Simulations of black holes in compactified spacetimes

(Work in progress, Phys.Rev.D81:084052)

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Why numerical relativity

Study of systems with strong and dynamical gravitational fields

- Gravitational radiation
  - Astrophysics, gravitational wave astronomy
- Mathematical and theoretical Physics:
  - Cosmic censorship
  - Instabilities (Black hole interior, Myers-Perry)
- High-energy particle systems:
  - AdS/CFT correspondence;
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High-energy particle systems

- Large extra dimensions scenarios:
  - fundamental Planck scale could be as low as the TeV: $\Rightarrow$ at the LHC particles collide at centre of mass energies above the fundamental Planck scale.
- *Matter does not matter*: for energies above Planck scale, $E = 2\gamma m_0 c^2 > E_{\text{Planck}}$
  - gravity is the dominant force;
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- have a richer phase structure and dynamics than in flat-space;
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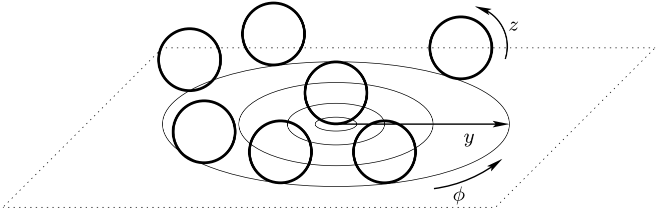
$D = 5$ black holes on a cylinder

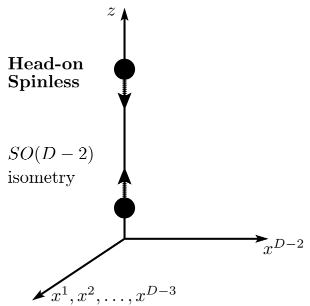
In the absence of black holes, we have $\mathbb{M}^{1,3} \times \mathcal{S}^1$:

$$ds^2 = \underbrace{-dt^2 + dx^2 + dy^2 + y^2 d\phi^2}_{\mathbb{M}^{1,3}} + \underbrace{dz^2}_{\mathcal{S}^1}$$

$z \in [-L, L]$, $\phi \in [0, 2\pi[$



Axial symmetry $SO(D-2)$ and $SO(D-3)$



- Highly symmetric systems;
- Can be reduced to effective $3+1$ systems;
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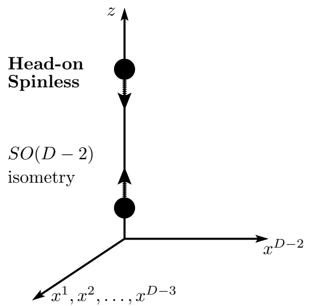

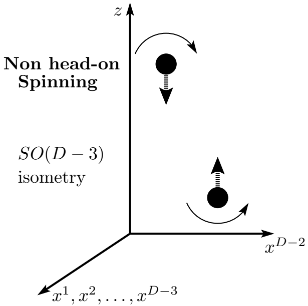

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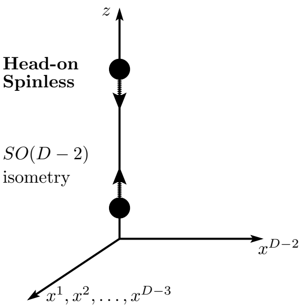

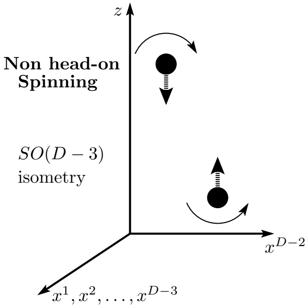

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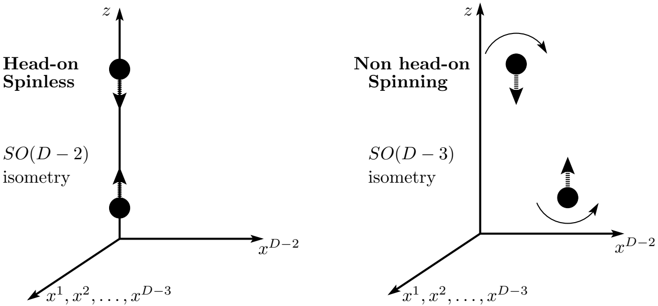

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Formalism

Most general metric element

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu + \lambda d\Omega^2_{D-4}$$

$\mu = 0, 1, 2, 3.$

D-dimensional vacuum Einstein equations imply

$$R_{\mu\nu} = \frac{D-4}{2\lambda}\left(\nabla_\mu \partial_\nu \lambda - \frac{1}{2\lambda}\partial_\mu \lambda \partial_\nu \lambda\right)$$

$$\nabla^\mu \partial_\mu \lambda = 2(D-5) - \frac{D-6}{2\lambda}\partial_\mu \lambda \partial^\mu \lambda$$

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Formalism

The resulting system is

$$(\partial_t - \mathcal{L}_\beta)\,\gamma_{ij} = -2\alpha K_{ij}$$

$$(\partial_t - \mathcal{L}_\beta)\,K_{ij} = -D_i\partial_j\alpha + \alpha\left({}^{(3)}R_{ij} + KK_{ij} - 2K_{ik}K^k{}_j\right) - \alpha\frac{D-4}{2\lambda}\left(D_i\partial_j\lambda - 2K_{ij}K_\lambda - \frac{1}{2\lambda}\partial_i\lambda\partial_j\lambda\right)$$

$$(\partial_t - \mathcal{L}_\beta)\,\lambda = -2\alpha K_\lambda$$

$$\frac{1}{\alpha}(\partial_t - \mathcal{L}_\beta)\,K_\lambda = -\frac{1}{2\alpha}\partial^i\lambda\partial_i\alpha + (D-5) + KK_\lambda + \frac{D-6}{\lambda}K_\lambda^2 - \frac{D-6}{4\lambda}\partial^i\lambda\partial_i\lambda - \frac{1}{2}D^k\partial_k\lambda$$

→ effective $3+1$ system with source terms

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$\rightarrow$ effective $3+1$ system with source terms

Formalism

Constraints

$$\mathcal{H} \equiv R + K^2 - K_{ij}K^{ij} - 16\pi E = 0$$

$$\mathcal{M}^i \equiv \nabla_j \left(K^{ij} - \gamma^{ij} K \right) - 8\pi p^i = 0$$

Outline

Initial data

Brill-Lindquist initial-data

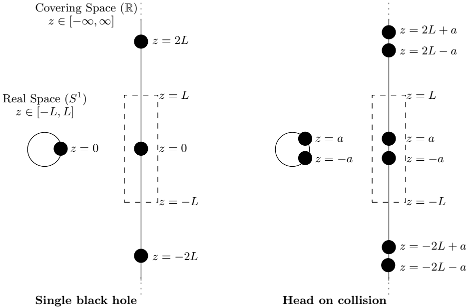
$$ds^2 = \psi^2 \left(dx^2 + dy^2 + dz^2 \right) + y^2 \psi^2 d\phi^2$$

$$K_\lambda = 0$$

"Standard" (asymptotically flat) case

$$\psi = 1 + \frac{r_S^2}{4\left[x^2 + y^2 + (z-a)^2\right]}$$

Initial data



Cylindrical case (Myers 1986)

$$\psi = 1 + \sum_{n=-\infty}^{+\infty} \frac{r_S^2}{4\left[x^2 + y^2 + (z - 2Ln)^2\right]}$$

$$= 1 + \frac{\pi r_S^2}{8L\rho} \frac{\sinh \frac{\pi\rho}{L}}{\cosh \frac{\pi\rho}{L} - \cos \frac{\pi z}{L}}, \qquad \rho^2 \equiv x^2 + y^2$$

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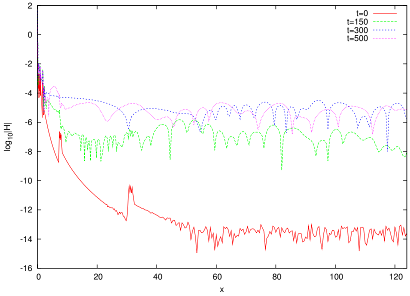

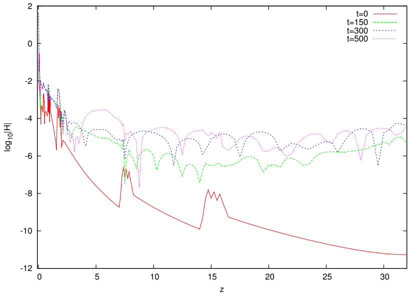

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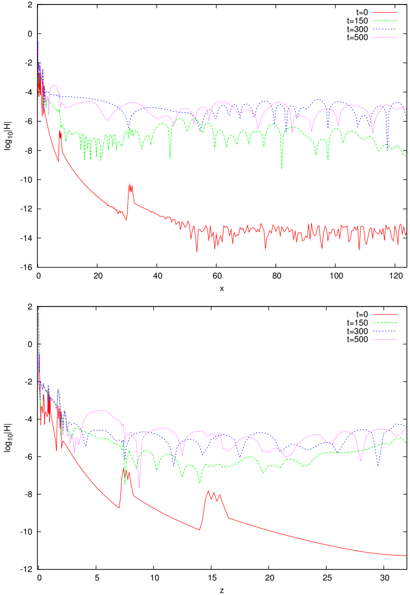

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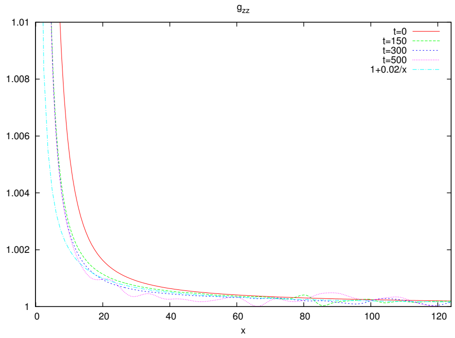
Metric falloff



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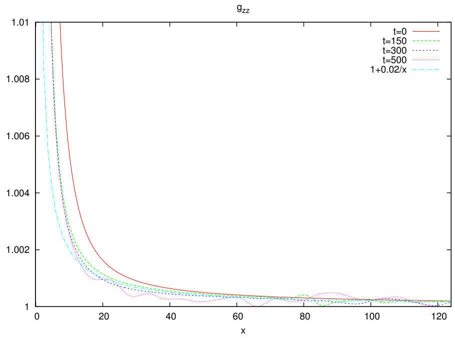

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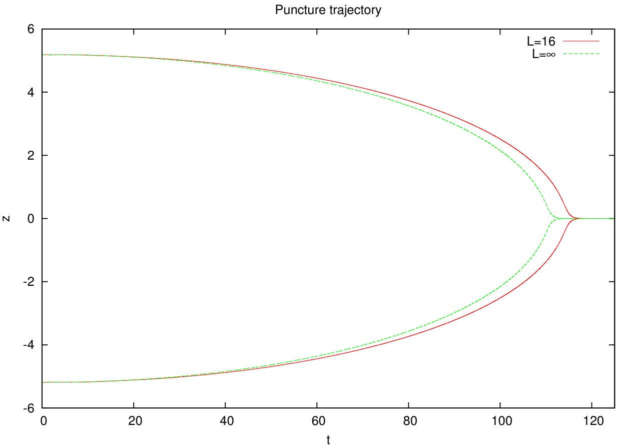

→ longer collision time for the cylindrical case

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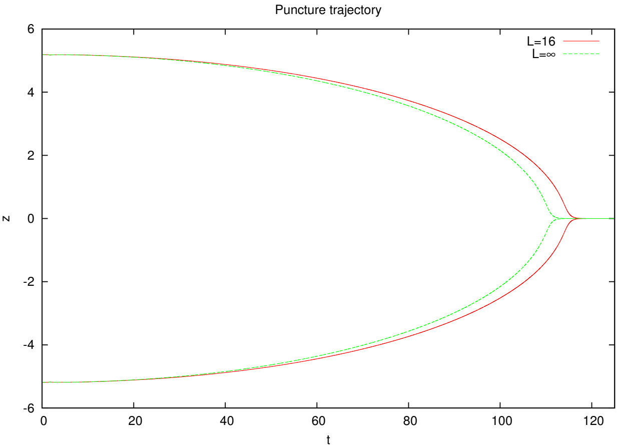

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1. We reduced the head-on collision of (non-spinning) black holes in cylindrical spacetimes (in any dimension) to an effective $3+1$ system with a scalar field;
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2. Radiated energy (along the lines of Witek et al, 2010);
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The group

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Vitor Cardoso (IST)

(Vitor Cardoso, looking for black holes in higher dimensions)

http://blackholes.ist.utl.pt/

the end