Topology, thermodynamics & dynamics of quantum vacuum in effective theory

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ERE- 2010

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THEORETICAL
PHYSICS

1. Introduction: effective quantum field theories

2. Quantum vacuum as topological medium

- Fermi surface as topological object
- Dirac (Fermi) points as topological objects
- emergent gravity & physical laws near Dirac point (Fermi point)

3. Quantum vacuum as self-sustained dynamic system

- effective variable for Lorentz invariant quantum vacuum
- gravitating & non-gravitating vacuum energy
- nullification of cosmological constant in equilibrium vacuum
- dynamics of vacuum: cosmology as relaxation to equilibrium

4. Conclusion: natural emergence from universality of topology & thermodynamics

3+1 sources of effective Quantum Field Theories in many-body system & quantum vacuum

I think it is safe to say that no one understands **Quantum Mechanics**

Richard Feynman

Thermodynamics is the only physical theory of universal content

Albert Einstein

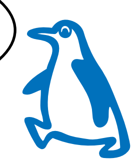
Symmetry: conservation laws, translational invariance, spontaneously broken symmetry, Grand Unification, ...

Topology: you can't comb the hair on a ball smooth, anti-Grand-Unification

Lev Landau

effective theories of quantum liquids: two-fluid hydrodynamics of superfluid ^{4}He & Fermi liquid theory of liquid ^{3}He

missing ingredient in Landau theories



Landau view on a many body system

many body systems are simple at low energy & temperature

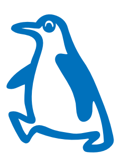
weakly excited state of liquid can be considered as system of "elementary excitations"

equally applied to:
superfluids,
solids,
&
relativistic quantum vacuum

Landau, 1941

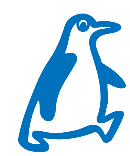
helium liquids

ground state + elementary excitations



ground state = vacuum

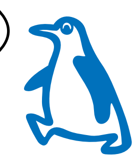
quasiparticles = elementary particles



Universe

vacuum + elementary particles

why is low energy physics applicable to our vacuum ?



characteristic high-energy scale in our vacuum is Planck energy

$$\boxed{E_{\rm P} = (hc^5/G)^{1/2} \sim 10^{19}\ \text{GeV} \sim 10^{32}\text{K}}$$

high-energy physics is extremely ultra-low energy physics

highest energy in accelerators

$$E_{\rm ew} \sim 1\ \text{TeV} \sim 10^{16}\text{K}$$

$$E_{\rm ew} \sim 10^{-16} E_{\rm Planck}$$

high-energy physics & cosmology belong to ultra-low temperature physics

T of cosmic background radiation

$$T_{\rm CMBR} \sim 1\ \text{K}$$

$$T_{\rm CMBR} \sim 10^{-32} E_{\rm Planck}$$

cosmology is extremely ultra-low frequency physics

cosmological expansion

$$v(r,t) = H(t)\, r$$

Hubble law

$$H \sim 10^{-60} E_{\rm Planck}$$

Hubble parameter

our Universe is extremely close to equilibrium ground state

We should study general properties of equilibrium ground states - quantum vacua

Why no freezing at low T?

natural masses of elementary particles
are of order of characteristic energy scale
the Planck energy

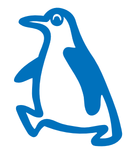
$$\boxed{m \sim E_{\mathrm{Planck}} \sim 10^{19}\ \mathrm{GeV} \sim 10^{32}\mathrm{K}}$$

even at highest temperature
we can reach

$$\boxed{T \sim 1\ \mathrm{TeV} \sim 10^{16}\mathrm{K}}$$

everything should be completely frozen out

$$\boxed{e^{-m/T} = 10^{-10^{16}} = 0}$$



10^{-123}, $10^{-10^{16}}$
another great challenge?

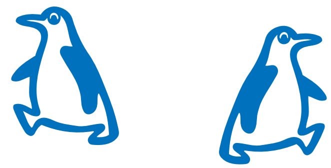
main hierarchy puzzle

$$m_{\mathrm{quarks}}\,,\ m_{\mathrm{leptons}} <<< E_{\mathrm{Planck}}$$

its emergent physics solution:

$$m_{\mathrm{quarks}} = m_{\mathrm{leptons}} = 0$$

reason:
momentum-space **topology**
of quantum vacuum



cosmological constant puzzle

$$\Lambda <<<< E^4_{\mathrm{Planck}}$$

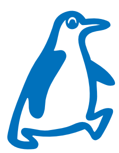
its emergent physics solution:

$$\Lambda = 0$$

reason:
thermodynamics
of quantum vacuum

Why no freezing at low T?

massless particles & gapless excitations are not frozen out



who protects massless excitations?

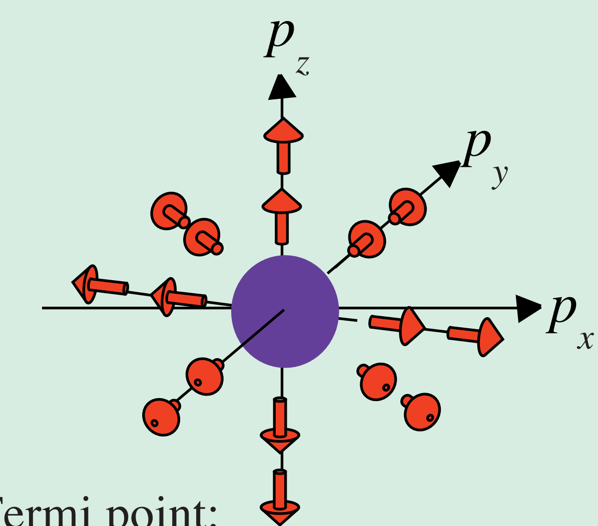
who protects massless fermions?

Topology

gapless fermions live
near
Fermi surface & Fermi point

no life without topology ?

we live because
Fermi point is the hedgehog
protected by topology



Fermi point:
hedgehog in momentum space

Life protection

hedgehog is stable:
one cannot comb the hair on a ball smooth

tools

Thermodynamics

responsible for properties
of vacuum energy

problems of cosmological constant:
perfect equilibrium Lorentz invariant vacuum
has $\Lambda = 0$;
perturbed vacuum has nonzero Λ
on order of perturbation

$$\boxed{\Lambda <<<< E_{\mathrm{Planck}}^4}$$

Topology in momentum space

responsible for properties of
fermionic and bosonic quantum fields
in the background of quantum vacuum

Fermi point in momentum space
protected by topology is a source of
massless Weyl fermions, gauge fields & gravity

$$\boxed{m_{\mathrm{quarks}}\,,\ m_{\mathrm{leptons}} <<< E_{\mathrm{Planck}}}$$

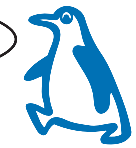
Topology:

Quantum vacuum as topological substance, universality classes

physics at low T is determined by gapless excitations

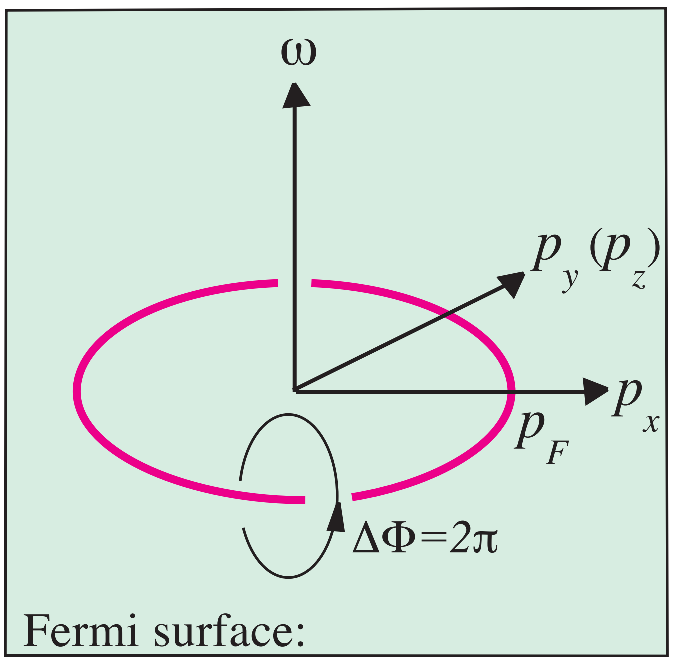
in metals low-lying excitations live near Fermi surface

or near Fermi point



universality classes of gapless vacua

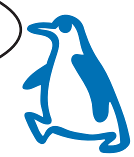
Fermi surface class

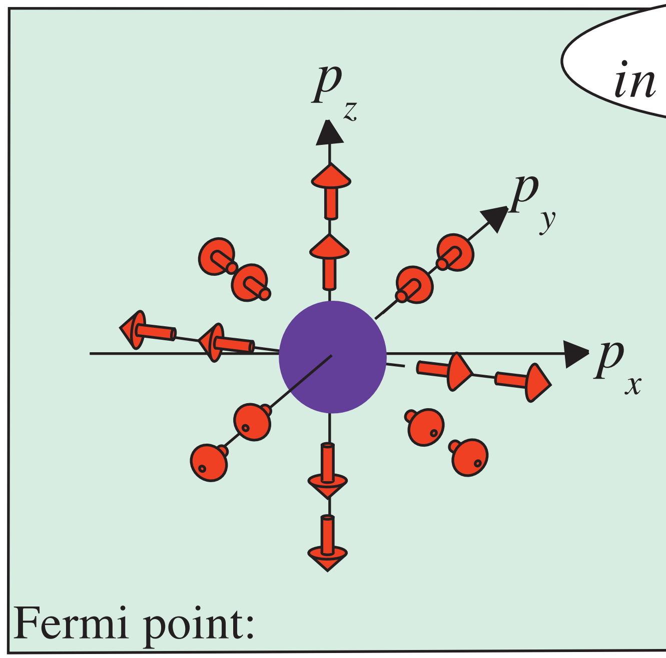


Fermi surface:
vortex line in **p**-space

Fermi point class

topology in momentum space





Fermi point:
hedgehog in **p**-space

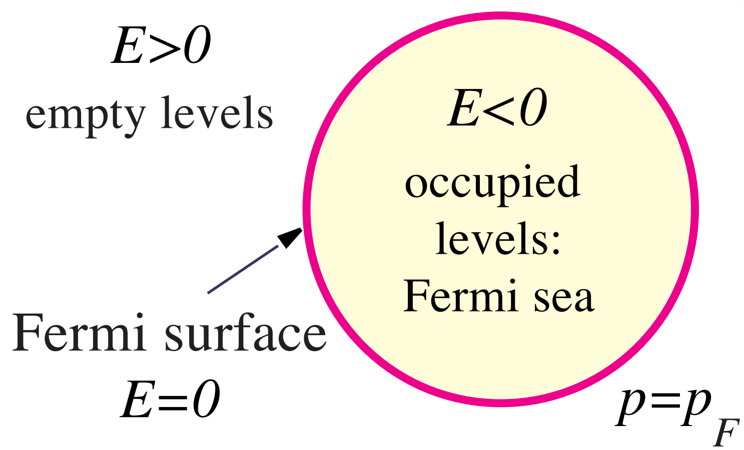
topology is robust to deformations: nodes in spectrum survive interaction

Horava, Kitaev, Ludwig, Schnyder, Ryu, Furusaki, ...

Topological stability of Fermi surface

Energy spectrum of
non-interacting gas of fermionic atoms

$$E(p) = \frac{p^2}{2m} - \mu = \frac{p^2}{2m} - \frac{p_F^{\,2}}{2m}$$

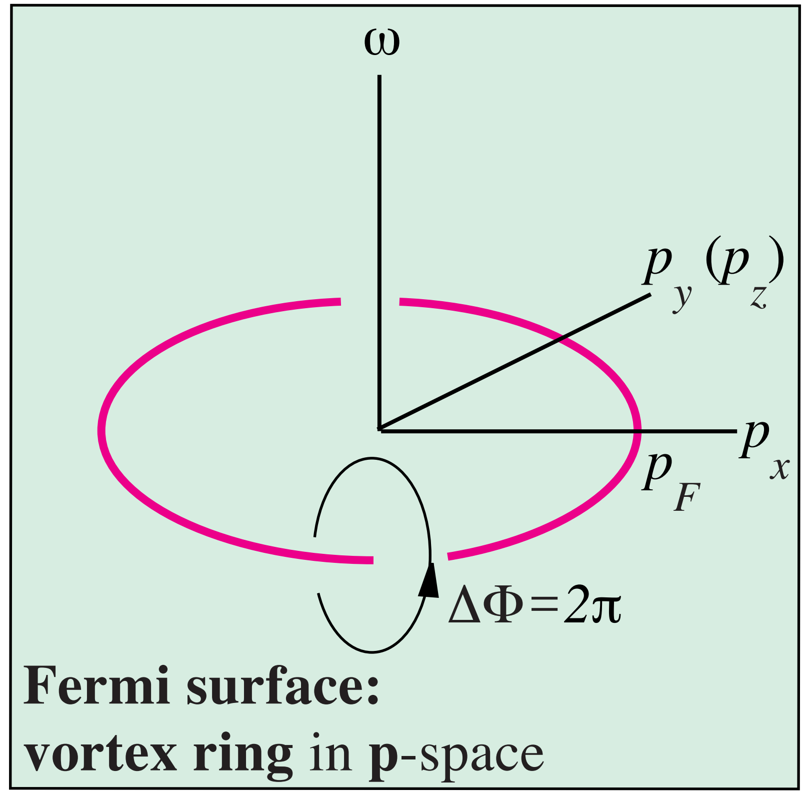


is Fermi surface a domain wall in momentum space?

no!
it is a **vortex ring**

Green's function

$$G^{-1} = i\omega - E(p)$$



phase of Green's function

$$G(\omega,\mathbf{p}) = |G| e^{\,i\Phi}$$

has winding number $N = 1$

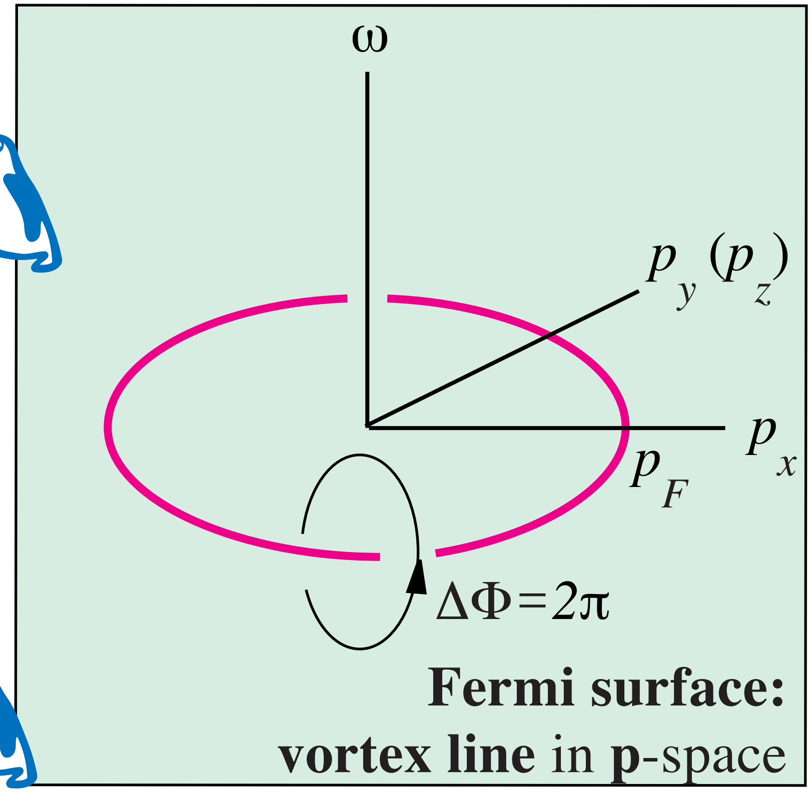
Route to Landau Fermi-liquid

is Fermi surface robust to interaction ?

Sure! Because of topology:
winding number $N=1$ cannot change continuously,
interaction cannot destroy singularity

then Fermi surface survives in Fermi liquid ?

Landau theory of Fermi liquid
is topologically protected & thus is universal



$$G(\omega,\mathbf{p})=|G|e^{\,i\Phi}$$

all metals have Fermi surface ...

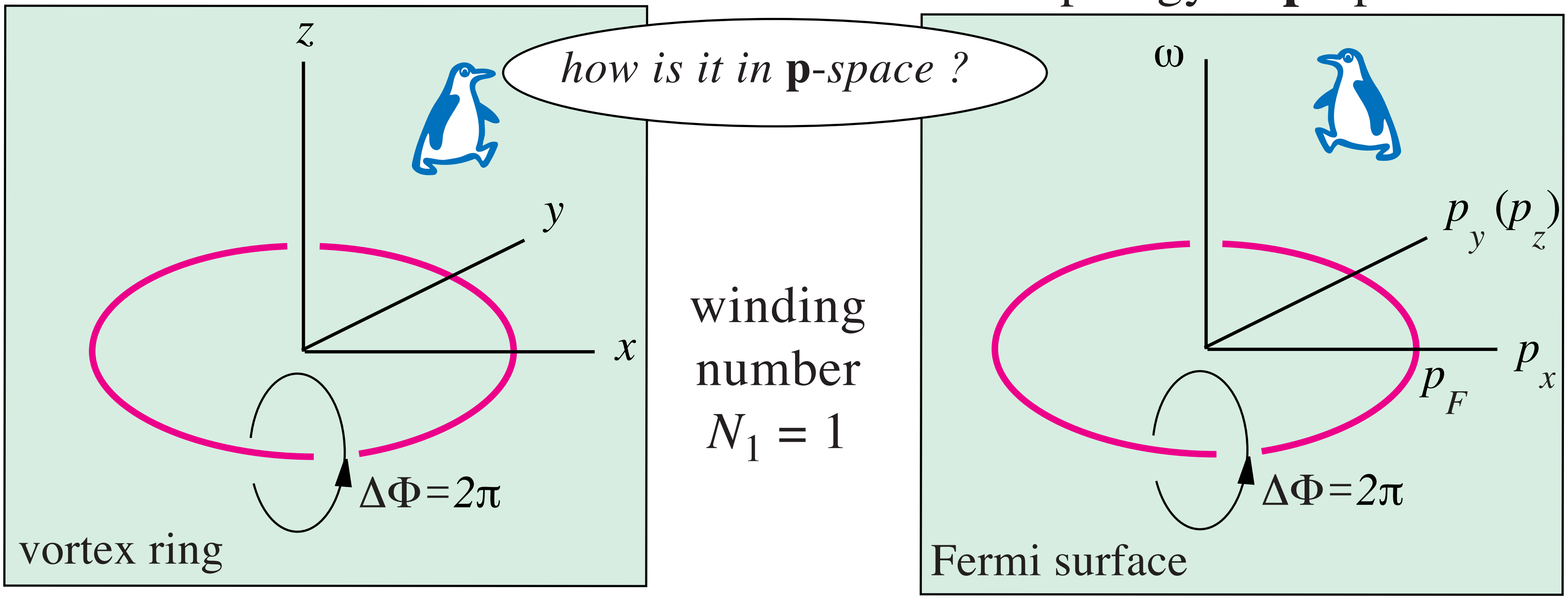
Not only metals.
Some superconductore too!

Stability conditions & Fermi surface topologies in a superconductor
Gubankova-Schmitt-Wilczek, *Phys.Rev.* **B**74 (2006) 064505

quantized vortex in **r**-space ≡ **Fermi surface** in **p**-space

homotopy group π_1

Topology in **r**-space | Topology in **p**-space



$\Psi(\mathbf{r}) = |\Psi|\, e^{i\Phi}$

scalar order parameter
of superfluid & superconductor

$G(\omega,\mathbf{p}) = |G| e^{i\Phi}$

Green's function (propagator)

classes of mapping $S^1 \rightarrow U(1)$

manifold of
broken symmetry vacuum states

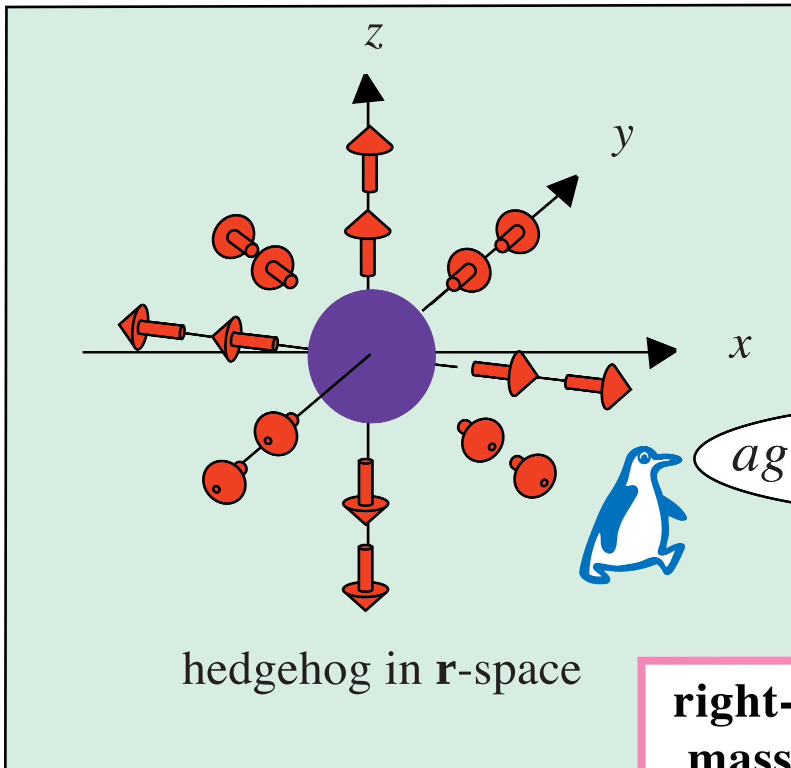
classes of mapping $S^1 \rightarrow GL(\mathrm{n,C})$

space of
non-degenerate complex matrices

Topology: Fermi (Dirac) point universality class

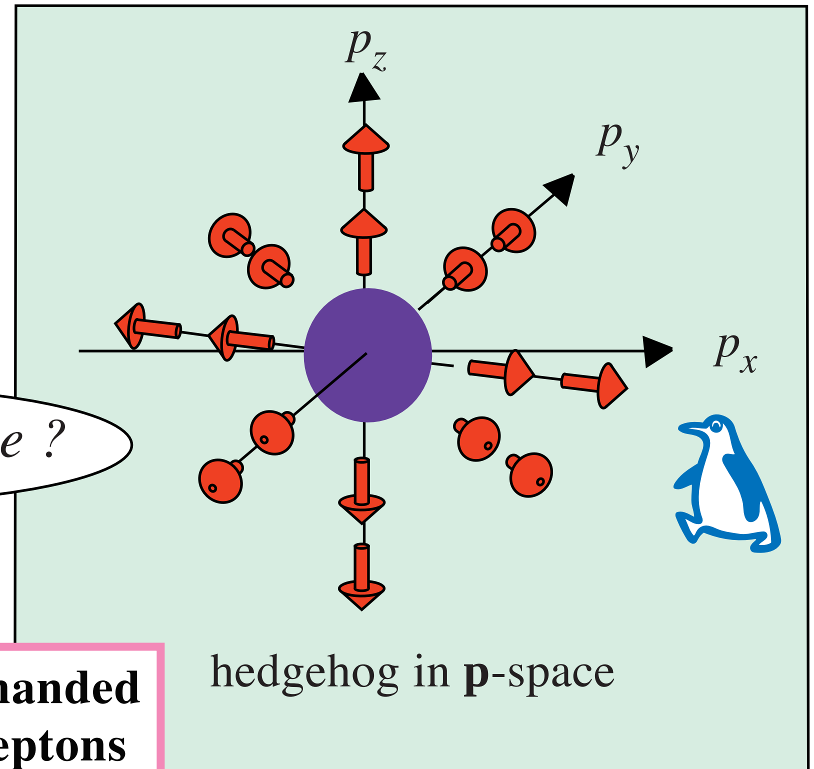
Superfluid ^{3}He-A, quantum vacuum of Standard Model, semimetal, graphene, ...

magnetic hedgehog vs right-handed electron



$\sigma(\mathbf{r}) = \hat{\mathbf{r}}$

again no difference ?



$\sigma(\mathbf{p}) = \hat{\mathbf{p}}$

right-handed and left-handed massless quarks and leptons are elementary particles in Standard Model

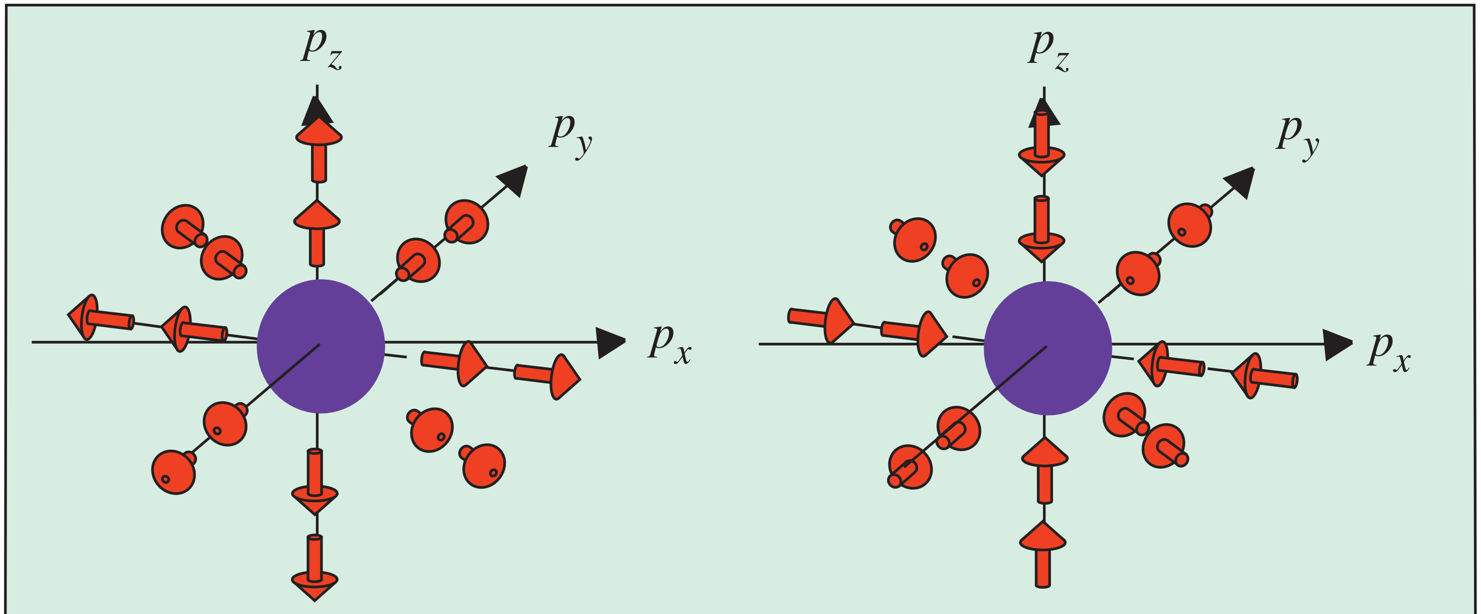
Landau CP symmetry is emergent

close to Fermi point

$$H = + c\, \sigma \cdot \mathbf{p}$$

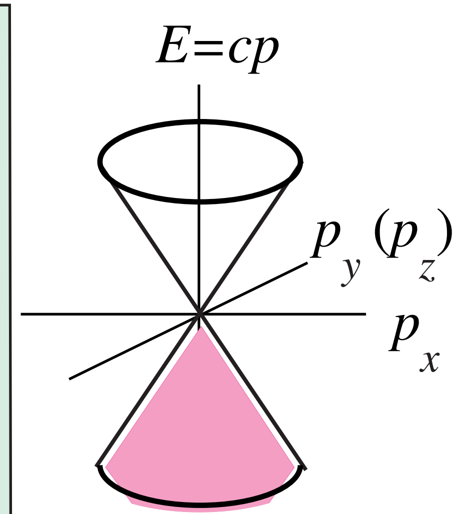
right-handed electron =
hedgehog in **p**-space with spines = spins

Topological invariant for right-handed and left-handed elementary particles



hedgehog with spines (spins) outward ($N_3 = +1$)

hedgehog with spines (spins) inward ($N_3 = -1$)



right neutrino

$$H = +c\,\sigma\cdot\mathbf{p}$$
$$\mathbf{g}(\mathbf{p}) = +c\mathbf{p}$$

$$\boxed{H = \sigma\cdot\mathbf{g}(\mathbf{p})}$$

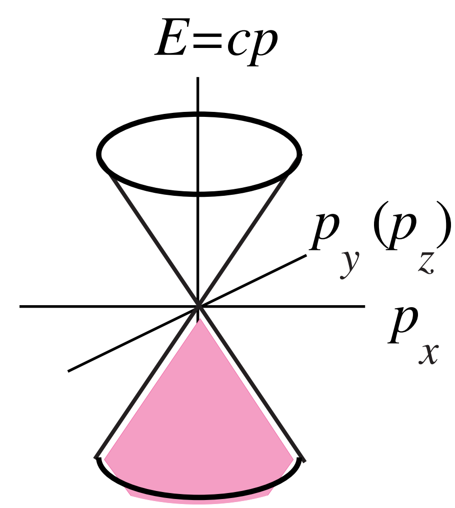
$$H = -c\,\sigma\cdot\mathbf{p}$$
$$\mathbf{g}(\mathbf{p}) = -c\mathbf{p}$$

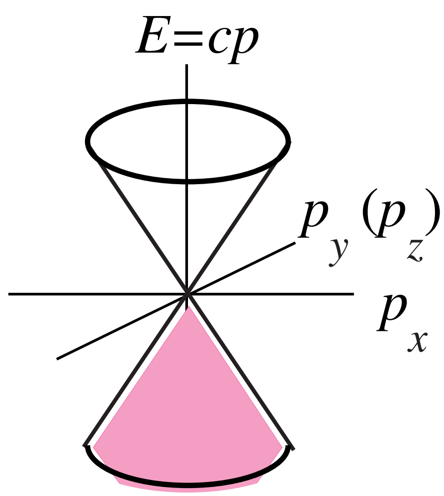
left neutrino

$$N_3 = \frac{1}{8\pi}\, e_{ijk} \int dS^i\, \hat{\mathbf{g}}\cdot(\partial^j \hat{\mathbf{g}} \times \partial^k \hat{\mathbf{g}})$$

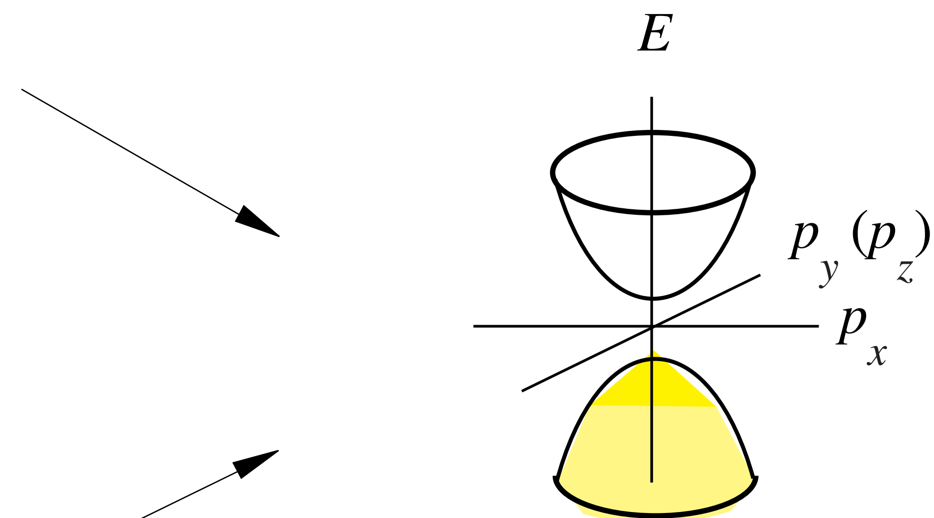
over 2D surface around Fermi point

where are Dirac particles?





Dirac particle - composite object made of left and right particles



$$E^2 = c^2p^2 + m^2$$

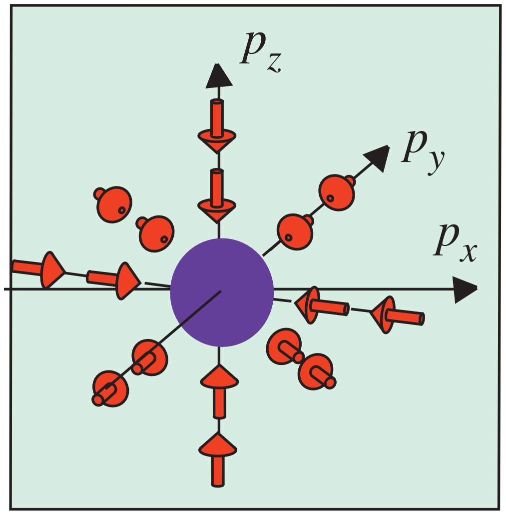
mixing of left and right particles is secondary effect, which occurs at extremely low temperature

$$T_{ew} \sim 1 \text{ TeV} \sim 10^{16}\text{K}$$

Chiral fermions in Standard Model

Family #1 of quarks and leptons

left particles



hedgehog with spines (spins) inward ($N_3 = -1$)

quarks

+2/3 u_L +1/6	-1/3 d_L +1/6
+2/3 u_L +1/6	-1/3 d_L +1/6
+2/3 u_L +1/6	-1/3 d_L +1/6

$SU(2)_L$

$SU(3)_C$

+2/3 u_R +2/3	-1/3 d_R -1/3
+2/3 u_R +2/3	-1/3 d_R -1/3
+2/3 u_R +2/3	-1/3 d_R -1/3

leptons

0 ν_L -1/2	-1 e_L -1/2

0 ν_R 0	-1 e_R -1

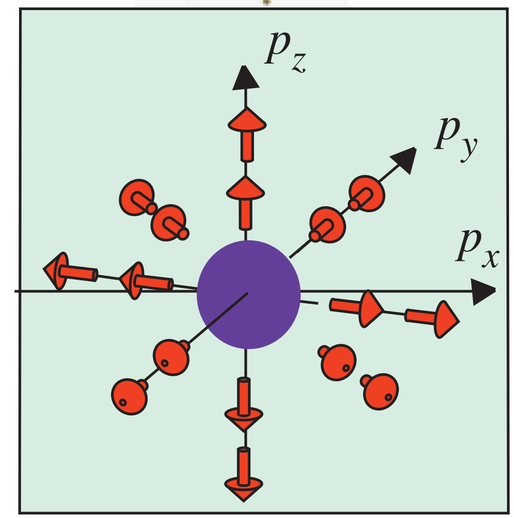
$$H = -c\, \sigma \cdot \mathbf{p}$$

$$N_3 = -1$$

$$H = +c\, \sigma \cdot \mathbf{p}$$

$$N_3 = +1$$

right particles



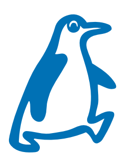
hedgehog with spines (spins) outward ($N_3 = +1$)

$$N_3 = \frac{1}{24\pi^2} e_{\mu\nu\lambda\gamma}\, \mathbf{tr} \int dS^\gamma\, \mathbf{G}^\mu\, \mathbf{G}^{-1}\, \mathbf{G}^\nu\, \mathbf{G}^{-1} \mathbf{G}^\lambda\, \mathbf{G}^{-1}$$

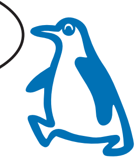
over 3D surface S in 4D momentum space

general topological invariant in terms of Green's function

life exists at low T because Fermi points are stable ?

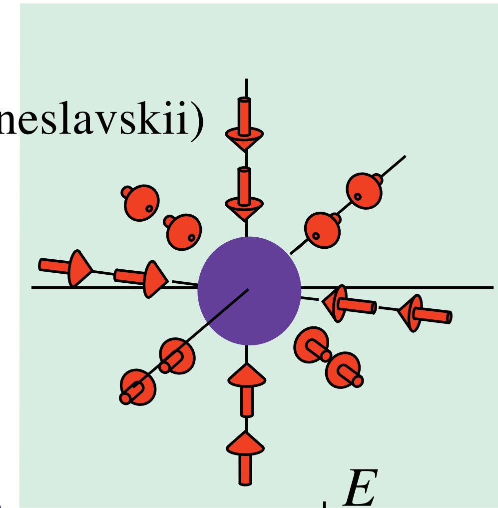


right !



Fermi (Dirac) points in 3+1 gapless topological matter

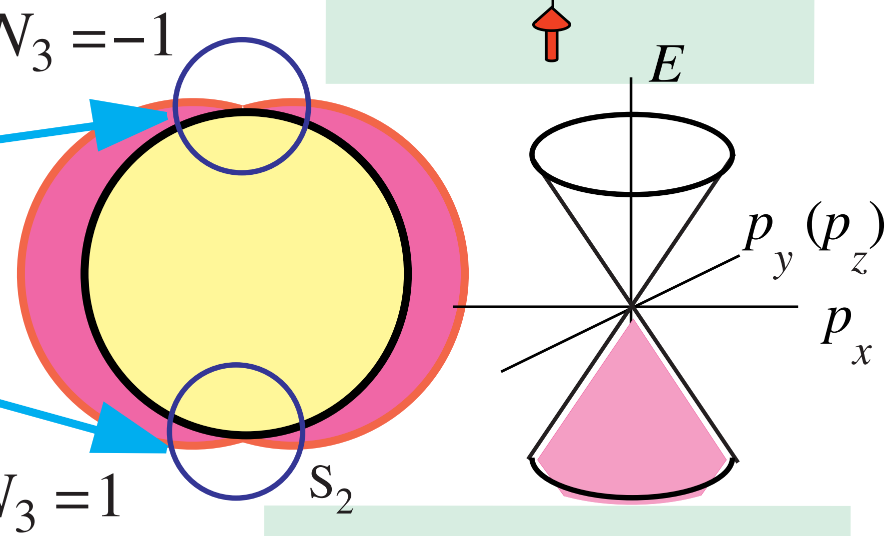
topologically protected point nodes in:
superfluid ^{3}He-A, triplet cold Fermi gases, semi-metal (Abrikosov-Beneslavskii)



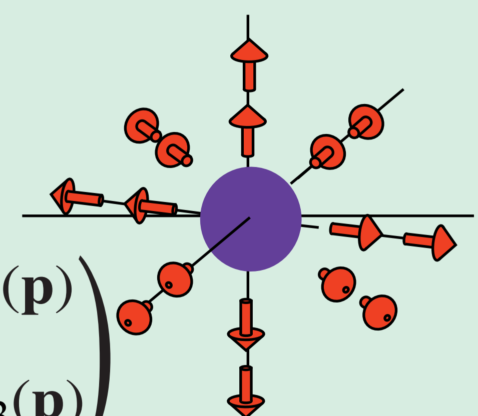
Gap node - Fermi point
(anti-hedgehog)

$$N_3 = \frac{1}{8\pi} e_{ijk} \int dS^k \; \hat{\mathbf{g}} \cdot (\partial_{p_i} \hat{\mathbf{g}} \times \partial_{p_j} \hat{\mathbf{g}})$$

over 2D surface S
in 3D p-space



Gap node - Fermi point
(hedgehog)



$$H = \begin{pmatrix} \frac{p^2}{2m} - \mu & c(p_x + ip_y) \\ c(p_x - ip_y) & -\frac{p^2}{2m} + \mu \end{pmatrix} = \begin{pmatrix} g_3(\mathbf{p}) & g_1(\mathbf{p}) + i\, g_2(\mathbf{p}) \\ g_1(\mathbf{p}) - i\, g_2(\mathbf{p}) & -g_3(\mathbf{p}) \end{pmatrix}$$

emergence of relativistic QFT near Fermi (Dirac) point

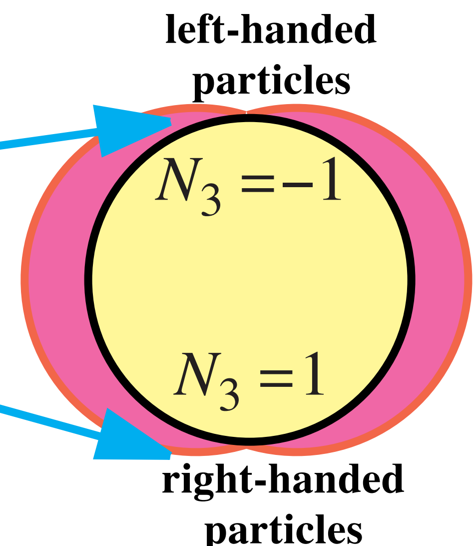
original non-relativistic Hamiltonian

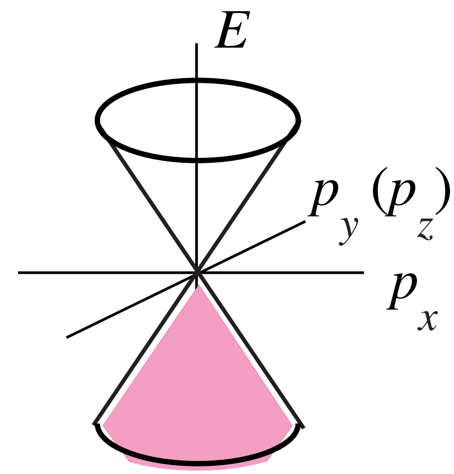
$$H = \begin{pmatrix} \frac{p^2}{2m} - \mu & c(p_x + ip_y) \\ c(p_x - ip_y) & -\frac{p^2}{2m} + \mu \end{pmatrix} = \begin{pmatrix} g_3(\mathbf{p}) & g_1(\mathbf{p}) + i\, g_2(\mathbf{p}) \\ g_1(\mathbf{p}) - i\, g_2(\mathbf{p}) & -g_3(\mathbf{p}) \end{pmatrix} = \boldsymbol{\tau} \cdot \mathbf{g}(\mathbf{p})$$

close to nodes, i.e. in low-energy corner
relativistic chiral fermions emerge

$$H = N_3\, c\, \boldsymbol{\tau} \cdot \mathbf{p}$$

$$E = \pm\, cp$$





chirality is emergent ??

what else is emergent ?

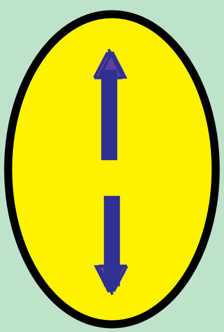
top. invariant determines chirality in low-energy corner

relativistic invariance as well

bosonic collective modes in two generic fermionic vacua

Landau theory of Fermi liquid

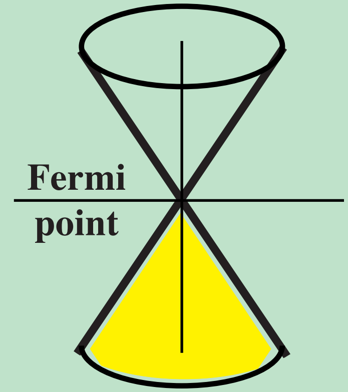
collective Bose modes
of fermionic vacuum:
propagating
oscillation of **shape**
of Fermi surface



Landau, ZhETF **32**, 59 (1957)

Standard Model + gravity

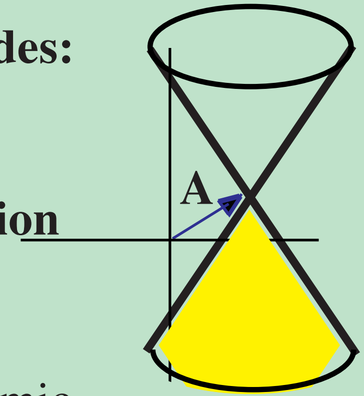
collective Bose modes:

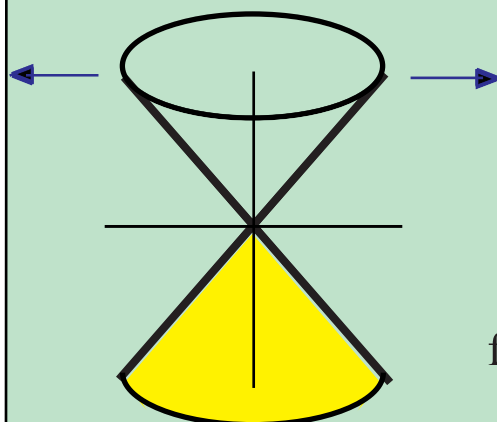


propagating
oscillation of **position**
of Fermi point

$$\mathbf{p} \rightarrow \mathbf{p} - e\mathbf{A}$$

form effective dynamic
electromagnetic field

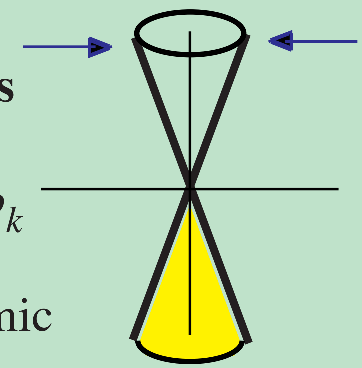




propagating
oscillation of **slopes**

$$E^2 = c^2 p^2 \rightarrow g^{ik} p_i p_k$$

form effective dynamic
gravity field



two generic quantum field theories of interacting bosonic & fermionic fields

relativistic quantum fields and gravity emerging near Fermi point

Atiyah-Bott-Shapiro construction:
linear expansion of Hamiltonian near the nodes in terms of Dirac Γ-matrices

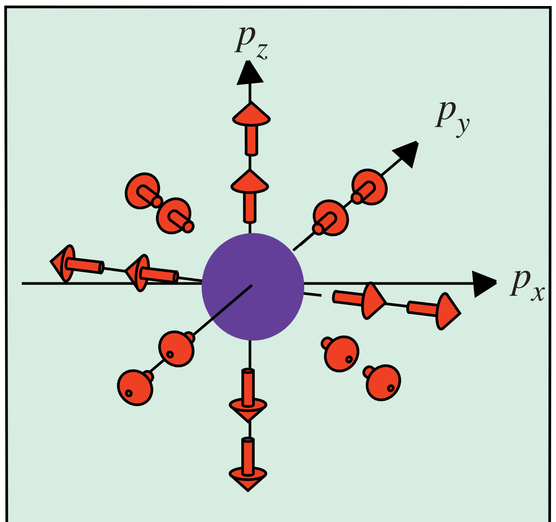
$$\boxed{E = v_F\,(p - p_F)}$$

linear expansion near Fermi surface

emergent relativity

$$\boxed{H = e_i^k\,\Gamma^i \cdot (p_k - p_k^0)}$$

linear expansion near Fermi point



hedgehog in **p**-space

$$\boxed{g^{\mu\nu}(p_\mu - eA_\mu - e\tau\cdot\mathbf{W}_\mu)(p_\nu - eA_\nu - e\tau\cdot\mathbf{W}_\nu) = 0}$$

effective metric: emergent gravity

effective electromagnetic field

effective $SU(2)$ gauge field

effective electric charge

$e = +1$ or -1

effective isotopic spin

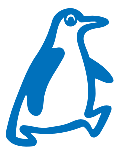
all ingredients of Standard Model :
chiral fermions & gauge fields
emerge in low-energy corner
together with spin, Dirac Γ−matrices, gravity & physical laws:
Lorentz & gauge invariance, equivalence principle, etc

gravity & gauge fields
are collective modes
of vacua with Fermi point

crossover from Landau 2-fluid hydrodynamics to Einstein general relativity

they represent two different limits of hydrodynamic type equations

equations for $g^{\mu\nu}$ depend on hierarchy of ultraviolet cut-off's:
Planck energy scale $E_{\rm Planck}$ vs Lorentz violating scale $E_{\rm Lorentz}$

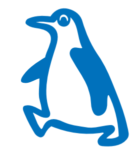


$E_{\rm Planck} >> E_{\rm Lorentz}$

emergent Landau two-fluid hydrodynamics

$E_{\rm Planck} << E_{\rm Lorentz}$

emergent general covariance & general relativity



^{3}He-A with Fermi point

$E_{\rm Lorentz} << E_{\rm Planck}$

$E_{\rm Lorentz} \sim 10^{-3}\, E_{\rm Planck}$

Universe

$E_{\rm Lorentz} >> E_{\rm Planck}$

$E_{\rm Lorentz} > 10^{9}\, E_{\rm Planck}$

Conclusion to topological medium part

Momentum-space topology determines:

universality classes of quantum vacua

effective field theories in these quantum vacua

topological quantum phase transitions (Lifshitz, plateau, etc.)

quantization of Hall and spin-Hall conductivity

topological Chern-Simons & Wess-Zumino terms

quantum statistics of topological objects

spectrum of edge states & fermion zero modes on walls & quantum vortices

chiral anomaly & vortex dynamics, etc.

3. Thermodynamics & dynamics of Lorentz invariant vacuum

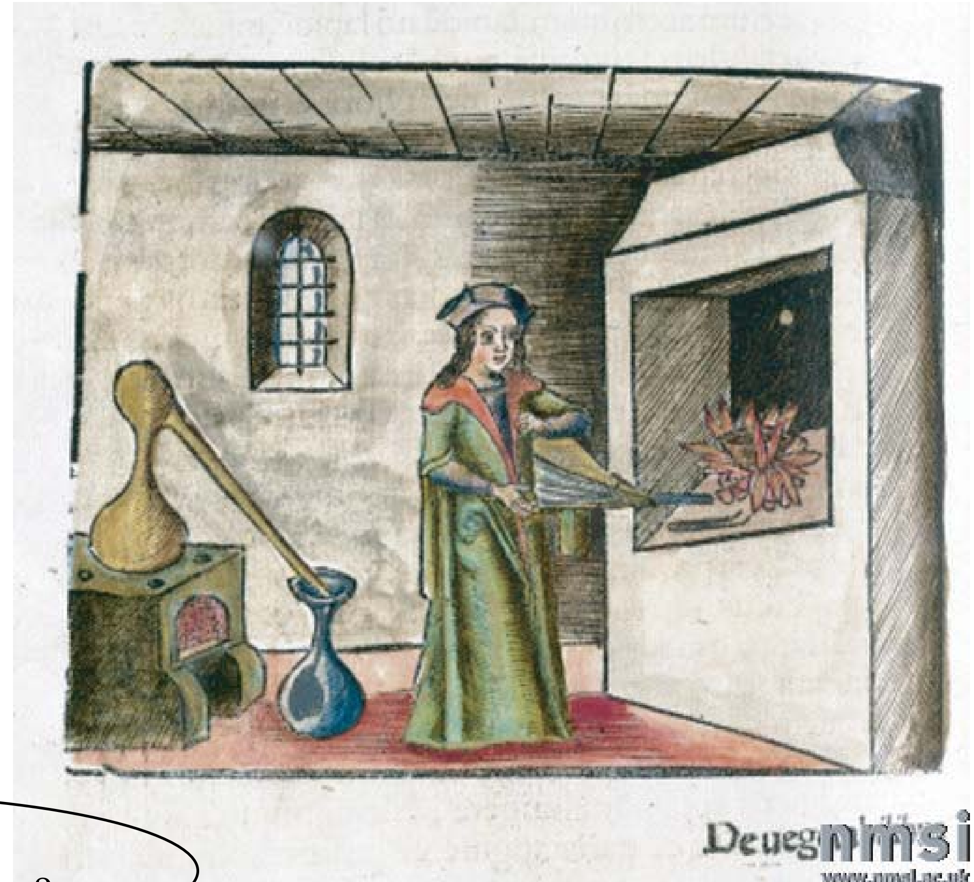
$\boxed{\text{quantum vacuum as self-sustained system}}$

$\boxed{\text{dynamics of cosmological `constant'}}$

dark energy problem

recipe to cook Universe

Dark Energy	70%
Dark Matter	30%
Baryonic Matter	4%
Visible Matter	0,4%



Dark and Dark!
What is the difference?

dark matter forms clusters
like ordinary matter

cosmological constant Λ is possible candidate for dark energy

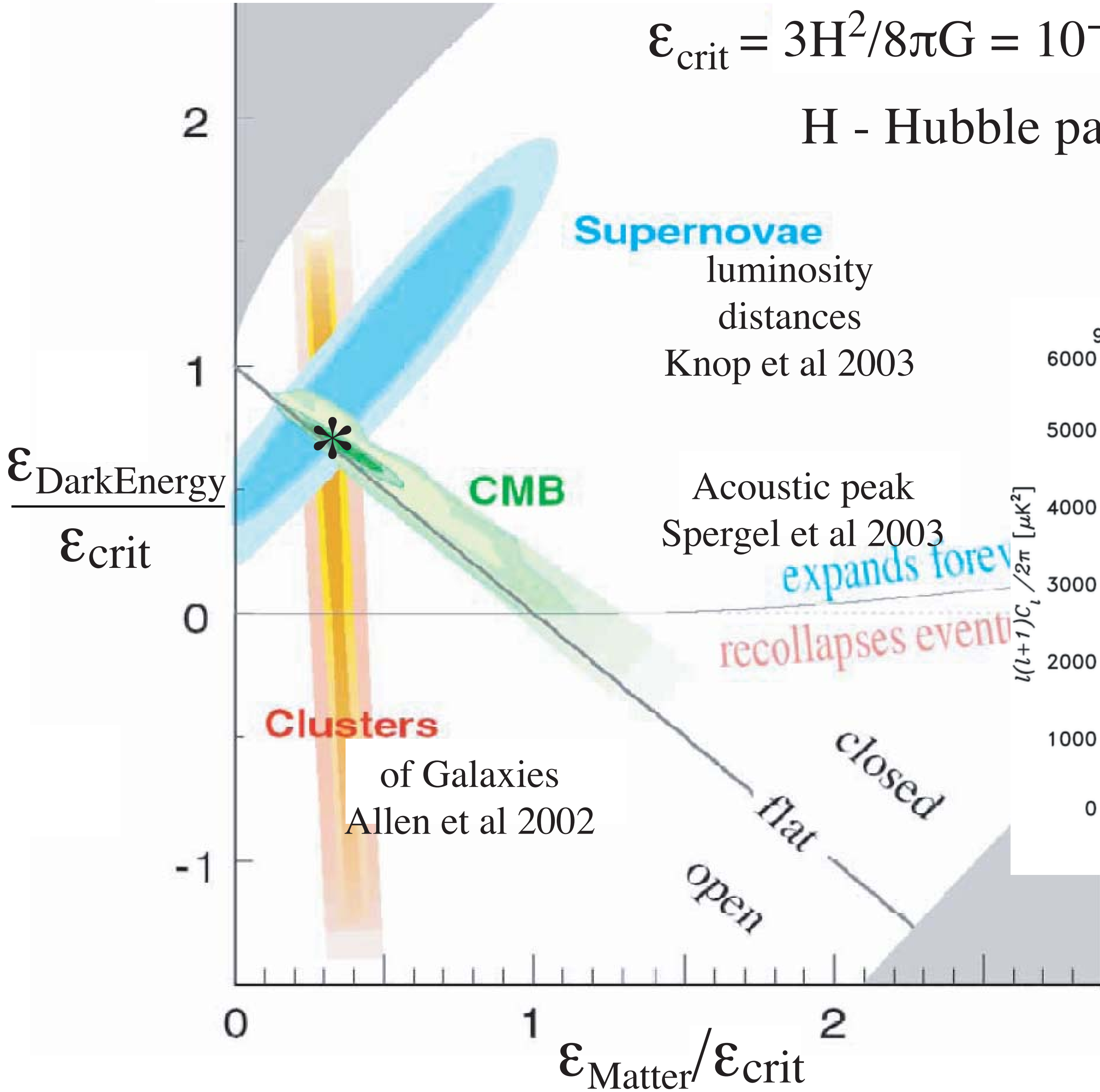
$$\Lambda = \varepsilon_{\text{Dark Energy}}$$

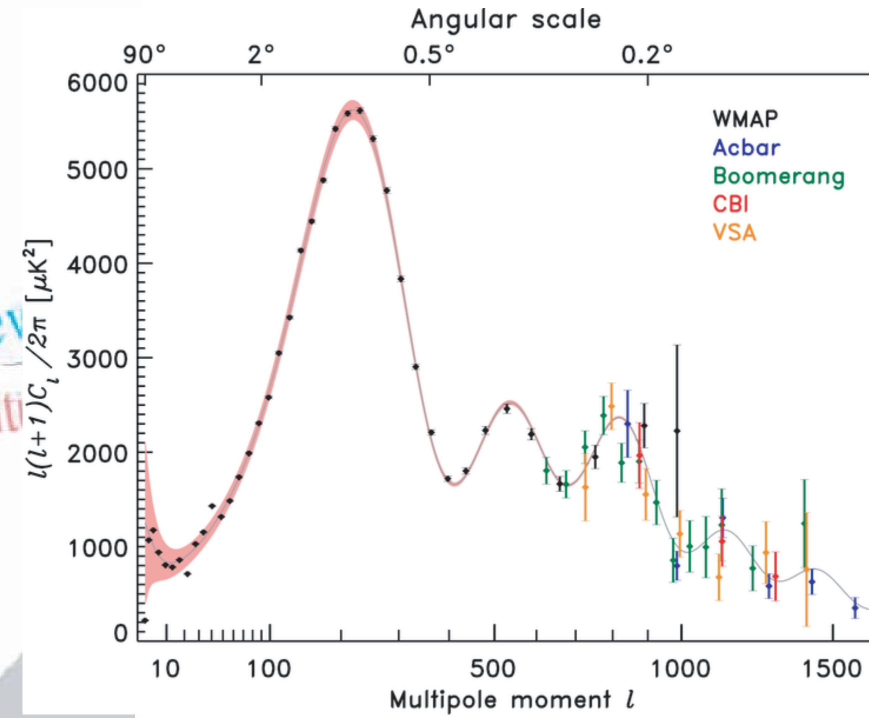
observational cosmology:

dark energy vs **dark matter**

$$\varepsilon_{crit} = 3H^2/8\pi G = 10^{-29} g/cm^3$$

H - Hubble parameter

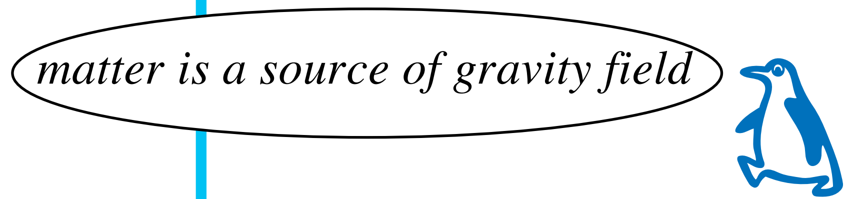




Cosmological Term

*** Original Einstein equations**

$$\boxed{\frac{1}{8\pi G}\left(R_{\mu\nu} - g_{\mu\nu}R/2\right) = T^{\text{Matter}}_{\mu\nu}}$$



*** 1917: Einstein added the cosmological term**

$$\boxed{\frac{1}{8\pi G}\left(R_{\mu\nu} - g_{\mu\nu}R/2\right) - \Lambda g_{\mu\nu} = T^{\text{Matter}}_{\mu\nu}}$$

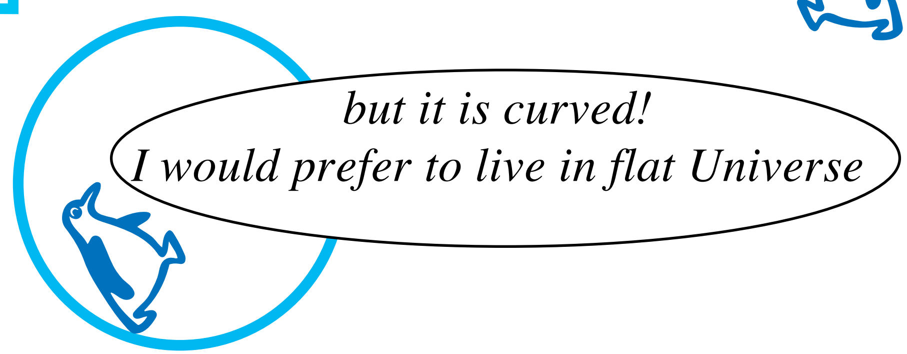
cosmological constant

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 = R^2$$

& obtained static solution: Universe as 3D sphere

$$\boxed{\Lambda = 0.5\ \varepsilon_{\text{Matter}}} = 1/(8\pi G R^2)$$

energy density of matter **radius of 3D sphere**



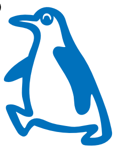
Estimation of cosmological constant

> **when I was discussing cosmological problems with Einstein, he remarked that the introduction of the cosmological term was the biggest blunder of his life**

-- George Gamow, My World Line, 1970

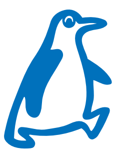
$$\boxed{\Lambda = 0.5\ \varepsilon_{\mathrm{Matter}}} = 1/(8\pi G R^2)$$

compare with observed $\Lambda = 2.3\ \varepsilon_{\mathrm{Matter}}$



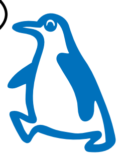
order of magnitude is OK, why blunder?

this was correct estimation of Λ



the first and the last one:

after 90 years nobody could improve it

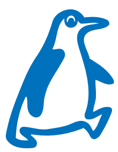


arguments against Λ !

* **1917: de Sitter found stationary solution of Einstein equations without matter**

$$\boxed{\frac{1}{8\pi G}\left(R_{\mu\nu} - g_{\mu\nu}R/2\right) - \Lambda g_{\mu\nu} = 0}$$

no source of gravity field !



What?
empty space gravitates !?

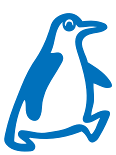
1923: expanding version of de Sitter Universe:

$$R = \exp\,(Ht) \qquad H^2 = 3/(8\pi\Lambda G)$$

* **1924: Hubble : Universe is not stationary**
* **1929: Hubble : recession of galaxies**

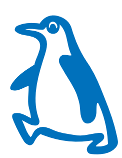
Wenn schon keine quasi-statische Welt, dann fort mit dem kosmologischen Glied.

A. Einstein $\longrightarrow$ H. Weyl, 23 Mai 1923



Away with curvature !
and with cosmological term !

Wait !



the question arises whether it is possible to represent the observed facts without introducing a curvature at all.

Einstein & de Sitter, PNAS **18** (1932) 213

Epoch of quantum mechanics: Λ *as vacuum energy*

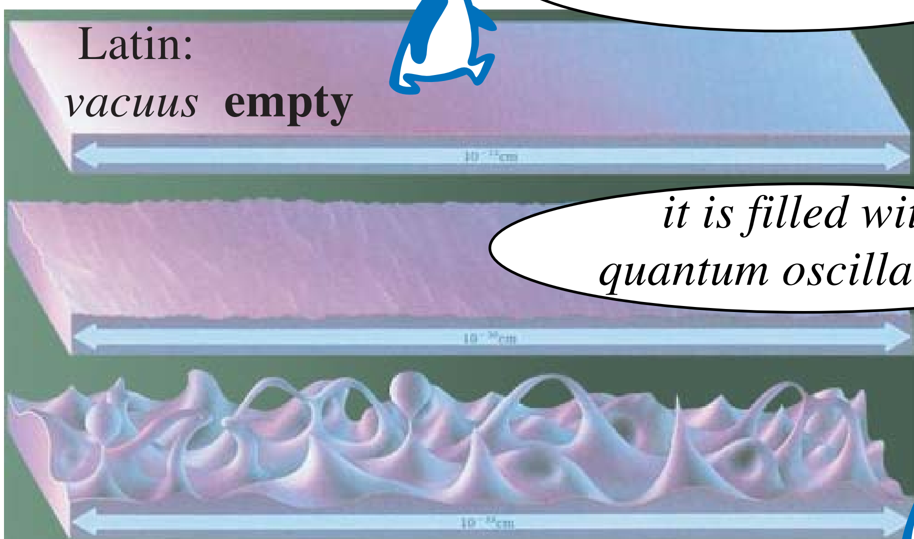
$$\frac{1}{8\pi G}\left(R_{\mu\nu} - g_{\mu\nu}R/2\right) - \Lambda g_{\mu\nu} = T_{\mu\nu}^{\mathrm{Matter}}$$

move Λ to the right

$$\frac{1}{8\pi G}\left(R_{\mu\nu} - g_{\mu\nu}R/2\right) = \Lambda g_{\mu\nu} + T_{\mu\nu}^{\mathrm{Matter}}$$

physical vacuum as source of gravity

vacuum is not empty ?



it is filled with quantum oscillations

quantum field is set of oscillators

$$E_n = h\nu\left(n + 1/2\right)$$

$n = 0$: **zero point energy of quantum fluctuations**

zero point energy has weight ?

you do not believe Einstein? energy must gravitate !

Equation of state of quantum vacuum

$$\frac{1}{8\pi G}\left(R_{\mu\nu} - g_{\mu\nu}R/2\right) = T^{\text{Vacuum}}_{\mu\nu} + T^{\text{Matter}}_{\mu\nu}$$

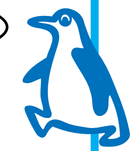
$$T^{\text{Vacuum}}_{\mu\nu} = \Lambda g_{\mu\nu}$$

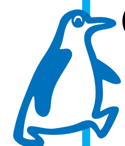
$$\Lambda = \varepsilon_{\text{vac}} = -p_{\text{vac}}$$

energy density of vacuum

pressure of vacuum

what is vacuum?

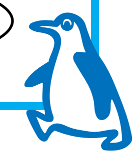




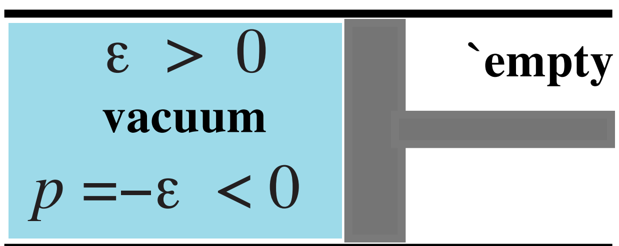
vacuum is medium with equation of state

$$\varepsilon = -p$$

right !



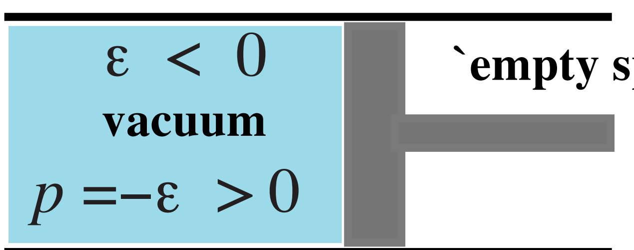
pumping the vacuum by piston



\`empty space'

applied external force

$F = \varepsilon A$, **negative pressure**



\`empty space'

applied external force

$F = \varepsilon A$, **positive pressure**

$$E_{\text{vac}} = \varepsilon_{\text{vac}} V$$

$$p_{\text{vac}} = -dE/dV = -\varepsilon_{\text{vac}}$$

How heavy is aether?

Luminiferous aether (photons)

$$\Sigma_{\mathbf{k}}\, (n(\mathrm{k}) + 1/2)\, ck$$

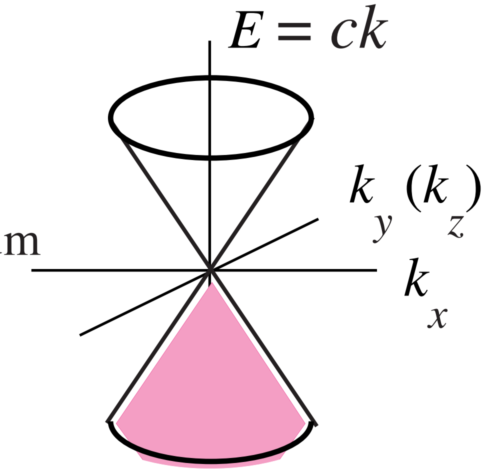
photonic vacuum:

$n(\mathrm{k})=0$

weight of photon vacuum

weight of Dirac vacuum

Dirac vacuum



occupied negative energy levels

$$\boxed{\varepsilon_{\text{zero point}} + \varepsilon_{\text{Dirac}} = (1/2)\sum_{\textbf{bosons}} E(\mathrm{k}) \ - \sum_{\textbf{fermions}} E(\mathrm{k}) \ = (\nu_{\mathbf{b}} - \nu_{\mathbf{f}})\mathrm{ck}^4_{\textbf{Planck}}}$$

zero-point energy

energy of Dirac vacuum

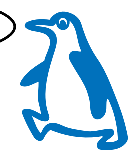
number of bosonic fields

number of Dirac fields

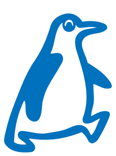
Planck scale

$$\boxed{\varepsilon_{\text{zero point}} \sim 10^{120}\Lambda_{\text{upper limit}}}$$

vacuum is too heavy !



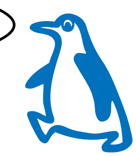
maybe $\Lambda = 0$ **?!**



supersymmetry:
symmetry between fermions and bosons

$$\nu_{\mathbf{b}} = \nu_{\mathbf{f}}$$

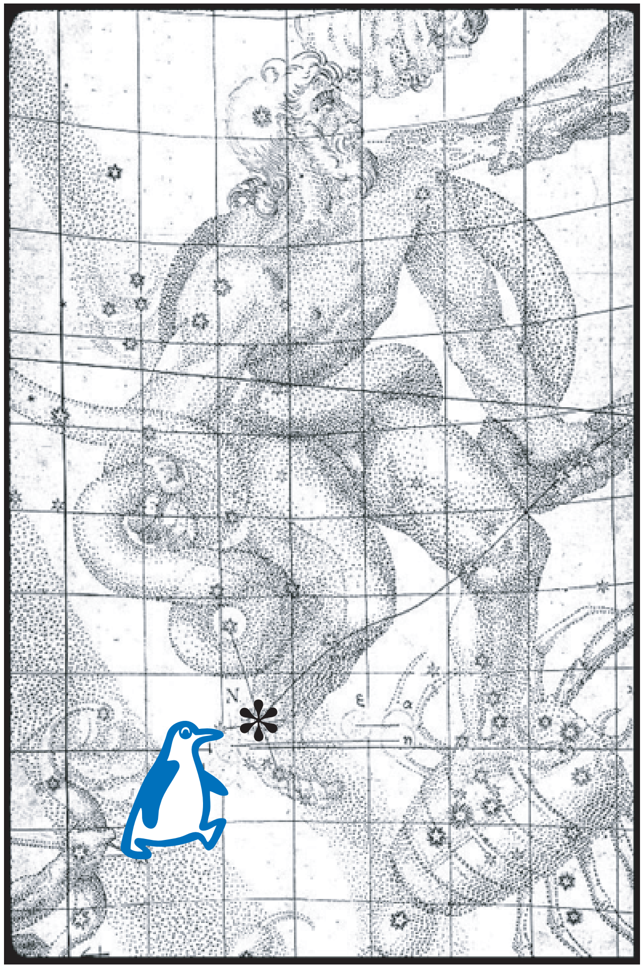
there is no supersymmetry below TeV



Λ in supernova era

Kepler's Supernova 1604

from `*De Stella Nova in Pede Serpentarii*'



distant supernovae: accelerating Universe

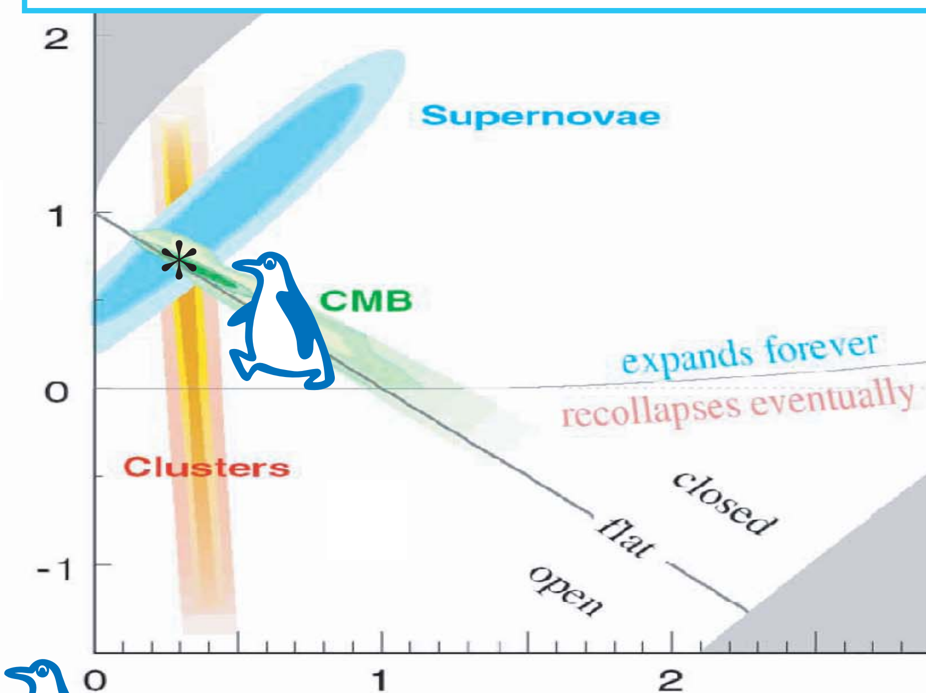
(Perlmutter et al., Riess et al.)

$$\Lambda_{exp} = 2\text{-}3\ \varepsilon_{\text{Dark Matter}} = 10^{-123}\varepsilon_{\text{zero point}}$$

$$\Lambda = \varepsilon_{vac} = \varepsilon_{\text{Dark Energy}} = 70\%$$

$$\varepsilon_{\text{Dark Matter}} = 30\%$$

$$\varepsilon_{\text{Visible Matter}} = 0{,}4\%$$



Universe is flat !

you are right, but Λ is not zero

$$\boxed{\Lambda_{\mathrm{exp}} = 2\text{-}3\ \varepsilon_{\mathrm{Dark\ Matter}} = 10^{-123}\varepsilon_{\mathrm{zero\ point}}}$$

it is easier to accept that $\Lambda = 0$
than 123 orders smaller

magic word: *regularization*

wisdom of particle physicist: $\boxed{\dfrac{1}{0} = 0}$

What can condensed matter physicist say on Λ ?

Why condensed matter ??!

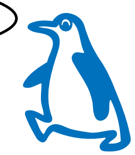
Cosmological constant paradox

$$\Lambda_{\text{observation}} = \varepsilon_{\text{Dark Energy}} \sim 2\text{–}3\ \varepsilon_{\text{DM}} \sim 10^{-47}\ \text{GeV}^4$$

$$\Lambda_{\text{theory}} = \varepsilon_{\text{zero point energy}} \sim E^4_{\text{Planck}} \sim 10^{76}\ \text{GeV}^4$$

$$\Lambda_{\text{observation}} \sim 10^{-123}\,\Lambda_{\text{Theory}}$$

too bad for theory

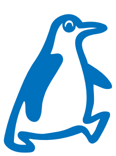


problems:

*** Why is vacuum not extremely heavy?**

*** Why is vacuum gravitating? Why is Λ non-zero?**

*** Why is vacuum as heavy as (dark) matter ?**



it is easier to accept that $\Lambda=0$
than 123 orders of magnitude smaller

$$\boxed{\Lambda_{exp} \sim 2\text{-}3\ \varepsilon_{\text{Dark Matter}} \sim 10^{-123}\Lambda_{bare}}$$

$\Lambda_{bare} \sim \varepsilon_{\text{zero point}}$

*it is easier to accept that $\Lambda = 0$ than 123 orders smaller

*magic word: *regularization*

wisdom of particle physicist: $\boxed{\frac{1}{0} = 0}$

*Polyakov conjecture: dynamical screeneng of Λ by infrared fluctuations of metric

A.M. Polyakov
Phase transitions and the Universe, UFN **136,** 538 (1982)
De Sitter space and eternity, Nucl. Phys. **B 797**, 199 (2008)

*Dynamical evolution of Λ similar to that of gap Δ in superconductors after kick

V. Gurarie, Nonequilibrium dynamics of weakly and strongly paired superconductors: 0905.4498
A.F. Volkov & S.M. Kogan, JETP **38**, 1018 (1974)
Barankov & Levitov, ...

what is natural value of cosmological constant ?

$$\boxed{\Lambda = E_{\mathrm{Planck}}^{4}}$$

$$\boxed{\Lambda = 0}$$

time dependent cosmological constant

$$\boxed{\Lambda \sim E_{\mathrm{Planck}}^{4}}$$

could be in early Universe

$$\boxed{\Lambda \sim 0}$$

should be in old Universe

how to describe quantum vacuum & vacuum energy Λ ?

* **quantum vacuum has equation of state w=−1**

$$\Lambda = \boxed{\varepsilon_{vac} = w_{vac} P_{vac}}$$

* **quantum vacuum is Lorentz-invariant**

$$\boxed{w_{vac} = -1}$$

* **quantum vacuum is a self-sustained medium, which may exist in the absence of environment**

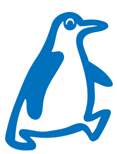
* **for that, vacuum must be described by conserved charge** q

q **is analog of particle density** n **in liquids**

q must be Lorentz invariant

$$\boxed{L\, q = q}$$

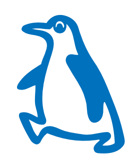
charge density n is not Lorentz invariant



$$\boxed{L\, n = \gamma(n + \mathbf{v}\cdot\mathbf{j})}$$

Hawking suggested to introduce special field which describes the vacuum only
Hawking, Phys. Lett. B **134**, 403 (1984)

does such q exist ?



relativistic invariant conserved charges q

possible

$$\boxed{\nabla_\alpha q^{\alpha\beta} = 0}$$

$$\boxed{\nabla_\alpha q^{\alpha\beta\mu\nu} = 0}$$

$$\boxed{q^{\alpha\beta} = q\, g^{\alpha\beta}}$$

$$\boxed{q^{\alpha\beta\mu\nu} = q\, e^{\alpha\beta\mu\nu}}$$

Duff & van Nieuwenhuizen
Phys. Lett. **B 94,** 179 (1980)

impossible

$$\boxed{\nabla_\alpha q^{\alpha} = 0}$$

$$\boxed{q^{\alpha} = \,?}$$

examples of vacuum variable q

4-form field

$$\boxed{F_{\kappa\lambda\mu\nu} = \nabla_{[\kappa} A_{\lambda\mu\nu]}}$$

$$\boxed{F_{\kappa\lambda\mu\nu} = q\,(-g)^{1/2}\, e_{\kappa\lambda\mu\nu}}$$

$$\boxed{q^2 = -\frac{1}{24} F_{\kappa\lambda\mu\nu} F^{\kappa\lambda\mu\nu}}$$

gluon condensates in QCD

$$\boxed{<\mathbf{G}_{\alpha\beta}\mathbf{G}_{\mu\nu}> = \frac{q}{12}(g_{\alpha\mu}g_{\beta\nu} - g_{\alpha\nu}g_{\beta\mu})}$$

$$\boxed{q = <\mathbf{G}_{\alpha\beta}\mathbf{G}^{\alpha\beta}>}$$

$$<\mathbf{G}_{\alpha\beta}> = 0$$

$$\boxed{<\mathbf{G}_{\alpha\beta}\mathbf{G}_{\mu\nu}> = \frac{q}{24}(-g)^{1/2}\, e_{\alpha\beta\mu\nu}}$$

$$\boxed{q = <\tilde{\mathbf{G}}_{\alpha\beta}\mathbf{G}^{\alpha\beta}>}$$

topological charge density

Einstein-aether theory (T. Jacobson, A. Dolgov)

$$\boxed{\nabla_\mu u_\nu = q\, g_{\mu\nu}}$$

thermodynamics in flat space

the same as in cond-mat

conserved charge Q

$$\boxed{Q = \int dV\, q}$$

thermodynamic potential

$$\boxed{\Omega = E - \mu Q = \int dV\, (\varepsilon\,(q) - \mu q)}$$

Lagrange multiplier or chemical potential μ

pressure

$$\boxed{\begin{array}{c} P = -\, dE/dV = -\varepsilon + q\, d\varepsilon/dq \\ E = V\,\varepsilon(Q/V) \end{array}}$$

$$d\Omega/dq\Big|_{\mu} = 0$$

equilibrium vacuum

$$\boxed{d\varepsilon/dq = \mu}$$

equilibrium self-sustained vacuum

$$\boxed{\begin{array}{l} d\varepsilon/dq = \mu \\ \varepsilon - q\, d\varepsilon/dq = -P = 0 \end{array}}$$

vacuum energy & cosmological constant

equilibrium self-sustained vacuum

$$d\varepsilon/dq = \mu$$

$$\varepsilon - q\, d\varepsilon/dq = -P = 0$$

$$q \sim \mu \sim E^2_{\rm Planck}$$

vacuum variable in equilibrium self-sustained vacuum

$$\varepsilon\,(q) \sim E^4_{\rm Planck}$$

energy of equilibrium self-sustained vacuum

pressure

$$P = -\varepsilon + q\, d\varepsilon/dq = -\Omega$$

$$\Lambda = \Omega = \varepsilon - \mu\, q$$

cosmological constant

$$P = -\Omega$$

equation of state

$$\Lambda = \underbrace{\varepsilon}_{E^4_{\rm Planck}} - \underbrace{\mu q}_{E^4_{\rm Planck}} = 0$$

cosmological constant in equilibrium self-sustained vacuum

***self-tuning*: two Planck-scale quantities cancel each other in equilibrium self-sustained vacuum**

dynamics of q in flat space

whatever is the origin of q the motion equation for q is the same

action

$$\boxed{S = \int dV\, dt\, \varepsilon(q)}$$

motion equation

$$\boxed{\nabla_\kappa \left(d\varepsilon/dq \right) = 0}$$

solution

$$\boxed{d\varepsilon/dq = \mu}$$

integration constant μ in dynamics
becomes chemical potential in thermodynamics

4-form field $F_{\kappa\lambda\mu\nu}$ as an example of conserved charge q in relativistic vacuum

$$\boxed{\begin{gathered} q^2 = -\frac{1}{24} F_{\kappa\lambda\mu\nu} F^{\kappa\lambda\mu\nu} \\ F_{\kappa\lambda\mu\nu} = \nabla_{[\kappa} A_{\lambda\mu\nu]} \\ F^{\kappa\lambda\mu\nu} = q\, e^{\kappa\lambda\mu\nu} \end{gathered}}$$

Maxwell equation

$$\boxed{\nabla_\kappa \left(F^{\kappa\lambda\mu\nu}\, q^{-1} d\varepsilon/dq \right) = 0} \longrightarrow \boxed{\nabla_\kappa \left(d\varepsilon/dq \right) = 0}$$

dynamics of q in curved space 4-form field and chiral condensate

action

$$\boxed{S = \int d^4x\, (-g)^{1/2} \left[\, \varepsilon(q) + K(q)R \,\right] + S_{\mathrm{matter}}}$$

gravitational coupling $K(q)$ is determined by vacuum
and thus depends on vacuum variable q

q becomes dynamical only when K depends on q

motion equation $d\varepsilon/dq + R\, dK/dq = \mu$ **integration constant**

Einstein equations

$$K\left(Rg_{\mu\nu} - 2R_{\mu\nu}\right) + \left(\varepsilon - \mu q\right)g_{\mu\nu} - 2\left(\nabla_\mu\nabla_\nu - g_{\mu\nu}\nabla^\lambda\nabla_\lambda\right)K = T_{\mu\nu}$$

Einstein tensor

cosmological term $\neq \varepsilon\, g_{\mu\nu}$

$$\boxed{\nabla_\mu T^{\mu\nu} = 0}$$

matter

case of *K=const* restores original Einstein equations

$$\boxed{K = \frac{1}{16\pi G}}$$

G - Newton constant

motion equation

$d\varepsilon/dq = \mu \qquad q = \text{const}$

original Einstein equations

$$\frac{1}{16\pi G}\left(R g_{\mu\nu} - 2R_{\mu\nu}\right) + \Lambda g_{\mu\nu} = T_{\mu\nu} \qquad \Lambda = \varepsilon - \mu q$$

Λ - cosmological constant

Minkowski solution

Maxwell equations

$$\boxed{d\varepsilon/dq + R\, dK/dq = \mu}$$

Einstein equations

$$\underbrace{K\left(Rg_{\mu\nu} - 2R_{\mu\nu}\right)}_{\text{Einstein tensor}} + \underbrace{\left(\varepsilon - \mu q\right)g_{\mu\nu}}_{\text{cosmological term}} - 2\left(\nabla_\mu\nabla_\nu - g_{\mu\nu}\nabla^\lambda\nabla_\lambda\right)K = \underbrace{T_{\mu\nu}}_{\text{matter}}$$

$$\boxed{\nabla_\mu T^{\mu\nu} = 0}$$

Minkowski vacuum solution

$$\boxed{\begin{array}{ll} R = 0 & d\varepsilon/dq = \mu \\ \Lambda = \varepsilon(q) - \mu q = 0 & \end{array}}$$

vacuum energy in action: $\varepsilon\,(q) \sim E^4_{\mathrm{Planck}}$

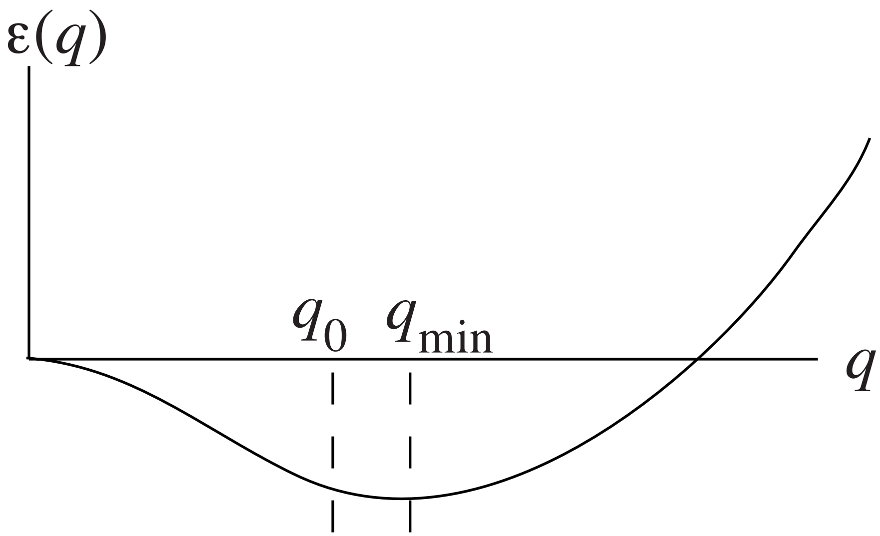
thermodynamic vacuum energy: $\varepsilon - \mu q = 0$

Model vacuum energy

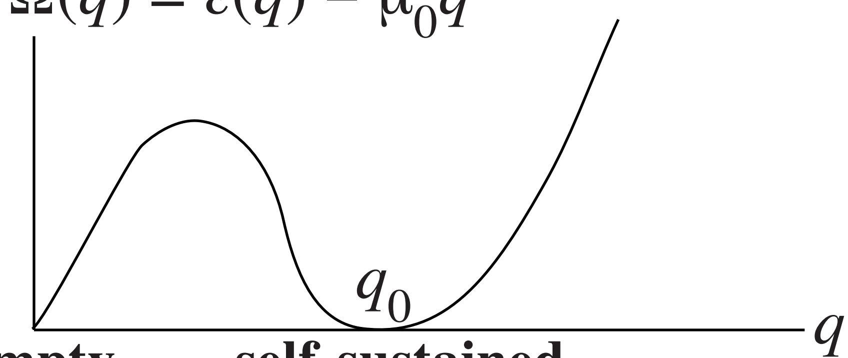
$$\varepsilon(q) = \frac{1}{2\chi}\left(-\frac{q^2}{q_0^2} + \frac{q^4}{3q_0^4}\right)$$

Minkowski vacuum solution

$$\boxed{d\varepsilon/dq = \mu \qquad \varepsilon - \mu q = 0} \rightarrow \boxed{q = q_0 \qquad \mu = \mu_0 = -\frac{1}{3\chi q_0}}$$



$$\Omega(q) = \varepsilon(q) - \mu_0 q$$



empty space $q = 0$

self-sustained vacuum $q = q_0$

vacuum compressibility

$$\boxed{\chi = -\frac{1}{V}\frac{dV}{dP}}$$

$$\boxed{\frac{1}{\chi} = \left(q^2\, d^2\varepsilon/dq^2\right)_{q=q_0} > 0}$$

vacuum stability

Minkowski vacuum (q-independent properties)

$$\Lambda = \Omega_{vac} = -P_{vac}$$

energy density of vacuum **pressure of vacuum**

$$P_{vac} = -\, dE/dV = -\,\Omega_{vac}$$

$$\chi_{vac} = -(1/V)\, dV/dP$$

compressibility of vacuum

$$\langle(\Delta P_{vac})^2\rangle = T/(V\chi_{vac})$$

$$\langle(\Delta\Lambda)^2\rangle = \langle(\Delta P)^2\rangle$$

pressure fluctuations

natural value of Λ *determined by macroscopic physics*

$$\boxed{\Lambda = 0}$$

natural value of χ_{vac} *determined by microscopic physics*

$$\boxed{1/\chi_{vac} = q^2\, d^2\varepsilon_{vac}/dq^2}$$

$$\boxed{\chi_{vac} \sim E^{-4}_{\mathbf{Planck}}}$$

volume of Universe is large:

$$\boxed{V > T_{CMB}/(\Lambda^2\chi_{vac})}$$

$$V > 10^{28}\, V_{hor}$$

dynamics of q in curved space: relaxation of Λ at fixed $\mu=\mu_0$

Maxwell equations $d\varepsilon/dq + R\, dK/dq = \mu_0$

Einstein equations $K\left(Rg_{\mu\nu} - 2R_{\mu\nu}\right) + g_{\mu\nu}\Lambda(q) - 2\left(\nabla_\mu\nabla_\nu - g_{\mu\nu}\nabla^\lambda\nabla_\lambda\right)K = T_{\mu\nu}^{\text{matter}}$

$$\Lambda(q) = \varepsilon(q) - \mu_0 q$$

dynamic solution

$$q(t) - q_0 \sim q_0 \frac{\sin \omega t}{t}$$

$$\Lambda(t) \sim \omega^2 \frac{\sin^2 \omega t}{t^2}$$

$$H(t) = \frac{\dot{a}(t)}{a(t)} = \frac{2}{3t}\left(1 - \cos \omega t\right)$$

$$\omega \sim E_{\text{Planck}}$$

similar to scalar field with mass $M \sim E_{\text{Planck}}$

A.A. Starobinsky, Phys. Lett. **B 91**, 99 (1980)

Relaxation of Λ (generic q-independent properties)

$$\Lambda(t) \sim \omega^2 \frac{\sin^2 \omega t}{t^2}$$

cosmological "constant"

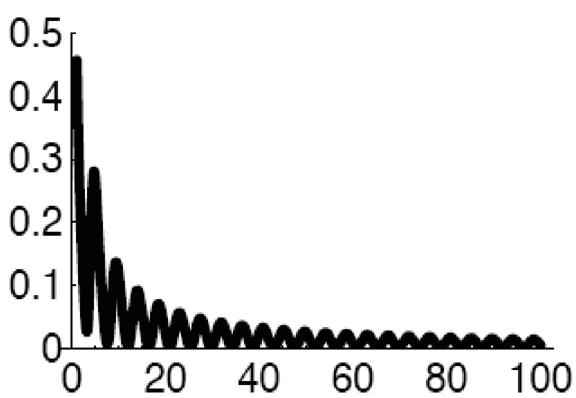
$$H(t) = \frac{\dot{a}(t)}{a(t)} = \frac{2}{3t}\left(1 - \cos \omega t\right)$$

Hubble parameter

$$G(t) = G_N \left(1 + \frac{\sin \omega t}{\omega t}\right)$$

Newton "constant"

$$\omega \sim E_{\mathrm{Planck}}$$



$$<\Lambda(t_{\mathrm{Planck}})> \sim E_{\mathrm{Planck}}^4$$

$$\Lambda(t = \infty) = 0$$

natural solution of the main cosmological problem ?

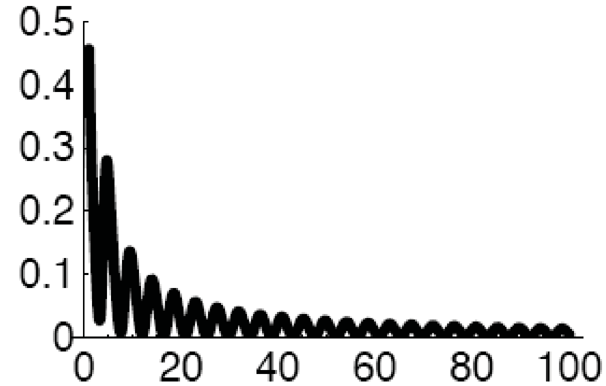
Λ relaxes from natural Planck scale value to natural zero value

present value of Λ

$$\Lambda(t) \sim \omega^2 \frac{\sin^2 \omega t}{t^2}$$

$$\omega \sim E_{\text{Planck}}$$

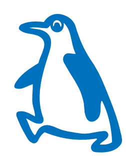
dynamics of Λ:
from Planck to present value



$$<\Lambda(t_{\text{Planck}})> \sim E_{\text{Planck}}^4$$

$$<\Lambda(t_{\text{present}})> \sim E_{\text{Planck}}^2 \,/\, t_{\text{present}}^2 \sim 10^{-120}\, E_{\text{Planck}}^4$$

coincides with present value of dark energy
something to do with coincidence problem ?



Dynamical evolution of Λ similar to that of gap Δ in superconductors after kick

dynamics of Λ in cosmology

$$\Lambda(t) \sim \omega^2 \frac{\sin^2 \omega t}{t^2}$$

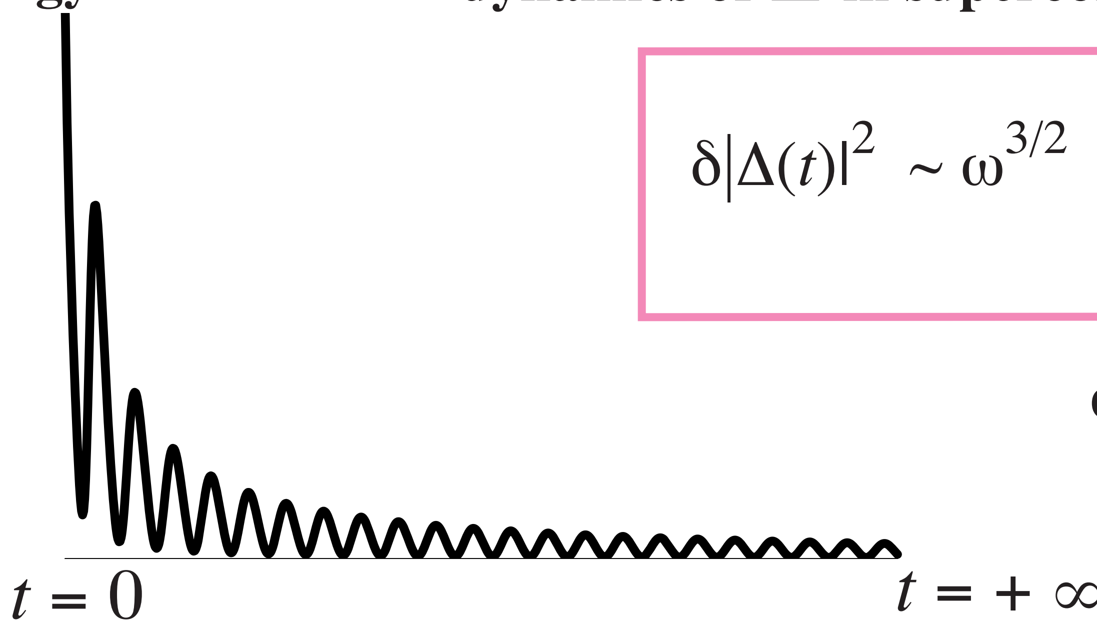
$\omega \sim E_{\rm Planck}$

F.R. Klinkhamer & G.E. Volovik
Dynamic vacuum variable & equilibrium approach in cosmology
PRD **78**, 063528 (2008)
Self-tuning vacuum variable & cosmological constant,
PRD **77**, 085015 (2008)

dynamics of Δ in superconductor

$$\delta|\Delta(t)|^2 \sim \omega^{3/2} \frac{\sin \omega t}{t^{1/2}}$$

$\omega = 2\Delta$



intial states:

nonequilibrium vacuum with $\Lambda \sim E^4_{\rm Planck}$

superconductor with nonequilibrium gap Δ

final states:

equilibrium vacuum with $\Lambda = 0$

ground state of superconductor

$$\varepsilon(t) - \varepsilon_{\rm vac} \sim \omega \frac{\sin^2 \omega t}{t}$$

V. Gurarie, Nonequilibrium dynamics of weakly and strongly paired superconductors: 0905.4498
A.F. Volkov & S.M. Kogan, JETP **38**, 1018 (1974)
Barankov & Levitov, ...

reversibility of the process

$$\boxed{\Lambda(t) \sim \omega^2 \frac{\sin^2 \omega t}{t^2}}$$

$$\omega \sim E_{\text{Planck}}$$

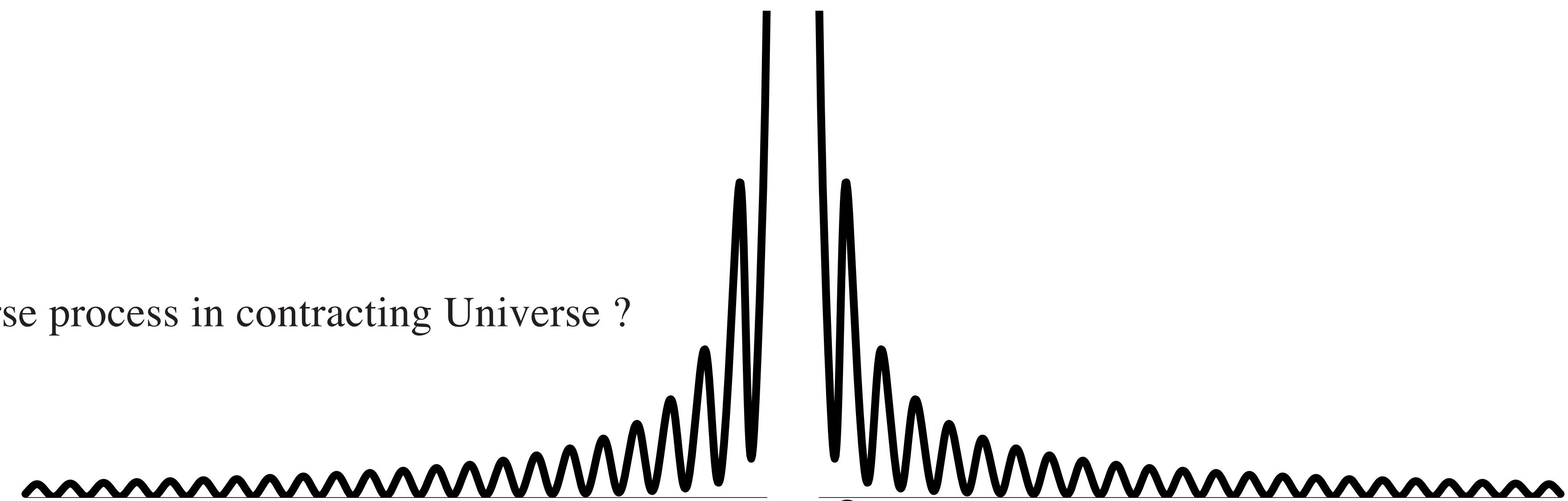
reversible energy transfer
from vacuum to gravity

$$\boxed{\delta|\Delta(t)|^2 \sim \omega^{3/2} \frac{\sin \omega t}{t^{1/2}}}$$

$$\omega = 2\Delta$$

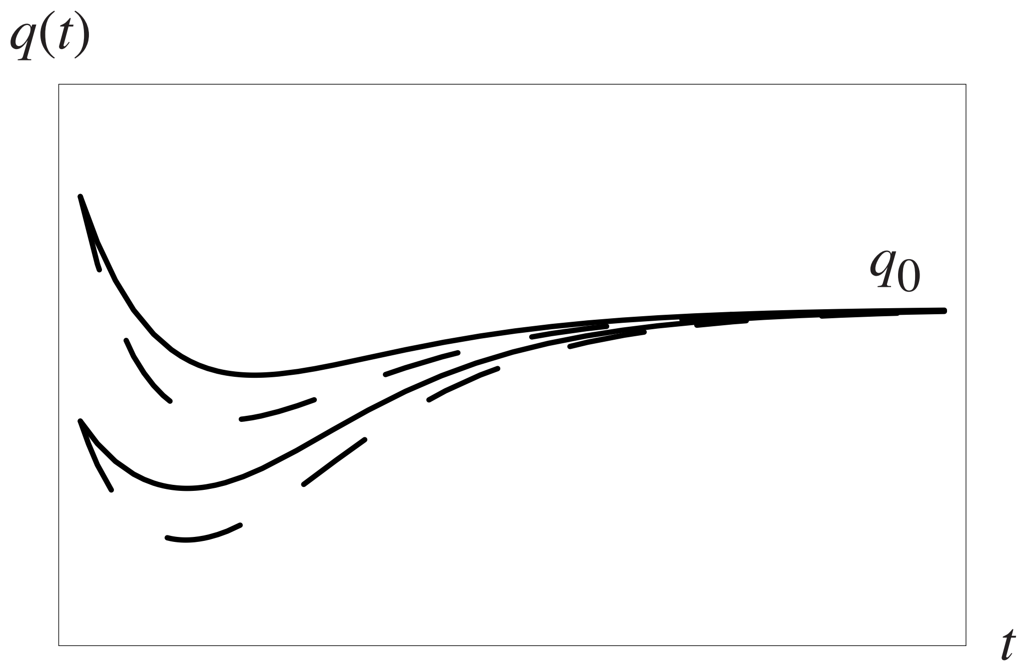
reversible energy transfer
from coherent degree of freedom (vacuum)
to particles (Landau damping)

inverse process in contracting Universe ?



Minkowski vacuum as attractor

allow μ to relax (Dolgov model)



both μ & q relax to equilibrium values μ_0 & q_0
cosmological constant Λ relaxes to zero

F. Klinkhamer & GV
Towards a solution of the cosmological constant problem
JETP Lett. **91**, 259 (2009)

Conclusion to thermodynamic part:

properties of relativistic quantum vacuum as a self-sustained system

* quantum vacuum is characterized by conserved charge q

  q has Planck scale value in equilibrium

$$\boxed{\varepsilon(q) \sim E^4_{\text{Planck}}}$$

* vacuum energy has Planck scale value in equilibrium

  but this energy is not gravitating

$$\boxed{T_{\mu\nu} = \Lambda g_{\mu\nu} \neq \varepsilon(q) g_{\mu\nu}}$$

* gravitating energy

  is thermodynamic vacuum energy

$$\boxed{\Omega(q) = \varepsilon - q\, d\varepsilon/dq}$$

$$\boxed{T_{\mu\nu} = \Lambda g_{\mu\nu} = \Omega(q) g_{\mu\nu}}$$

* thermodynamic energy

  of equilibrium vacuum

$$\boxed{\Omega(q_0) = \varepsilon(q_0) - q_0\, d\varepsilon/dq_0 = 0}$$

* cosmology as relaxation to equilibrium vacuum

general conclusion

* emergent physics must be based on universal features, which do not depend on details of microscopic (Planck) physics

* topology & thermodynamics are necessary ingredients, they are robust to perurbations

* if gravity is emergent, it must emerge together with all other physics (except maybe quantum mechanics)

* is quantum mechanics emergent? If yes, what is the scenario?

Challenge:

origin of small residual vacuum energy

q-vacuum and structure of black-hole interior

q-theory and problem of stability of de Sitter vacuum

derivation of Standard Model from underlying discrete symmetry

origin of QM: if it is effective, its emergence should not depend on details