Numerical Relativity in D-dimensional Spacetimes Collisions of Black Holes and Wave Extraction

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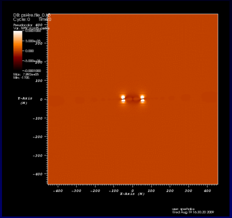
Black Holes in High Energy Physics

- Hoop - Conjecture (Thorne '72):
  - black hole formation if:
    circumference of particle $< 2\pi r_S$
- in high energy collisions: $E = 2\gamma m_0 c^2 > E_{Planck}$
  - gravity is dominant
  - particular nature of particle **not** important for understanding of process



Sperhake et al. '09

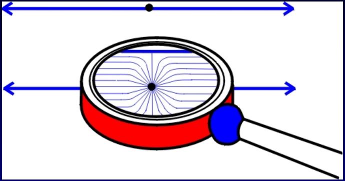
$\Rightarrow$ high energy collisions of particles are well described by BH collisions

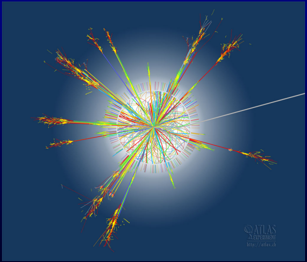
- ultra relativistic collision of solitons: black hole formation if boost $\gamma_c \geq 2.9$ (Choptuik & Pretorius '10)
- head-on collisions of highly boosted BHs: $E_\infty/M \approx 14 \pm 3\%$ (Sperhake et al. '08)
- grazing collisions and scattering processes of highly boosted BHs: $E_\infty/M \leq 35 \pm 5\%$ (Shibata et al. '08, Sperhake et al. '09)

$\Rightarrow$ compute cross-section and energy loss in high energy scattering process

Black Holes in High Energy Physics

- above the Planck scale:
  gravity is dominant interaction
- in $D = 4$:
  $m_{EW} \sim 10^3 \, GeV$, $M_{Pl} \sim 10^{18} \, GeV$
  $\Rightarrow$ “hierarchy problem”



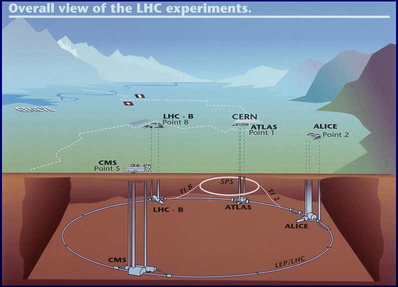


- consider theories of gravity in higher dimensions
  - flat, compact
    (Arkani-Hamed, Dimopoulos & Dvali ’98)
  - warped (Randall & Sundrum ’99)
  - flat, non-compact
    (Dvali, Gabadadze & Porrati ’00)
- in $D > 4$: lowering of Planck scale
  - $M_{Pl,D} \sim m_{EW} \Rightarrow M_{Pl,4}^2 \sim M_{Pl,D}^{D-2} R^{D-4}$
  - $M_{Pl,6} \sim TeV$

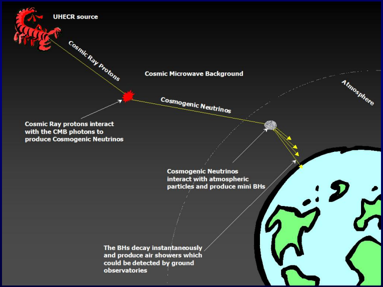
TeV gravity scenarios

⇒ black hole production in high energy collision of particles

- at the Large Hadron Collider
- in Cosmic Rays interactions



http://lhc.web.cern.ch/lhc/



http://www.phy.olemiss.edu/GR/

⇒ compute cross-section of BH production and energy emitted in gravitational radiation for BH event generators

Numerical Relativity in D Dimensions

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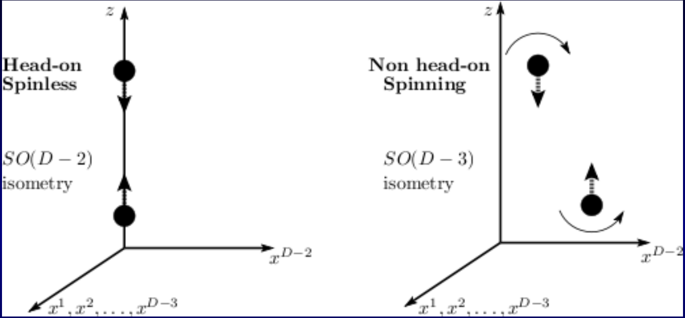


- consider highly symmetric problems
- dimensional reduction by isometry on a (D-4)-sphere
- D dimensional vacuum Einstein Eqs. $\Rightarrow$ 4D Einstein Eqs. plus scalar field
- different higher dimensions manifest in scalar field

Wave Extraction in $D > 4$

Generalization of Regge-Wheeler-Zerilli formalism by Kodama & Ishibashi '03

Master function

$$\Phi_{,t} = (D-2) r^{(D-4)/2} \frac{2rF_{,t} - F^{r}_{t}}{k^2 - D + 2 + \frac{(D-2)(D-1)}{2} \frac{r_S^{D-3}}{r^{D-3}}}\,, \qquad k = l(l+D-3)$$

Energy flux & radiated energy

$$\frac{dE_l}{dt} = \frac{(D-3)k^2(k^2-D+2)}{32\pi(D-2)} (\Phi^{l}_{,t})^2\,, \qquad E = \sum_{l=2}^{\infty} \int_{-\infty}^{\infty} dt \frac{dE_l}{dt}$$

Momentum flux & recoil velocity in $D = 5$

$$\frac{dP}{dt} = \frac{1}{4\pi} \Phi^{l=3}_{,t} \left(5\Phi^{l=2}_{,t} + 21\Phi^{l=4}_{,t}\right)\,, \qquad v_{recoil} = \left| \int_{-\infty}^{\infty} dt \frac{dP}{dt} \right|$$

Head-on collisions in $D = 5$

Numerical Setup

- use Sperhake's extended LEAN code (Zilhão et al. '10):
  - 3+1 Einstein equations with scalar field
  - BSSN system with moving puncture approach
    dynamical variables: χ, $\tilde{\gamma}_{ij}$, K, $\tilde{A}_{ij}$, $\tilde{\Gamma}^i$, ζ, K_ζ
  - modified puncture gauge

$$\partial_t \alpha = -2\alpha(\eta_K K + \eta_{K_\zeta} K_\zeta) + \beta^k \partial_k \alpha$$
$$\partial_t \beta^i = \frac{3}{4}\tilde{\Gamma}^i - \eta_\beta \beta^i + \beta^k \partial_k \beta^i$$

- Brill Lindquist type initial data

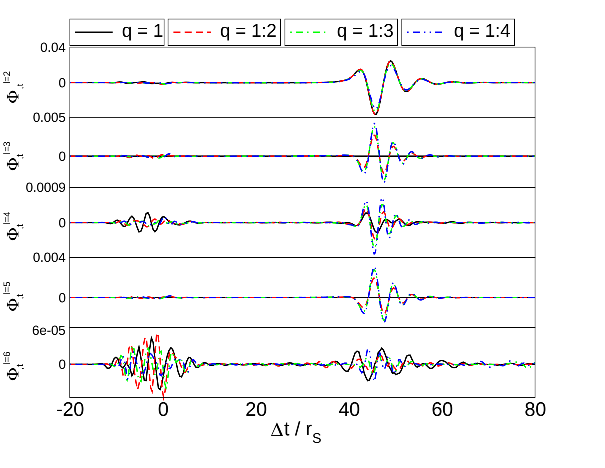
$$\psi = 1 + r_{S,1}^{D-3}/4r_1^{D-3} + r_{S,2}^{D-3}/4r_2^{D-3}$$

- inital positions: $z_1 = -z_2 = 3.185 r_S$
- unequal mass head-on with mass ratios $q = r_{S,1}^{D-3}/r_{S,2}^{D-3} = 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$
- measure lengths in terms of r_S with

$$r_S^{D-3} = \frac{16\pi}{(D-2)A^{S^{D-2}}} M$$

Head-on in $D = 5$ - Gravitational Waves

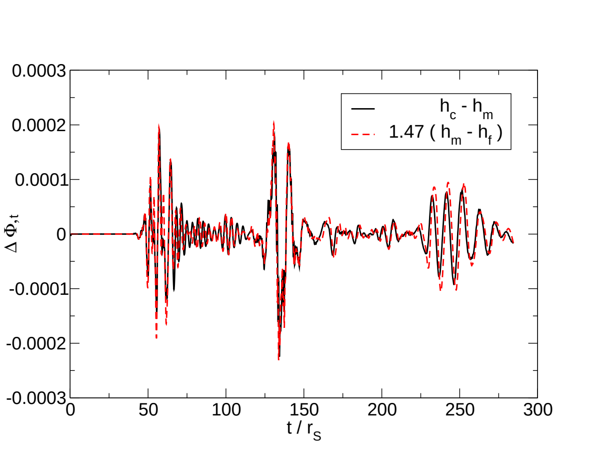
Modes of $\Phi_{,t}$



- characteristic ringdown frequency for $l = 2$ mode of $q = 1$: $r_S\omega = 0.955 \pm 0.005 - \imath(0.255 \pm 0.005)$ ($r_S\omega = 0.9477 + \imath 0.2561$, e.g., Berti et al. '09)

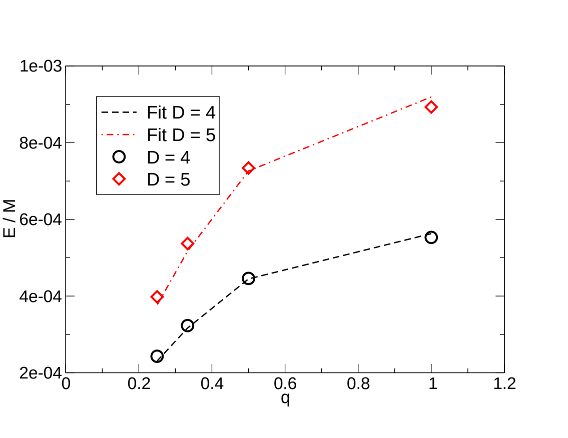
Head-on in $D = 5$ - Convergence Test

$\Delta\Phi^{l=2}_{,t}$ for $q = 1:4$



- simulations at resolutions $h_c = 1/72$, $h_m = 1/78$, $h_f = 1/84$
- obtain 4^{th} order convergence $\Rightarrow \Delta\Phi_{,t}/\Phi_{,t} = 1.5\%$

Head-on Collision in $D = 5$ - Radiated Energy



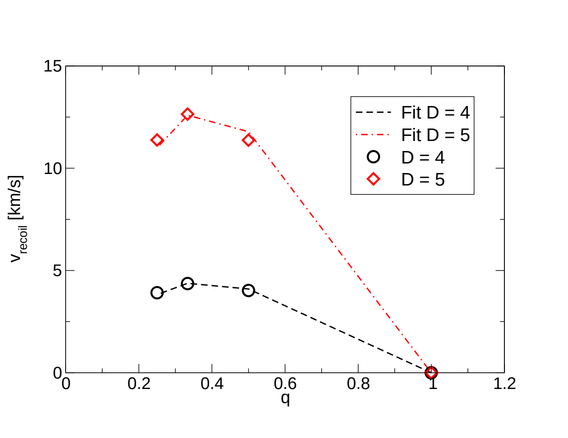
maximum energy at $q = 1$

- $D = 4$: $E_{max}/M = 0.055\%$
- $D = 5$: $E_{max}/M = 0.089\%$

- Fitting function (see M.Lemos '10, MSc thesis, http://blackholes.ist.utl.pt/)

$$E/M = A_E \frac{q^2}{(1+q)^4}\,, \qquad A_E = 0.009\,(D=4) \text{ and } A_E = 0.014\,(D=5)$$

Head-on Collision in $D = 5$ - Recoil Velocity



maximum kick velocity at $q = 0.38$

- $D = 4$: $v_{recoil} = 4.4 km/s$
- $D = 5$: $v_{recoil} = 12.8 km/s$

- Fitting function (see Fitchett '83; M.Lemos '10, MSc thesis, http://blackholes.ist.utl.pt/)

$$v = A_v q^2 \frac{1-q}{(1+q)^5}\,, \qquad A_v = 248 km/s\,(D=4) \text{ and } A_v = 716 km/s\,(D=5)$$

Conclusions and Outlook

- evolution of equal mass head-on in $D = 5$
  - quasinormal ringdown with characteristic frequency $r_S\omega = 0.955 \pm 0.005 - \imath(0.255 \pm 0.005)$
  - radiated energy $E^{rad}/M = 0.089\%$
- evolution of unequal mass head-on collisions in $D = 5$ with $q = 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$
  - maximum kick velocity at $q = 0.38$ with
    $v^{max}_{recoil} = 4.4 km/s$ $(D = 4)$
    $v^{max}_{recoil} = 12.8 km/s$ $(D = 5)$
- ToDo:
  - dependence on initial separation (work in progress)
  - higher mass ratios (work in progress)
  - numerical simulations of black hole collisions in $D \geq 6$
  - study head-ons of BHs with non-zero initial velocity and spinning BHs

http://blackholes.ist.utl.pt