligned} S_{\mathrm{LV}} \;=\; & \zeta^2 \int \Big\{ \xi\, K^2 - g_2\, \zeta^{-2}\, R^2 - g_3\, \zeta^{-2}\, R_{ij} R^{ij} \\ & - g_4\, \zeta^{-4}\, R^3 - g_5\, \zeta^{-4}\, R(R_{ij} R^{ij}) \\ & - g_6\, \zeta^{-4}\, R^i{}_j R^j{}_k R^k{}_i - g_7\, \zeta^{-4}\, R \nabla^2 R \\ & - g_8\, \zeta^{-4}\, \nabla_i R_{jk}\, \nabla^i R^{jk} \Big\} \sqrt{g}\; N\; \mathrm{d}^d x\; \mathrm{d}t. \end{aligned}$$

Physical units:

- From the normalization of the Einstein–Hilbert term:

$$(16\pi G_{\rm Newton})^{-1} = \zeta^2; \qquad\qquad \Lambda = \frac{g_0\,\zeta^2}{2};$$

  so that ζ is identified as the Planck scale.
- The cosmological constant is determined by the free parameter g_0, and observationally $g_0 \sim 10^{-123}$ (renormalized after including vacuum energy contributions).
- In particular, the way we have set this up we are free to choose the Newton constant and cosmological constant independently (and so to be compatible with observation).

Physical units:

- The Lorentz violating term in the kinetic energy leads to an extra scalar mode for the graviton, with fractional $O(\xi)$ effects at all momenta.
- Phenomenologically, this behaviour is potentially dangerous and should be carefully investigated.
- The various Lorentz-violating terms in the potential become comparable to the spatial curvature term in the Einstein–Hilbert action for physical momenta of order

$$\zeta_{\{2,3\}} = \frac{\zeta}{\sqrt{|g_{\{2,3\}}|}}; \qquad \zeta_{\{4,5,6,7,8\}} = \frac{\zeta}{\sqrt[4]{|g_{\{4,5,6,7,8\}}|}}.$$

Physical units:

- The Planck scale ζ is divorced from the various Lorentz-breaking scales $\zeta_{\{2,3,4,5,6,7,8\}}$.
- We can drive the Lorentz breaking scale arbitrarily high by suitable adjustment of the dimensionless couplings $g_{\{2,3\}}$ and $g_{\{4,5,6,7,8\}}$.
- Based on his intuition coming from "analogue spacetimes", Grisha Volovik has been asserting for many years that the Lorentz-breaking scale should be much higher than the Planck scale.
- This model naturally implements that idea.

Hořava gravity — Problems and pitfalls:

Where are the bodies buried?

- Projectability: This yields a spatially integrated Hamiltonian constraint rather than a super-Hamiltonian constraint.
- Prior structure: Is the preferred foliation "prior structure"? Or is it dynamical?
- Scalar graviton: As long as $\xi \neq 0$ there is a spin-0 scalar graviton, in addition to the spin-2 tensor graviton.
- Hierarchy problem? (Renormalizability may still require fine tuning.)
- Beta functions? Beyond power-counting? RG flow? (Very limited explicit calculations so far.)

Potential for violent conflicts with empirical reality.

Graviton propagator linearized around flat space:

Conformal point	GR
$\lambda = 1/3$	$\lambda = 1$

$$\ldots\ldots\ldots\ldots\ldots\star\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\star\ldots\ldots\ldots\ldots\ldots$$

$\xi = -2/3$	$\xi = 0$

- The scalar mode has negative kinetic energy all the way from the conformal point to the GR point.
- RG flow through this zone does not seem practicable.
- The scalar mode is elliptic (unstable), at least in the usual projectable version.

Graviton propagator linearized around flat space:

The spin-0 scalar graviton is potentially dangerous:

- Binary pulsar?
  (Extra energy loss mechanism?)
- PPN physics?
  (Detailed solar system tests?)
- Eötvös experiments?
  (Universality of free-fall?)
- Negative kinetic energy?
  (Between the UV conformal and IR general relativistic limits.)

Lots of careful thought still needed...

Spin-zero graviton:

The "Healthy Extension" of Horava-Lifshitz gravity.

- Consistent Extension of Horava Gravity.
  D. Blas, O. Pujolas, S. Sibiryakov. Sep 2009.
  Published in Phys.Rev.Lett.104:181302,2010.
  e-Print: arXiv:0909.3525 [hep-th]
- Models of non-relativistic quantum gravity:
  the good, the bad and the healthy.
  Diego Blas, Oriol Pujolas, Sergey Sibiryakov. Jul 2010.
  e-Print: arXiv:1007.3503 [hep-th]

Key idea: Make the "preferred foliation" dynamical.
(Stably causal spacetime with preferred "cosmic time".)

Spin-zero graviton:

Conformal point	GR
$\lambda = 1/3$	$\lambda = 1$

$$\dots\dots\dots\dots\dots\star\dots\dots\dots\dots\dots\dots\dots\dots\dots\star\dots\dots\dots\dots\dots$$

$\xi = -2/3$	$\xi = 0$

- The "healthy extension" has $\lambda > 1$ ($\xi > 0$).
- Need to go beyond projectability to make the spin zero mode hyperbolic.
- RG flow down to $\xi = 0$?

Spin-zero graviton:

Search for "analogue spacetime" hints:

- Emergent gravity at a Lifshitz point from a Bose liquid on the lattice.
  Cenke Xu, Petr Horava. Mar 2010.
  Published in Phys.Rev.D81:104033,2010.
  e-Print: arXiv:1003.0009 [hep-th]

Add an extra gauge symmetry:

- General covariance in quantum gravity at a Lifshitz point.
  Petr Horava, Charles M. Melby-Thompson. Jul 2010.
  e-Print: arXiv:1007.2410 [hep-th]

Extra symmetry not related to diffeomorphism invariance.

Spin-zero graviton:

Projectability?

- Projectable Horava-Lifshitz gravity in a nutshell.
  Silke Weinfurtner, Thomas P. Sotiriou, Matt Visser. Feb 2010.
  Published in J.Phys.Conf.Ser.222:012054,2010.
  e-Print: arXiv:1002.0308 [gr-qc]

Other?

- CDT meets Horava-Lifshitz gravity.
  J. Ambjorn, A. Gorlich, S. Jordan, J. Jurkiewicz, R. Loll. Feb 2010.
  Published in Phys.Lett.B690:413-419,2010.
  e-Print: arXiv:1002.3298 [hep-th]

Spin-zero graviton:

The choices seem to be:

- Approximately decouple the scalar mode?
- Kill the scalar mode with more symmetry?
- Kill the scalar mode with more a priori restrictions on the metric? (Even more restrictions than projectability.)

None of these approaches is as yet fully satisfying.

FLRW spacetimes:

The (generalized) Hořava model naturally provides:

- Dark radiation;
- Dark stiff matter.

From the Hamiltonian/ super-Hamiltonian distinction, can potentially get:

- Dark dust.

Summary:

- Still a very active field...
- (Initial feeding frenzy somewhat subsided)...
- Very real physics challenges remain...

End: