Quasi-local energy flux through the dynamical cosmological horizons

Hoi-Lai YU *IPAS, 2010-Sept-6*

- **Quasi-local energy flux expression for dynamically evolving spacetime**
- **Flux formula to FRW cosmology where dynamical horizon is $S^2 \times R$ and always timelike**
- **Cosmo horizon expands dynamically as energy flux flows through**
- **During exponent cosmo inflation, the energy flux through the horizon vanishes, energy of the universe is dynamically conserved**
- **Duality between black-hole isolated/dynamical horizon and cosmological isolated/dynamical horizon under spacelike and timelike hypersurfaces interchanges**
- **Zeroth, First law and Effective temperature on the cosmological horizons**

Quasi-local energy flux for spacetime perturbation

$$S = \frac{1}{16\pi}\int_M \mathcal{L} = \frac{1}{16\pi}\int_M R^{ab}\wedge\eta_{ab} + \mathcal{L}_{matter}, \tag{1}$$

$$R^{ab} = d\omega^{ab} + \omega^a{}_c\wedge\omega^{cb} \tag{2}$$

$$g = \vartheta_a \otimes_s \vartheta_b \tag{3}$$

$$\eta_{ab} = *(\vartheta_a\wedge\vartheta_b) = \frac{1}{2}\epsilon_{abcd}\vartheta^c\wedge\vartheta^d. \tag{4}$$

A conserved Hamiltonian Noether current 3-form $\mathcal{H}(\xi)$ can be defined,

$$\mathcal{H}(\xi) = \pounds_\xi\omega^{ab}\wedge\eta_{ab} - i_\xi\mathcal{L}, \tag{5}$$

such that the Noether current $\mathcal{H}(\xi)$ is closed ($d\mathcal{H}(\xi)\simeq 0$) when the field equations are satisfied.

Locally there exist a 2-form (called the Noether charge)

$$Q(\xi) = i_\xi\omega^{ab}\wedge\eta_{ab} \tag{6}$$

such that, on solution, $\mathcal{H}(\xi)\simeq dQ(\xi)$.

When integrated on a 3-space Σ, it gives a "Hamiltonian"

$$H(\xi) = \frac{1}{16\pi}\int_\Sigma \mathcal{H}(\xi) = \frac{1}{16\pi}[\int_\Sigma \xi^\mu \mathcal{H}_\mu + \oint_{\partial\Sigma} Q], \tag{7}$$

where $\mathcal{H}_\mu$ are constraints including matter fields contributions.
The value of $H(\xi)$ is therefore given by the boundary term

$$B = \frac{1}{16\pi}\oint_{\partial\Sigma} Q(\xi). \tag{8}$$

Although $H(\xi)$ is sometimes called "Hamiltonian" in the literature, it may not be functionally differentiable to define conserved quantities along the displacement vector field ξ which generates diffeomorphism invariant transformations.
We justify the functional differentiability of $H(\xi)$ as the total Hamiltonian by further varying $H(\xi)$,

$$\delta H(\xi) \quad = \quad \Omega(\pounds_\xi, \delta) + \frac{1}{16\pi}\oint_{\partial\Sigma} i_\xi(\delta\omega^{ab} \wedge \eta_{ab}), \tag{9}$$

where Ω is the symplectic 3-form,

$$\Omega(\pounds_\xi, \delta) = \frac{1}{16\pi}\int_\Sigma \pounds_\xi \omega^{ab} \wedge \delta\eta_{ab} - \delta\omega^{ab} \wedge \pounds_\xi \eta_{ab}. \tag{10}$$

If the symplectic 3-form is a total variation $\Omega(\pounds_\xi, \delta) = \delta \widetilde{H}(\xi)$, then $\widetilde{H}(\xi)$ is the (well-defined) functionally differentiable Hamiltonian. In this case, replacing δ by $\pounds_\xi$, we obtain $\pounds_\xi \widetilde{H}(\xi) = 0$. The value of $\widetilde{H}(\xi)$ gives the dynamically conserved quantities associated with the vector field ξ. On shell, we have $\delta \widetilde{H} = \frac{1}{16\pi} \oint_S \delta Q(\xi) - i_\xi(\delta\omega^{ab} \wedge \eta_{ab})$. Again, replacing δ by $\pounds_\xi$, we arrive at a general definition of the stationary spacetime boundary condition by requiring that

$$\pounds_\xi \widetilde{H} \quad = \quad \frac{1}{16\pi} \left[\oint_S i_\xi \omega^{ab} \wedge \pounds_\xi \eta_{ab} + \pounds_\xi \omega^{ab} \wedge i_\xi \eta_{ab} \right] = 0. \qquad (11)$$

This can be satisfied by the boundary conditions on a bifurcate Killing horizons for stationary black holes or the Isolated Horizons boundary conditions, such that $\pounds_\xi \vartheta^a|_S = 0, \pounds_\xi \omega^{ab}|_S = 0$ (which implies $\pounds_\xi \widetilde{H}|_S = 0$).

In a general dynamical situation, the energy is not conserved, there is no such a Hamiltonian $\widetilde{H}(\xi)$. However, we can still make sense of a quasi-local energy flux. Like the notion of the energy in General Relativity, the quasi-local energy flux is also tied to a choice of a vector field ξ.

Apply perturbation Δ to a spacetime region with stationary boundary conditions such that in the initial boundary, the vector ξ satisfy the stationary boundary conditions.

$$\delta\mathbb{E}(\xi) \equiv \Delta\Omega(\pounds_\xi, \delta) + \oint_S i_\xi \left(\delta\omega^{ab} \wedge \Delta\eta_{ab} - \Delta\omega^{ab} \wedge \delta\eta_{ab}\right) \tag{12}$$

where

$$\mathbb{E} = \Delta H(\xi) - \oint_S i_\xi(\Delta\omega^{ab} \wedge \eta_{ab}) \tag{13}$$

is a perturbation of H with a correction term in order to have a symplectic structure in the boundary term expression and identifying δ with $\pounds_\xi$.
We obtain the corresponding quasi-local energy flux,

$$\mathbb{F}(\xi) \equiv \pounds_\xi \mathbb{E}(\xi), \tag{14}$$

associated to the perturbation Δ,

$$\mathbb{F}(\xi) = \frac{1}{16\pi} \oint_S i_\xi \left(\pounds_\xi\omega^{ab} \wedge \Delta\eta_{ab} - \Delta\omega^{ab} \wedge \pounds_\xi\eta_{ab}\right), \tag{15}$$

where conditions $\pounds_\xi\eta_{ab}|_S = 0$ and $\pounds_\xi\omega^{ab}|_S = 0$ define the stationary cases and the corresponding quasi-local energy flux vanishes.

Vaidya spacetime example, Bondi type energy flux and First law

Vaidya spacetime which describes a spherically symmetric collapse of null dust (radiation).

$$ds^2 = -e^{2\psi}dt^2 + e^{-2\psi}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{16}$$

where $\psi = \psi(t, r)$ **and** $e^{2\psi} = 1 - 2m(t, r)/r$**.**

In this coordinate, the marginally trapped surfaces are given by $r = 2m(t, r)$**. For constant** $m(t, r)$**, this is just the standard Schwarzschild metric. Now consider a perturbation** $\Delta m(t, r)$ **away from the stationary solution,**

$$m(t, r) = m_0 + \Delta m(t, r), \tag{17}$$

because m_0 **is a constant, this implies** $m' = \partial_r(\Delta m)$ **and** $\dot{m} = \partial_t(\Delta m)$**. In terms of the orthonormal frames, the natural choice is,**

$$\vartheta^0 = e^{\psi}dt, \qquad \vartheta^1 = e^{-\psi}dr,$$
$$\vartheta^2 = r d\theta, \qquad \vartheta^3 = r\sin\theta d\phi, \tag{18}$$

with corresponding basis vectors:

$$
\begin{aligned}
e_0 &= e^{-\psi}\partial_t, & e_1 &= e^{\psi}\partial_r, \\
e_2 &= \frac{1}{r}\partial_\theta, & e_3 &= \frac{1}{r\sin\theta}\partial_\phi.
\end{aligned} \tag{19}
$$

For $\xi = c_1\partial_t + c_2\partial_r$, the Lie derivatives of ϑ^a are

$$
\begin{aligned}
\pounds_\xi \vartheta^0 &= \frac{-1}{re^\psi}\left[c_1\dot{m} + c_2\left(m' - \frac{m}{r}\right)\right]dt, \\
\pounds_\xi \vartheta^1 &= \frac{1}{re^{3\psi}}\left[c_1\dot{m} + c_2\left(m' - \frac{m}{r}\right)\right]dr, \\
\pounds_\xi \vartheta^2 &= c_2 d\theta, \\
\pounds_\xi \vartheta^3 &= c_2 \sin\theta d\phi.
\end{aligned} \tag{20}
$$

The spin-connection ω^{ab} has the following nonvanishing terms:

$$
\begin{aligned}
\omega^{01} &= \frac{1}{re^{4\psi}}\dot{m}dr - \frac{1}{r}\left(m' - \frac{m}{r}\right)dt = -\omega^{10}, \\
\omega^{12} &= -e^\psi d\theta = -\omega^{21}, \\
\omega^{13} &= -e^\psi \sin\theta d\phi = -\omega^{31}, \\
\omega^{23} &= -\cos\theta d\phi = -\omega^{32}.
\end{aligned} \tag{21}
$$

The corresponding Lie derivatives of ω^{ab} have the follow

nonvanishing terms,

$$
\begin{aligned}
\pounds_\xi \omega^{01} &= \frac{1}{e^{4\psi}}\left[\frac{c_1}{r}\left(\ddot{m} + \frac{4\dot{m}^2}{re^{2\psi}}\right)\right] dr \\
&\quad + \frac{1}{e^{4\psi}}\left[\frac{c_2}{r}\left(\dot{m}' + \frac{4\dot{m}m'}{re^{2\psi}} - \frac{\dot{m}}{r} - \frac{4m\dot{m}}{r^2 e^{2\psi}}\right)\right] dr \\
&\quad - \left[\frac{c_1}{r}\left(\dot{m}' - \frac{\dot{m}}{r}\right) + \frac{c_2}{r}\left(m'' - \frac{2m'}{r} + \frac{2m}{r^2}\right)\right] dt \\
&= -\pounds_\xi \omega^{10}, \\
\pounds_\xi \omega^{12} &= \frac{1}{re^{\psi}}\left[c_1 \dot{m} + c_2\left(m' - \frac{m}{r}\right)\right] d\theta \\
&= -\pounds_\xi \omega^{21}, \\
\pounds_\xi \omega^{13} &= \frac{1}{re^{\psi}}\left[c_1 \dot{m} + c_2\left(m' - \frac{m}{r}\right)\right] \sin\theta d\phi \\
&= -\pounds_\xi \omega^{31}, \\
\pounds_\xi \omega^{23} &= 0 = -\pounds_\xi \omega^{32}.
\end{aligned} \tag{22}
$$

The perturbation of the orthonormal tetrad the spin-connection

have the following forms respectively,

$$\begin{aligned}\Delta\vartheta^0 &= -\frac{1}{re^{\psi}}\Delta m\,dt,\\ \Delta\vartheta^1 &= \frac{1}{re^{3\psi}}\Delta m\,dr,\\ \Delta\vartheta^2 &= 0,\\ \Delta\vartheta^3 &= 0,\end{aligned}\tag{23}$$

$$\begin{aligned}\Delta\omega^{01} &= \left(\frac{4\dot{m}\Delta m}{re^{3\psi}}+\frac{1}{re^{4\psi}}\Delta\dot{m}\right)dr-\frac{1}{r}\left(\Delta m'-\frac{\Delta m}{r}\right)dt\\ &= -\Delta\omega^{10}\\ \Delta\omega^{12} &= \frac{1}{re^{\psi}}\Delta m\,d\theta=-\Delta\omega^{21}\\ \Delta\omega^{13} &= \frac{1}{re^{\psi}}\Delta m\,\sin\theta d\phi=-\Delta\omega^{31}\\ \Delta\omega^{23} &= 0=-\Delta\omega^{32}\end{aligned}\tag{24}$$

Many of the terms vanish in the energy flux $\mathbb{F}(\xi)$, the remaining

nonvanishing terms that will contribute are,

$$
\begin{aligned}
& 2 \oint i_\xi(\pounds_\xi \omega^{12} \wedge \Delta\eta_{12} + \pounds_\xi \omega^{13} \wedge \Delta\eta_{13}) \\
= \quad & -16\pi c_1 \frac{\Delta m}{r e^{2\psi}} \left[c_1 \dot{m} + c_2 \left(m' - \frac{m}{r}\right)\right], \qquad (25)
\end{aligned}
$$

$$
\begin{aligned}
& -2 \oint i_\xi(\Delta\omega^{12} \wedge \pounds_\xi \eta_{12} + \Delta\omega^{13} \wedge \pounds_\xi \eta_{13}) \\
= \quad & 16\pi c_1 \left[\frac{\Delta m}{r e^{2\psi}} \left[c_1 \dot{m} + c_2 \left(m' - \frac{m}{r}\right)\right] - c_2 \frac{\Delta m}{r}\right], \qquad (26)
\end{aligned}
$$

and

$$
\begin{aligned}
& -2 \oint i_\xi(\Delta\omega^{01} \wedge \pounds_\xi \eta_{01}) \\
= \quad & 16\pi c_2 \left[\left(\Delta m' - \frac{\Delta m}{r}\right) c_1 - \left(\frac{4\dot{m}\Delta m}{e^{3\psi}} + \frac{\Delta \dot{m}}{e^{4\psi}}\right) c_2\right]. \qquad (27)
\end{aligned}
$$

Finally, the total energy flux is

$$\begin{aligned}\mathbb{F}(\xi) &= \pounds_\xi \mathbb{E}_{dyn}(\xi) \\ &= 16\pi c_2 \left[\left(\Delta m' - \frac{2\Delta m}{r}\right) c_1 - \left(\frac{4\dot{m}\Delta m}{e^{3\psi}} + \frac{\Delta \dot{m}}{e^{4\psi}}\right) c_2\right].\end{aligned} \tag{28}$$

Taking $u = t - r = const$ and $t, r \to \infty$ to approach the null infinity and dropping the term contains $\dot{m}\Delta m$ which is of higher order in Δ, we arrive at the Bondi type energy flux

$$\mathbb{F}(\xi) = 16\pi c_2(c_1 m' - c_2 \dot{m}) = -16\pi \partial_u m(u), \tag{29}$$

Where, $c_1 = 1$ and $c_2 = 1$ by requiring that ξ and Δ defines the same direction of mass changes for the consistency of interchanging Δ and $\pounds_\xi$.

The same energy flux result, equation was also obtained long ago by Lindquist, Schwartz and Misner using Landau-Lifshitz stree-energy pseudotensor. Such energy flux $-\partial_u m(u)$ has the interpretation as the luminosity of the star as seen by an observer at null infinity.

Notice that the nonvanishing term come only from the following equations

$$\begin{aligned}\mathbb{F}(\xi) &= -\oint i_\xi[\Delta\omega^{01}] \wedge \pounds_\xi(\vartheta^2 \wedge \vartheta^3) \\ &\quad -\oint (i_\xi \vartheta^1)[\Delta\omega^{12} \wedge \pounds_\xi \vartheta^3 - \Delta\omega^{13} \wedge \pounds_\xi \vartheta^2] \\ &= \oint_{\partial\Sigma} i_\xi[-\Delta\omega^{01} + \frac{\Delta m}{r^2 e^\psi}\vartheta^1] \wedge \pounds_\xi(\vartheta^2 \wedge \vartheta^3) \qquad (30)\end{aligned}$$

where $\vartheta^2 \wedge \vartheta^3$ is the area element.

This indicates the first law for general spacetime regions. For stationary spacetimes where $\pounds_\xi(\vartheta^2 \wedge \vartheta^3) = 0$, the energy flux vanishes. The appearance of the first law provides a nontrivial consistent check of our energy flux expression.

Quasi-local energy flux expression for FRW Cosmology

Robertson-Walker metric with $k = 0$,

$$ds^2 = -dt^2 + a(t)^2 \left[dr^2 + r^2 d\Omega^2\right] . \tag{31}$$

with

$\vartheta^0 = dt$, $\vartheta^1 = a(t)dr$, $\vartheta^2 = a(t)rd\theta$, **and** $\vartheta^3 = a(t)r\sin\theta d\phi$.

The Cosmological Dynamical Horizon (CDH) can now be defined as hypersurfaces being foliated by family of closed two-dimensional surfaces, such that on each leaf, the expansion of one future directed null normal is zero.

Given a 2-surface S **in a four-dimensional spacetime** M**, one can introduce a set of orthonormal vectors** e_0, e_1, e_2, **and** e_3 **adapted to the 2-surface** S**, with** e_0 **and** e_1 **being the set of timelike and spacelike unit normals to** S**, and** $e_A = [e_2, e_3]$ **being tangent to** S**.**

The extrinsic curvatures of S with respect to e_0 and e_1 directions are:

$$k(e_0)_{AB} = g(e_B, \nabla_A e_0) = \Gamma_{0BA} \tag{32}$$

$$k(e_1)_{AB} = g(e_B, \nabla_A e_1) = \Gamma_{1BA}. \tag{33}$$

Here $\Gamma_{abc} = g(e_b, \nabla_c e_a)$ are Ricci rotation coefficients ($\omega^{ab} = \Gamma^{ab}{}_c \vartheta^c$) and the null expansion parameters ρ and μ of the CDH for FRW cosmology becomes,

$$\rho = \frac{1}{2}[k(e_0)+k(e_1)] = -\left(\frac{\dot{a}}{a} + \frac{1}{ar}\right), \quad \textbf{and} \quad \mu = \frac{1}{2}[k(e_0)-k(e_1)] = -\left(\frac{\dot{a}}{a} - \frac{1}{ar}\right). \tag{34}$$

The future pointing CDH for expanding universe are defined by $\mu = 0$ and $\rho < 0$,

$$H \equiv \frac{\dot{a}}{a} = \frac{1}{ar} = \frac{1}{R}. \tag{35}$$

For contracting universe, the dynamical cosmological horizons are defined by $\mu > 0$ and $\rho = 0$.

The CDH is $S^2 \times R$, and in this way are always timelike in contrasting to the BH dynamical horizons which are spacelike.

There is a duality between the cosmological isolated/dynamical horizons and the black hole isolated/dynamical horizons under interchanges between timelike and spacelike hypersurfaces.

Let *generic dynamical horizons* Ξ**, be a sub-manifold of spacetime that bears topology** $S^2 \times \mathbb{R}$ **with every cross-section of** Ξ **being marginally trapped(this condition ensures that** $\pounds_\xi \eta_{ab}|_\Xi = 0$**),**

and Ξ_S **be a** *generic isolated horizon* **with,**

$$\pounds_\xi \omega^{ab}|_{\Xi_S} = 0,$$

and having the null energy conditions being satisfied on both Ξ **and** Ξ_S**.**

These boundary conditions of the generic isolated horizon Ξ_S **is to guarantee the vanishing of the variation of the Hamiltonian and the existence of a functionally differentiable Hamiltonian.**

Applying to the Robertson-Walker metric, it leads to the displacement null vector orthogonal to surface $\partial\Xi_S$ being

$\xi = \chi \left(\partial_t - \frac{1}{a}\partial_r\right)$

(where χ is the conformal freedom for the choice of the null vector ξ)

provided

$$\chi = \textbf{constant}, \quad \textbf{and} \quad \dot{H} = 0. \tag{36}$$

This is the case for constant Hubble parameter H_0(i.e. during exponent inflation).

During inflation, the energy flux flowing in and out of the CDH are balanced out and the system is being dynamically conserved. We shall call this generic isolated horizon Ξ_S when applied to cosmology as the *cosmological isolated horizon(CIH)***.**

Establish the zeroth law for the CIH on Ξ_S like the zeroth law on the isolated horizon of black holes.

The key arises from the requirement that $\pounds_\xi \omega^{ab}|_{\Xi_S} = 0$.

In the case of Robertson-Walker metric, $i_\xi d\omega^{ab}|_{\Xi_S} = 0$ leads to $di_\xi \omega^{ab}|_{\Xi_S} = 0$ by the identity $\pounds_\xi = i_\xi d + di_\xi$.

Therefore, $i_\xi \omega^{ab}$ is a constant on the CIH, Ξ_S, and the only non-vanishing coefficient is the just surface gravity

$$\kappa|_{\Xi_S} \equiv i_\xi \omega^{01}|_{\Xi_S} = \chi \frac{\dot{a}}{a} = \textbf{constant} \equiv H_0. \qquad (37)$$

The constancy of κ on the CIH Ξ_S establishes the zeroth law for the CIH.

Extending to CDH, we have to consider the energy flux for a perturbation of the above stationary spacetime, in particular, a perturbation, Δ on $a(t)$, along the CDH.

An example is the slow-rolling period during cosmo inflation such that $\dot{H} \ll H^2$.

Introducing a one parameter family λ of the change of the scaling factor, $a(t,\lambda)$ along the CDH, then the perturbation Δ is given by

$$\Delta a(t) := \lim_{\lambda \to 0} \frac{a(t,\lambda) - a(t,0)}{\lambda} \equiv \left. \frac{da(t,\lambda)}{d\lambda} \right|_{\lambda=0} = \dot{a}(t). \tag{38}$$

Similarly $\Delta \dot{a}(t) = \ddot{a}(t)$.

With this perturbation Δ on $a(t)$, we obtain,

$\Delta\vartheta^0 = 0$, $\Delta\vartheta^1 = (\Delta a)dr$, $\Delta\vartheta^2 = (\Delta a)rd\theta$, $\Delta\vartheta^3 = (\Delta a)r\sin\theta d\phi$,

and the perturbation of the spin-coefficients,

$\Delta\omega^{01} = (\Delta\dot{a})dr$, $\Delta\omega^{02} = (\Delta\dot{a})rd\theta$, $\Delta\omega^{03} = (\Delta\dot{a})r\sin\theta d\phi$,
$\Delta\omega^{12} = \Delta\omega^{13} = \Delta\omega^{23} = 0$.

The total energy flux flowing across the CDH for a perturbation away from the stationary case becomes:

$$\mathbb{F} = -\chi^2 \dot{H} \left(2R + HR^2\right)\Big|_{R_1}^{R_2} = -3\dot{H}\chi^2(R_2 - R_1), \qquad (39)$$

using the fact that on CDH, the expansion of one future directed null normal is zero, i.e. $H = 1/R$**.**

Note that our null vector ξ **was defined with respect to the past light cone of the co-moving observer.**

For spacetime satisfies the null energy condition with ϵ **being the energy density,**

$\epsilon + \frac{p}{c^2} \geq 0,$

$\dot{H} = -\frac{4\pi G}{c^4}(\epsilon + \frac{p}{c^2})$ **leads to** $\dot{H} \leq 0$**,**

A negative sign in the gravitational energy flux indicated an inward energy flowing.

The importance of this results indicates that it is this inward flowing flux that gives rises to the cosmo acceleration in a linear relation. The flux expression in (39) is dual to the energy balance expression for dynamical black hole horizons under timelike and spacelike hypersurfaces interchanges. Note also, during inflation, no energy flux will go through the constant Hubble radius.

To complete the analysis, we shall compare our results of the quasi-local energy flux with results obtained from the Friedmann energy equations and the continuity equation in FRW cosmology. We start by noting that the continuity equation is,

$$\dot{\epsilon} + 3(\epsilon + \frac{p}{c^2})H = 0, \tag{40}$$

and the rate of change in total energy E **that gives the first law of thermodynamics is,**

$$\frac{dE}{dt} = -\frac{p}{c^2}\frac{d}{dt}R^3. \tag{41}$$

To investigate how energies are changing through the CDH

hypersurface Ξ, we apply $\pounds_\xi$ to the energy $E = \frac{4\pi}{3}\epsilon R^3$ to obtain

$$\begin{aligned} \chi(\partial_t - \frac{1}{a}\partial_r)E &= -\chi\frac{4\pi}{3}(\epsilon + \frac{p}{c^2})R^2 \\ &= \frac{\chi c^4}{G}\dot{H}R^2 \end{aligned} \tag{42}$$

and note the change in total energy is proportional to $\dot{H}$ along ξ. On another hand, integrating flux expression (39) along the null hypersurface with $dt = dR$, we obtain the same changes in total energy

$$\int -3\dot{H}\chi^2 R(dt + dR)\Big|_{\Xi} = -3\dot{H}\chi^2 R^2, \tag{43}$$

and therefore fixing the normalization for the conformal factor at $\chi = -\frac{c^4}{3G}$.

To establish the corresponding first law on the CDH, we compare results in the dynamical horizons balance laws from with the flux expression in (39),

$$dE = -3\dot{H}\chi^2 dR \qquad (44)$$

(for $\dot{H}$ **being approximately constant as in realistic cases). Note that with null energy condition,** $\dot{H} \leq 0$**, by defining effective surface gravity** κ **by**

$$\kappa = -\frac{12\dot{H}\chi^2 G}{Rc^2}, \qquad (45)$$

this yields a generalized first law, $dE = \frac{\kappa c^2}{8\pi G} dA$**, on the CDH.**

In general, as the universe is accelerating, the effective surface gravity will vary with time, and depend on the conformal factor χ**. Again, as in the black hole thermodynamics, we obtain the effective local temperature on the CDH being,**

$$T = \frac{\hbar\kappa}{2\pi k_B c} = \frac{-2\hbar\dot{H}c^5}{3\pi k_B RG}. \qquad (46)$$

This temperature is much smaller than the 2.7 K temperature of the universe determined by the cosmic microwave background radiation.

We end the investigation of CDH with the following conclusions.

- **During inflation, no flux flowing in/out of the horizon, effectively, spacetime of the cosmo was developed from vacuum density of a "single" point.**
- **The cosmo acceleration is linearly proportional to the inwardly flowing energy flux across the CDH. It is the nonzero energy flux that causes the cosmo acceleration.**
- **There exist a dual picture between black hole isolated and dynamical horizons and the CIS(during exponent inflation) and CDH during upon interchanges between time like and space hypersurfaces. More precisely, the isolated horizon and dynamical horizons of dynamical black holes are mapped into cosmological horizon and Hubble horizon during inflation corresponding by identifying the corresponding light cone structures.**